

The Economics of Commercial ISAC for Wide-Area Sensing

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Abstract—Wide-area integrated sensing and communications (ISAC) could provide government sensing capabilities by leveraging commercial wireless infrastructure. Commercial deployment requires compensating service providers (SPs) for both ISAC deployment costs and the opportunity cost of communications resources diverted to sensing. We develop a Cournot competition framework to characterize this opportunity cost and its dependence on sensing timing, provider bandwidth, and market structure. We show that spreading sensing over a longer period reduces communications opportunity cost for a fixed sensing time-bandwidth product. We then characterize cost-minimizing government allocation of sensing requirements across heterogeneous SPs and show that the resulting allocation favors providers with lower marginal sensing costs.

I. INTRODUCTION

Integrated sensing and communications (ISAC) has emerged as a promising technology for enabling wireless networks to perform sensing functions in addition to their traditional communications role [1], [2]. Much of the ISAC literature has focused on the technical challenges of jointly designing sensing and communications waveforms [3], allocating radio resources [4], and characterizing sensing performance [5]. Less attention has been devoted to a fundamental economic question that arises with large-scale ISAC deployment: *how should the costs of providing sensing services be allocated when sensing uses commercially operated communications infrastructure?*

This question is particularly important for military and homeland-security applications such as the detection and tracking of drones [6], [7]. Persistent drone detection can support the protection of military installations, critical infrastructure, airports, and other sensitive locations. Commercial cellular networks already provide extensive geographic coverage and substantial radio infrastructure that could potentially be leveraged for this purpose. Rather than constructing a separate nationwide sensing network, the government could therefore obtain sensing services from commercial wireless providers that augment their networks with ISAC capabilities.¹

Commercial deployment, however, creates an economic problem beyond the capital cost of adding sensing capabilities. A service provider (SP) must incur the costs of deploying and

operating ISAC equipment, but sensing also consumes network resources that could otherwise support communications. Allocating radio resources, transmit power, computing capacity, or other resources to sensing can therefore reduce communications revenue or service quality. These forgone benefits represent an *opportunity cost* of sensing that could be substantial even after the physical sensing equipment has been deployed.

The opportunity-cost problem is closely related to the economics of temporal sharing of commercial spectrum with a higher-priority incumbent user, as in the Citizens Broadband Radio Service (CBRS) [9]. In [10], temporal sharing is studied for a setting in which spectrum becomes periodically unavailable to commercial users.² This creates congestion and an economic cost that depends on provider size and the reliability of spectrum availability. ISAC creates a related but distinct problem: sensing activity can intermittently divert communications resources, with the resulting opportunity cost depending on the timing of sensing and the capacity of the provider's remaining network. Unlike the exogenous intermittency considered in [10], however, the allocation of resources to sensing is endogenous and can be influenced by the government's sensing requirements and compensation mechanism.

The distinction between deployment cost and resource-use cost is central to the economics of commercial ISAC. Deployment costs depend on the architecture used to add sensing capabilities to existing networks. Operating costs include the incremental costs of processing and maintaining those capabilities. The opportunity cost of sensing, in contrast, depends on how sensing resources interact with communications demand. Sensing performed on a lightly loaded network may have little economic cost, whereas the same sensing activity on a highly loaded network can result in substantial cost.

In this paper, we develop an economic framework for evaluating commercial provision of wide-area ISAC sensing services for government applications. We characterize the communications opportunity cost of sensing using a Cournot competition model, derive the minimum compensation required for voluntary participation, and study government allocation of sensing requirements across heterogeneous providers. We show that, for a fixed sensing time-bandwidth product, spreading sensing over

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¹ISAC-equipped networks could also generate commercial sensing revenue (e.g., logistics, smart infrastructure, automotive applications) that offsets some of these costs, which we do not account for here [8].

²In the context of spectrum sharing, this type of model has been generalized in several ways including sharing multiple intermittent bands [11] and sharing spectrum with delay tolerant traffic [12].

a longer period reduces communications opportunity cost, and that optimal procurement favors providers with lower marginal sensing costs.

II. MODEL

We begin by describing our model for competition among N commercial SPs offering traditional communication services. We use the Cournot competition model developed in [10], in which SPs compete by simultaneously choosing quantities to serve (rather than prices), and the market price adjusts to clear demand. This model provides a tractable representation of competition for communications customers and allows us to quantify the communications value of network resources diverted to sensing. Each SP i chooses a target quantity of non-atomic customers, x_i , to serve, i.e., x_i is treated as a continuous quantity, a standard approximation when modeling a large customer base. Let $X = \sum_i x_i$ denote the total quantity of customers served by all SPs. The price per customer for SP i is determined by the difference between a downward-sloping inverse demand function, $P(X)$, and a congestion cost $l_i(x_i)$.

We assume a normalized linear inverse demand function,

$$P(X) = 1 - X.$$

Here, $P(X)$ is the price per customer when the total quantity of customers served is X . Demand is downward-sloping, and the normalization sets both the maximum price and the total market size to 1.³ We further assume a linear congestion cost for SP i given by

$$l_i(x_i) = \frac{x_i}{B_i},$$

where B_i represents the bandwidth of SP i .⁴

Each SP i seeks to maximize its revenue, given by

$$R_i = (P(X) - l_i(x_i)) x_i.$$

Because all SPs face the same demand curve, their revenue-maximization decisions are interdependent. The resulting market outcome is the Nash equilibrium of the Cournot game, in which no SP can unilaterally change its customer quantity and increase its revenue.

We next extend this framework to the case in which SPs provide ISAC services in addition to traditional communications services. We focus on the economic cost of ISAC resource allocation rather than on the physical-layer design of the ISAC waveform. In particular, our baseline model represents sensing activity as consuming a portion of the network's communications resources. This captures architectures in which sensing and communications are separated in time, frequency, or other resources. More tightly integrated ISAC architectures can reduce this opportunity cost by allowing sensing and communications to share the same radio resources.

³Linear demand is a standard simplifying assumption in Cournot models that yields closed-form equilibrium expressions while preserving the qualitative tradeoffs of interest.

⁴This linear form is a tractable reduced-form proxy for network congestion: latency increases with the density of customers per unit of available bandwidth, capturing the qualitative effect of congestion.

We characterize the ISAC activity of SP i by two parameters. Let $\alpha_i \in [0, 1]$ denote the fraction of time during which SP i performs sensing, and let $\beta_i \leq B_i$ denote the amount of bandwidth allocated to sensing when sensing occurs. Thus, during the fraction α_i of time devoted to sensing, SP i has only $B_i - \beta_i$ units of bandwidth available for communications. During the remaining fraction $1 - \alpha_i$ of time, all B_i units are available for communications.⁵

The time-varying nature of sensing creates a time-varying congestion cost for an SP's network. We assume that each SP must always serve any customers using its network and so when the SP is sensing, these customers will experience a higher congestion. We further assume that customers are sensitive to the average congestion that they experience over time. Hence, with ISAC employed, the average congestion experienced by SP i 's customers is given by

$$\bar{l}_i(x_i) = (1 - \alpha_i) \frac{x_i}{B_i} + \alpha_i \frac{x_i}{B_i - \beta_i}. \quad (1)$$

We consider a model where each SP receives a fixed payment F_i from the government for sensing and incurs an incremental cost C_i^{ISAC} for deploying and operating ISAC. The profit of SP i for sensing and communications is then

$$\Pi_i = (P(X) - \bar{l}_i(x_i)) x_i + F_i - C_i^{ISAC}. \quad (2)$$

III. EQUIVALENT BANDWIDTHS WITH SENSING

Letting

$$\frac{1}{B_{i,e}} = (1 - \alpha_i) \frac{1}{B_i} + \alpha_i \frac{1}{B_i - \beta_i}, \quad (3)$$

we can re-write (1) as $\bar{l}_i(x_i) = \frac{x_i}{B_{i,e}}$. We refer to $B_{i,e}$ as the equivalent bandwidth available for communications; it converts the time-varying bandwidth due to sensing into a single bandwidth measure that produces the same average congestion. The equivalent bandwidth captures the impact of sensing on the congestion experienced by communications customers.

Importantly, $B_{i,e}$ generally differs from the time-average bandwidth $B_i - \alpha_i \beta_i$. Because congestion is inversely related to available bandwidth, intermittent reductions in bandwidth can impose a larger congestion cost than an equivalent reduction that is spread uniformly over time.

Equation (3) can equivalently be written as

$$\frac{B_i}{B_{i,e}} = (1 - \alpha_i) + \alpha_i \frac{1}{1 - \frac{\beta_i}{B_i}}. \quad (4)$$

This shows that the equivalent bandwidth as a fraction of the total bandwidth only depends on α_i and the fraction of the total bandwidth used for sensing. Figure 1 plots the equivalent bandwidth B_e for an arbitrary SP normalized by its bandwidth B as a function of α , where $\frac{\beta}{B}$ is fixed for each curve so that the fraction of resources used for sensing is constant. Note

⁵Our objective is not to model the physical-layer sensing performance of a particular ISAC waveform or architecture. Instead, we use α and β to represent the network resources consumed by sensing and focus on the resulting economic tradeoffs.

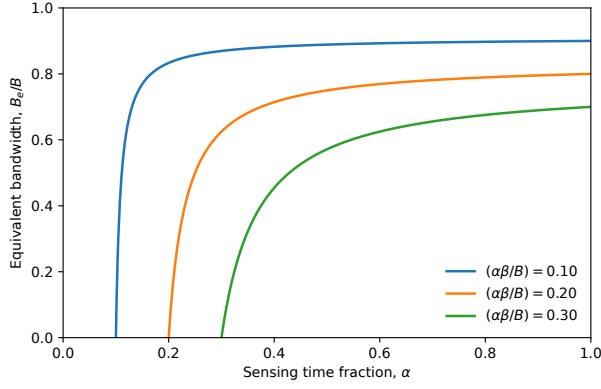


Fig. 1. Equivalent bandwidth as a fraction of the original bandwidth B as a function of α , where for each curve $\frac{\alpha\beta}{B}$ is constant.

that the curves increase in α , which shows that given a fixed value of $\frac{\beta\alpha}{B}$, a larger α and smaller β (i.e., sensing using less bandwidth for longer durations) would improve the congestion experienced by the SP's commercial traffic.

Equivalent bandwidth characterizes the impact of ISAC (i.e., of α and β) on communications. The parameters α and β both impact the sensing performance as well. The bandwidth allocated, β , impacts the range/delay resolution. The time, α , impacts the Doppler/velocity resolution [13]. As a simple benchmark, suppose that sensing quality is proportional to the time-bandwidth product,

$$Q_i = k\alpha_i\beta_i,$$

where k is a proportionality constant. We can then view each curve in Fig. 1 as corresponding to choices of α and β that meet a fixed sensing target.

Let $q_i = Q_i/k$ denote a normalized sensing target so that

$$\alpha_i\beta_i = q_i. \quad (5)$$

Define the normalized sensing load

$$\rho_i = \frac{q_i}{B_i}.$$

Substituting $\beta_i = q_i/\alpha_i$ into (4) gives

$$\begin{aligned} \frac{B_i}{B_{i,e}} &= 1 - \alpha_i + \frac{\alpha_i}{1 - \frac{\rho_i}{\alpha_i}} \\ &= 1 + \rho_i + \frac{\rho_i^2}{\alpha_i - \rho_i}. \end{aligned} \quad (6)$$

For a feasible sensing configuration, $\alpha_i > \rho_i$. Differentiating the right-hand side of (6) gives

$$\frac{\partial}{\partial \alpha_i} \left(\frac{B_i}{B_{i,e}} \right) = -\frac{\rho_i^2}{(\alpha_i - \rho_i)^2} < 0.$$

Therefore, for a fixed time-bandwidth product q_i , the equivalent communications bandwidth $B_{i,e}$ is increasing in α_i as in the examples in Fig. 1. Hence, within this simplified sensing-quality model, it is preferable from the perspective of communications

performance to spread sensing activity over a longer period while using less bandwidth at any instant. Concentrating the same total sensing resources into short periods with a large value of β_i produces a greater congestion penalty.

At the limiting case $\alpha_i = 1$, sensing is continuously spread over time with $\beta_i = q_i$, and the equivalent bandwidth becomes

$$B_{i,e} = B_i - q_i. \quad (7)$$

This is exactly the bandwidth that would remain if the sensing resources were removed uniformly rather than intermittently. Any more intermittent allocation with the same time-bandwidth product produces a smaller equivalent bandwidth.

This result provides an economic interpretation of Fig. 1. Even when two sensing configurations use the same average amount of radio resources, they need not impose the same communications opportunity cost. Temporal concentration of sensing creates additional congestion because customers experience periods in which the available communications bandwidth is particularly small.

Of course, the sensing system may impose additional constraints on α_i and β_i . For example, a minimum instantaneous sensing bandwidth may be needed to achieve a desired range or delay resolution, while temporal requirements may constrain the sensing duration or revisit rate. The government therefore cannot in general select α_i and β_i solely to minimize communications costs. Nevertheless, (6) illustrates how the physical sensing requirements translate into an economic cost and provides a basis for comparing sensing architectures with different resource-use patterns.

In the following, we assume that α and β are fixed when an SP performs sensing. These fixed values could be either determined by the government or optimized by the SPs to meet a target sensing requirement specified by the government.

IV. MARKET EQUILIBRIUM

We next consider the market equilibrium in which each SP seeks to maximize its profit in (2) given (α_i, β_i) . Because the ISAC payment and incremental ISAC costs do not depend on the SP's choice of x_i , the equilibrium quantities can be obtained by maximizing

$$\left(1 - X_{-i} - x_i - \frac{x_i}{B_{i,e}} \right) x_i,$$

where $X_{-i} = \sum_{j \neq i} x_j$ so that $X = X_{-i} + x_i$. The first-order optimality condition gives that the equilibrium quantity, x_i^* , satisfies

$$x_i^* = \frac{1 - X_{-i}}{2 + \frac{2}{B_{i,e}}}.$$

Simplifying, this gives

$$x_i^* = \frac{B_{i,e}}{B_{i,e} + 2} (1 - X).$$

Letting $A = \sum_{i=1}^N \frac{B_{i,e}}{B_{i,e} + 2}$, we further have

$$x_i^* = \frac{B_{i,e}}{B_{i,e} + 2} \frac{1}{1 + A}. \quad (8)$$

Note that the parameter A depends on the equivalent bandwidths of all SPs, not just that of SP i , and so the equilibrium quantity of an SP will also depend on these.

The equilibrium price an SP receives is given by

$$p_i^* = 1 - X - \frac{x_i^*}{B_{i,e}} = x_i^* \left(1 + \frac{1}{B_{i,e}} \right)$$

and so the SP's revenue from communications is

$$R_i^* = (x_i^*)^2 \left(1 + \frac{1}{B_{i,e}} \right). \quad (9)$$

If an SP does not employ ISAC, then the same expressions apply with $B_{i,e}$ replaced by B_i .

We also specialize to the case where all SPs are symmetric, i.e., they have the same bandwidth B and the same sensing parameters, resulting in the same equivalent bandwidth B_e .

In this case, the equilibrium quantities for all SPs become

$$x^* = \frac{B_e}{2 + (N+1)B_e}$$

and the revenue from communications is

$$R_i^* = \frac{B_e(B_e + 1)}{(2 + (N+1)B_e)^2}. \quad (10)$$

V. OPPORTUNITY COST OF SENSING

We now use the equilibrium characterization from the previous section to examine the communications opportunity cost of ISAC. We define the opportunity cost of sensing as the equilibrium communications revenue forgone when network resources are diverted from communications to sensing. Because an SP's equilibrium revenue depends not only on its own equivalent bandwidth but also on the equivalent bandwidths of its competitors, the opportunity cost depends on the sensing adopted by the other SPs.

Let $\mathbf{B}_e = (B_{1,e}, \dots, B_{N,e})$ denote a vector of equivalent bandwidths and let $R_i^*(\mathbf{B}_e)$ denote SP i 's equilibrium communications revenue given these equivalent bandwidths. If SP j does not perform sensing, then, $B_{j,e} = B_j$.

For SP i , let $\mathbf{B}_{-i,e}$ denote the equivalent bandwidths of the other SPs. We define the opportunity cost to SP i from moving from $B_{i,e}^0$ to $B_{i,e}^1$ conditional on $\mathbf{B}_{-i,e}$ as

$$\begin{aligned} OC_i(B_{i,e}^0 \rightarrow B_{i,e}^1 | \mathbf{B}_{-i,e}) \\ = R_i^*(B_{i,e}^0, \mathbf{B}_{-i,e}) - R_i^*(B_{i,e}^1, \mathbf{B}_{-i,e}). \end{aligned} \quad (11)$$

If $B_{i,e}^0$ represents no sensing and $B_{i,e}^1$ represents sensing, then OC_i is the opportunity cost of sensing for SP i , holding the competitors equivalent bandwidths fixed.

Figure 2 illustrates the opportunity cost for a market consisting of a large SP ($B_1 = 2$) and a small SP ($B_2 = 1$). Two cases are illustrated, corresponding to only the indicated SP sensing, with the sensing parameters satisfying $\alpha\beta = 0.2$. For each SP, we calculate the opportunity cost relative to the counterfactual in which that SP does not sense and the other SP remains non-sensing (i.e. $B_{-i,e} = B_{-i}$). The same sensing requirement imposes substantially different opportunity costs

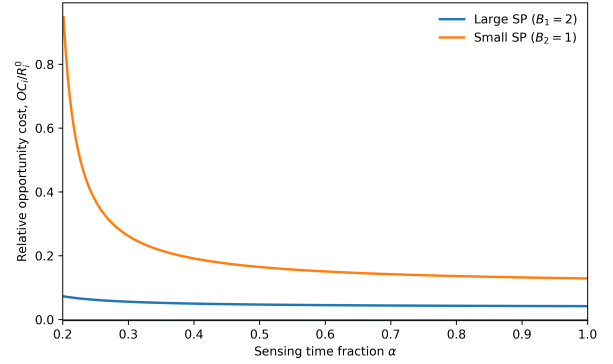


Fig. 2. Ratio of communications opportunity cost with ISAC to revenue without ISAC for a market consisting of two SPs with bandwidths $B_1 = 2$ and $B_2 = 1$. In each curve, only the indicated SP performs sensing. The sensing parameters satisfy $\alpha\beta = 0.2$ held fixed.

on the two providers. The smaller SP experiences a much larger relative opportunity cost than the larger SP because diverting a given amount of bandwidth from a smaller network produces a larger increase in communications congestion.⁶ The opportunity cost also depends on the temporal allocation of sensing: for a fixed $\alpha\beta$, spreading sensing over more time with less bandwidth reduces it.

In Fig. 2 both SPs have positive opportunity costs for communications. The following result shows that this holds in general.

Proposition 1. *For any market with N SPs and any fixed profile of competitors' equivalent bandwidths, any SP i 's equilibrium communications revenue is strictly increasing in its own equivalent bandwidth. Consequently, any reduction in $B_{i,e}$ due to sensing imposes a positive communications opportunity cost on SP i .*

Proof. Using (9) and (8), we can write the equilibrium revenue of SP i as

$$R_i^* = \frac{B_{i,e}(B_{i,e} + 1)}{[B_{i,e}(A_{-i} + 2) + 2(A_{-i} + 1)]^2}, \quad (12)$$

where $A_{-i} = \sum_{j \neq i} \frac{B_{j,e}}{B_{j,e} + 2}$. Differentiating with respect to $B_{i,e}$ gives

$$\frac{\partial R_i^*}{\partial B_{i,e}} = \frac{3B_{i,e}A_{-i} + 2B_{i,e} + 2A_{-i} + 2}{[B_{i,e}(A_{-i} + 2) + 2(A_{-i} + 1)]^3} > 0.$$

Thus, when a single SP devotes resources to sensing, its communications revenue necessarily decreases. $\square$

The effect on competing SPs is in the opposite direction. For any $j \neq i$, a decrease in $B_{i,e}$ decreases A_{-j} while leaving all other terms in (12) unchanged. It therefore increases both x_j^* and R_j^* . Sensing by one provider consequently creates a competitive spillover: part of the communications business displaced from the sensing provider is captured by its competitors.

⁶The same conclusions hold for the opportunity cost without normalizing by the revenue.

This observation is important when deciding which SPs should be contracted to provide sensing. The private opportunity cost borne by a SP can differ from the overall change in communications-sector revenue, since part of the SP's lost revenue may be transferred to competing SPs. To highlight this effect, consider the revenue for N symmetric providers as in (10). Differentiating this with respect to B_e gives

$$\frac{dR}{dB_e} = \frac{2 - (N - 3)B_e}{[2 + (N + 1)B_e]^3}.$$

For $N \leq 3$, this is always positive, so an SP's communications revenue increases when all SPs gain equivalent bandwidth. For $N > 3$, however, revenue is maximized at $B_e = \frac{2}{N-3}$ and decreases with B_e for larger values of B_e . The reason for this is that increasing B_e reduces congestion, which can also lead the SPs to expand their market quantities, leading to a lower equilibrium price and thus less revenue. Thus, in a sufficiently competitive market, reducing all SPs' effective bandwidths through ISAC can increase their communications revenue by softening competition, enabling prices to increase.

VI. GOVERNMENT PROCUREMENT OF ISAC SERVICES

In this section we turn to the problem facing a government that seeks to procure ISAC services.

Suppose that SP i is asked to provide a specified sensing service resulting in the equivalent-bandwidth $B_{i,e}$. Its profit when providing ISAC is

$$\Pi_i^{ISAC} = R_i^*(B_{i,e}, \mathbf{B}_{-i,e}) + F_i - C_i^{ISAC}.$$

If it does not participate, its baseline profit from communications is

$$\Pi_i^0 = R_i^*(B_i, \mathbf{B}_{-i,e}).$$

Therefore, SP i is willing to provide the sensing service if

$$R_i^*(B_{i,e}, \mathbf{B}_{-i,e}) + F_i - C_i^{ISAC} \geq R_i^*(B_i, \mathbf{B}_{-i,e}).$$

Equivalently, the government payment must satisfy

$$F_i \geq C_i^{ISAC} + OC_i. \quad (13)$$

Equation (13) separates the compensation required for commercial ISAC into two distinct components.⁷ The first, C_i^{ISAC} , captures the incremental cost of deploying and operating the sensing capability. The second, OC_i , compensates the provider for the communications value displaced by sensing.

This distinction can be important. Two providers may face similar equipment and operating costs but very different opportunity costs because of differences in their bandwidth holdings, market shares, or the amount of sensing traffic they are required to support. Compensation based only on the incremental engineering cost of the sensing equipment may therefore provide insufficient incentives for participation.

⁷While we focus on monetary compensation F_i , the government could alternatively (or additionally) compensate SPs by granting access to additional spectrum, which would raise B_i and, by Lemma 1 below, reduce the marginal opportunity cost of sensing directly.

A. Aggregate Sensing Requirement

The sensing performance needed by the government may depend on the aggregate sensing provided by each SP within a given area. Abstractly, let the aggregate sensing performance be given by $Q^G = G(Q_1, Q_2, \dots, Q_N)$, where Q_i is the sensing performance of SP i . Given a requirement $Q^G \geq \bar{Q}$, the government must determine which SPs to procure sensing from and the sensing parameters for each provider. As a simple benchmark, we assume additive sensing performance,

$$Q^G = \sum_i Q_i.$$

If the government pays each SP its minimum compensation in (13), its cost minimization problem is then

$$\begin{aligned} \min_{q_i} \sum_i [C_i^{ISAC}(q_i) + OC_i(q_i)] \\ \text{s.t. } \sum_i q_i \geq \bar{Q}. \end{aligned}$$

Here, $C_i^{ISAC}(q_i)$ is the cost of providing sensing level q_i for SP i and $OC_i(q_i)$ is the communications opportunity cost of SP i for adopting sensing with quality q_i , assuming that all other SPs are not adopting sensing.

Continuing to assume that the sensing requirement is given as in (5) and that the SPs are free to choose any α and β to meet this, we have

$$OC_i(q_i) = R_i^*(B_i, \mathbf{B}_{-i}) - R_i^*(B_i - q_i, \mathbf{B}_{-i}),$$

where $\mathbf{B}_{-i} = (B_j)_{j \neq i}$. The following lemma characterizes the convexity of $OC_i(q_i)$.

Lemma 1. *Fix the equivalent bandwidths of all SPs other than i at their non-sensing levels B_j . Then $OC_i(q_i)$ is strictly convex in q_i on $[0, B_i]$.*

Proof. Write $f(B) \equiv R_i^*(B)$ with the competing SPs' contribution to the market held fixed at $S_i = \sum_{j \neq i} B_j / (B_j + 2)$, and let $a = S_i + 2$, $b = 2(S_i + 1)$, so that

$$f(B) = \frac{B(B+1)}{(aB+b)^2}. \quad (14)$$

Differentiating twice and simplifying,

$$f''(B) = \frac{-2[(3S_i+2)(S_i+2)B + 4(S_i+1)]}{(aB+b)^4}. \quad (15)$$

Since $S_i \geq 0$ and $B > 0$, every term in the bracket is strictly positive, so $f''(B) < 0$ for all $B > 0$ and all $S_i \geq 0$ – i.e., R_i^* is strictly concave in SP i 's own equivalent bandwidth, independent of the number of competitors N . Because $OC_i(q_i) = f(B_i) - f(B_i - q_i)$ and $f(B_i)$ is a constant in q_i , $OC_i''(q_i) = -f''(B_i - q_i) > 0$, proving the claim. $\square$

Lemma 1 shows that the convexity of the opportunity cost curves shown in Fig. 2 is not an artifact of the two-SP example but a structural feature of the Cournot equilibrium, holding for any market size N .

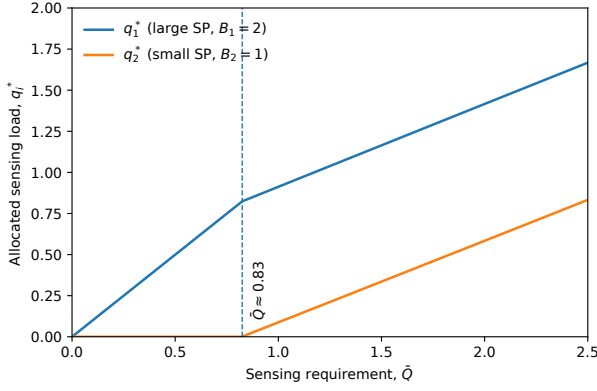


Fig. 3. Cost-minimizing sensing allocation (q_1^*, q_2^*) as a function of the government's sensing requirement $\bar{Q}$, for $B_1 = 2$, $B_2 = 1$, and linear deployment cost $c = 0.02$.

Proposition 2. Suppose $C_i^{ISAC}(q_i)$ is weakly convex for each SP i . Then the allocation $\{q_i^*\}$ that minimizes $\sum_i [C_i^{ISAC}(q_i) + OC_i(q_i)]$ subject to $\sum_i q_i \geq \bar{Q}$ is characterized by

$$h'_i(q_i^*) = \lambda \text{ for all } i \text{ with } q_i^* > 0, \quad h'_i(0) \geq \lambda \text{ otherwise,}$$

where $h_i(q_i) \equiv C_i^{ISAC}(q_i) + OC_i(q_i)$ and $\lambda \geq 0$ is the Lagrange multiplier for the sensing requirement.

Proof. By Lemma 1, $OC_i(q_i)$ is strictly convex; adding the weakly convex $C_i^{ISAC}(q_i)$ preserves strict convexity of h_i , so the problem is a convex program with a linear constraint, and the stated first-order (KKT) conditions are necessary and sufficient for a global optimum. $\square$

Proposition 2 has a water-filling-type interpretation: providers are utilized in increasing order of marginal cost $h'_i(0)$ as $\bar{Q}$ rises, each contributing sensing capacity up to the point where its marginal total cost equals λ .

Because Fig. 2 shows the large SP's opportunity-cost curve lies uniformly below the small SP's, Proposition 2 implies that an efficient sensing procurement should load the large SP first and turn to the small SP only once the large SP's marginal cost has risen to match. To illustrate this, we extend the example of Fig. 2 ($B_1 = 2$, $B_2 = 1$, $\alpha\beta = 0.2$) with a linear deployment cost $C_i^{ISAC}(q_i) = c q_i$, $c = 0.02$. Figure 3 shows the resulting allocation: the large SP alone supplies sensing until $\bar{Q} \approx 0.83$, after which the small SP also contributes, with the large SP's allocation remaining consistently larger. At $\bar{Q} = 1.5$, for example, the large SP absorbs more than three times the sensing load of the small SP despite having only twice its bandwidth, and total government compensation is roughly 9–11% lower than under a naive equal split $q_1 = q_2 = \bar{Q}/2$ across the range $\bar{Q} \in [1, 2]$.

We note that, since sensing by one SP raises the equilibrium revenue of its rivals, computing each OC_i under the assumption that rivals are not sensing may overstate the true joint cost whenever multiple SPs sense simultaneously. Proposition 2

characterizes the cost-minimizing allocation under this baseline approximation; accounting for the resulting cross-provider spillovers directly is left for future work.

VII. CONCLUSION

This paper developed an economic framework for commercial provision of wide-area ISAC sensing services, distinguishing the deployment cost of ISAC equipment from the opportunity cost of communications resources diverted to sensing. Within a Cournot competition model, we showed that this opportunity cost depends on how sensing is distributed over time, favoring longer, lower-bandwidth sensing over short, intensive bursts, and that it can differ substantially across providers of different size. We derived the minimum government compensation needed to induce voluntary participation and showed that, when procuring sensing from multiple providers, the cost-minimizing allocation follows a water-filling structure that favors larger, lower-cost providers. Future work includes accounting for cross-provider competitive spillovers in the allocation problem and incorporating geographic coverage constraints on sensing requirements.

REFERENCES

- [1] X. Luo, Q. Lin, R. Zhang, H.-H. Chen, X. Wang, and M. Huang, "Isac—a survey on its layered architecture, technologies, standardizations, prototypes, and testbeds," *IEEE Communications Surveys & Tutorials*, vol. 28, pp. 485–526, 2025.
- [2] Z. Wei, H. Qu, Y. Wang, X. Yuan, H. Wu, Y. Du, K. Han, N. Zhang, and Z. Feng, "Integrated sensing and communication signals toward 5g-a and 6g: A survey," *IEEE Internet of Things Journal*, vol. 10, no. 13, pp. 11 068–11 092, 2023.
- [3] W. Zhou, R. Zhang, G. Chen, and W. Wu, "Integrated sensing and communication waveform design: A survey," *IEEE Open Journal of the Communications Society*, vol. 3, pp. 1930–1949, 2022.
- [4] C. Dou, N. Huang, Y. Wu, L. Qian, and T. Q. Quek, "Channel sharing aided integrated sensing and communication: An energy-efficient sensing scheduling approach," *IEEE Trans. on Wireless Communications*, vol. 23, no. 5, pp. 4802–4814, 2023.
- [5] K. Meng, C. Masouros, A. P. Petropulu, and L. Hanzo, "Cooperative isac networks: Performance analysis, scaling laws, and optimization," *IEEE Trans. on Wireless Communications*, vol. 24, no. 2, pp. 877–892, 2024.
- [6] Ericsson, "Drone detection with ISAC for defense," Ericsson, 2026, accessed August 7, 2026. [Online]. Available: <https://www.ericsson.com/en/industries/defense/drone-detection-isac>
- [7] U.S. Department of Defense, "Integrated Sensing and Communications (ISAC): Sensing the Future Battlefield," FutureG Office, Office of the Under Secretary of Defense for Research and Engineering, 2025.
- [8] Y. Cui, F. Liu, X. Jing, and J. Mu, "Integrating sensing and communications for ubiquitous iot: Applications, trends, and challenges," *IEEE Network*, vol. 35, no. 5, pp. 158–167, 2021.
- [9] R. Berry, T. W. Hazlett, M. Honig, and J. N. Laneman, "Evaluating the CBRS experiment," *Available at SSRN 4528763*, 2023.
- [10] R. Berry, M. Honig, T. Nguyen, V. Subramanian, and R. Vohra, "The value of sharing intermittent spectrum," *Management Science*, vol. 66, no. 11, pp. 5242–5264, 2020.
- [11] K. Mu and R. Berry, "Market impacts of pooling intermittent spectrum," in *2024 IEEE International Symposium on Dynamic Spectrum Access Networks (DySPAN)*, 2024, pp. 189–196.
- [12] M. Felix and R. Berry, "Markets for intermittent service using intermittent spectrum," in *International Conference on Game Theory for Networks*. Springer, 2025, pp. 3–17.
- [13] P. M. Woodward, *Probability and Information Theory, with Applications to Radar*. London: Pergamon Press, 1953.