

Solution of Regular Problems from Chapter 2:

2-10. (a) Let A = Top branch Fails B = Bottom branch Fails, F = System Fails

$$F = A \cap B \quad P(F) = P(A)P(B)$$

$$A = F_1 \cup F_2 \cup F_3 \quad \text{Therefore } \bar{A} = \bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3$$

$$\text{Hence we have } P(\bar{A}) = P(\bar{F}_1)P(\bar{F}_2)P(\bar{F}_3) = (1 - p_1)(1 - p_2)(1 - p_3)$$

$$\text{So that } P(A) = 1 - (1 - p_1)(1 - p_2)(1 - p_3)$$

$$B = F_4 \cup F_5 \quad P(B) = 1 - P(\bar{B}) = 1 - (1 - p_4)(1 - p_5)$$

$$P(F) = [1 - (1 - p_1)(1 - p_2)(1 - p_3)][1 - (1 - p_4)(1 - p_5)]$$

For the values provided:

$$= [1 - 0.9 \bullet 0.95 \bullet 0.95][1 - 0.85 \bullet 0.85] = 0.18775 \bullet 0.2775 = 0.0521$$

(b) It is exactly the same as (a) except instead of #5 in (a) we have two in parallel, and also in place of #3 in (a).

Let A = Top branch Fails B = Bottom branch Fails

$$C = F_4 \cap F_5 \quad D = F_6 \cap F_7 \quad \text{Then } P(C) = p_4 p_5 \quad P(D) = p_6 p_7$$

$$P(A) = P(F_1 \cup C) = p_1 + P(C) - p_1 P(C) = p_1 + p_4 p_5 - p_1 p_4 p_5$$

$$P(B) = 1 - P(\bar{B}) = 1 - P(\bar{F}_2 \cap \bar{F}_3 \cap \bar{D})$$

$$= 1 - P(\bar{F}_2)P(\bar{F}_3)P(\bar{D}) = 1 - (1 - p_2)(1 - p_3)(1 - p_6 p_7)$$

$$P(F) = P(A \cap B) = P(A)P(B) = (p_1 + p_4 p_5 - p_1 p_4 p_5)[1 - (1 - p_2)(1 - p_3)(1 - p_6 p_7)]$$

For the values provided:

$$= (0.1 + 0.15 \bullet 0.15 - 0.1 \bullet 0.15 \bullet 0.15)(1 - 0.95 \bullet 0.95 \bullet 0.96) = 0.12025 \bullet 0.1336$$

$$= 0.016065$$

2-11. Let D = {component is defective}

A₁ = {component is from Mfg. I}

A₂ = {Component is from Mfg. II}

A₃ = {Component is from Mfg. III}

From the information provided we have:

$$\begin{array}{lll}
 P(A_1) = 0.5 & P(A_2) = 0.4 & P(A_3) = 0.1 \\
 P(D | A_1) = 0.001 & P(D | A_2) = 0.005 & P(D | A_3) = 0.01
 \end{array}$$

(a) $P(D) = P(D | A_1)P(A_1) + P(D | A_2)P(A_2) + P(D | A_3)P(A_3)$
 (by the rule of total probability)
 $= 0.001 \cdot 0.5 + 0.005 \cdot 0.4 + 0.01 \cdot 0.1 = 0.0035$

(b) Use Bayes' Rule:

$$P(A_1 | D) = \frac{P(D | A_1)P(A_1)}{P(D)} = \frac{0.001 \cdot 0.5}{0.0035} = 0.1429$$

$$P(A_2 | D) = \frac{P(D | A_2)P(A_2)}{P(D)} = \frac{0.005 \cdot 0.4}{0.0035} = 0.5714$$

$$P(A_3 | D) = \frac{P(D | A_3)P(A_3)}{P(D)} = \frac{0.01 \cdot 0.1}{0.0035} = 0.2857$$

- 2-12. (a) If the components are mixed before being used in a circuit, then the event that a component is defective is independent of the event that another in the same circuit is defective.

Let D_1 and D_2 be the event that the first and the second components in a circuit are defective.

Then in this case we have that D_1 and D_2 are independent:

$P(\text{Both are defective}) =$

$$P(D_1 \cap D_2) = P(D_1)P(D_2) = (0.0035)^2 = 1.225 \cdot 10^{-5}$$

- (b) Since in each circuit **both** components come from the same manufacturer, they both are either from I, or II, or III, and hence they are conditionally independent:

$$\begin{aligned}
 P(D_1 \cap D_2) &= P(D_1 \cap D_2 | A_1)P(A_1) + \\
 &+ P(D_1 \cap D_2 | A_2)P(A_2) + \\
 &+ P(D_1 \cap D_2 | A_3)P(A_3)
 \end{aligned}$$

$$P(D_1 \cap D_2 | A_1) = P(D_1 | A_1)P(D_2 | A_1) = (0.001)^2 = 10^{-6}$$

$$P(D_1 \cap D_2 | A_2) = P(D_1 | A_2)P(D_2 | A_2) = (0.005)^2 = 2.5 \cdot 10^{-5}$$

$$P(D_1 \cap D_2 | A_3) = P(D_1 | A_3)P(D_2 | A_3) = (0.01)^2 = 10^{-4}$$

$$P(D_1 \cap D_2) = 10^{-6} \cdot 0.5 + 2.5 \cdot 10^{-5} \cdot 0.4 + 10^{-4} \cdot 0.1 = 2.05 \cdot 10^{-5}$$

(twice as high as in (a))

2-13.

$$(a) \quad P(R_u) = P(R_u | T_o)P(T_o) + P(R_u | T_1)P(T_1) \\ = 0.09 \bullet \frac{1}{2} + 0.09 \bullet \frac{1}{2} = 0.09 \quad (\text{We used Total Probability rule})$$

$$(b) \quad (\text{Error}) = \{ \text{"1" is sent but "0" is received} \} \\ \text{or } \{ \text{"0" is sent but "1" is received} \}$$

$$\text{We have } P(R_o | T_o) = 0.9 \quad P(R_u | T_o) = 0.09 \Rightarrow P(R_1 | T_o) = 0.01 \\ (\text{The total must add up to 1.0})$$

Similarly, if "1" is sent:

$$P(R_1 | T_1) = 0.9 \quad , \quad P(R_u | T_1) = 0.09 \Rightarrow P(R_o | T_1) = 0.01$$

$$P(\text{Error}) = P(R_o \cap T_1) + P(R_1 \cap T_o) \\ = P(R_o | T_1)P(T_1) + P(R_1 | T_o)P(T_o) \\ = 0.01 \bullet \frac{1}{2} + 0.01 \bullet \frac{1}{2} = 0.01$$

$$(c) \quad P(T_o | R_o) = \frac{P(R_o | T_o)P(T_o)}{P(R_o)} \quad \text{Bayes' Rule}$$

$$P(R_o) = P(R_o | T_1)P(T_1) + P(R_o | T_o)P(T_o) = \\ = 0.01 \bullet \frac{1}{2} + 0.9 \bullet \frac{1}{2} = 0.91 \bullet 0.5 = 0.455$$

$$P(T_o | R_o) = \frac{0.9 \bullet \frac{1}{2}}{0.455} = 0.989$$

$$P(T_1 | R_o) = \frac{P(R_o | T_1)P(T_1)}{P(R_o)} = \frac{0.01 \bullet 0.5}{0.455} = 0.011$$

$$(d) \quad P(T_o) = 0.6 \quad P(T_1) = 0.4 \\ P(R_u) = 0.09 \bullet 0.6 + 0.09 \bullet 0.4 = 0.09 \\ P(\text{Error}) = P(R_o | T_1)P(T_1) + P(R_1 | T_o)P(T_o) = 0.01 \bullet 0.4 + 0.01 \bullet 0.6 = 0.01 \\ P(R_o) = P(R_o | T_1)P(T_1) + P(R_o | T_o)P(T_o) = 0.01 \bullet 0.4 + 0.9 \bullet 0.6 = 0.544 \\ P(T_o | R_o) = \frac{P(R_o | T_o)P(T_o)}{P(R_o)} = \frac{0.9 \bullet 0.6}{0.544} = 0.99265 \\ P(T_1 | R_o) = \frac{P(R_o | T_1)P(T_1)}{P(R_o)} = \frac{0.01 \bullet 0.4}{0.544} = 0.00735$$

2-14. $p = 0.6$ (Probability of a hit per one launch); Number of missiles launched = $n = 5$

$$P\{\text{at least 1 hit}\} = 1 - P\{0 \text{ hits in 5 missiles}\}$$

$$= 1 - (0.6)^0 (0.4)^5 = 1 - 0.01024 = 0.98976$$

Failure = {0 or 1 hits in 5 launches}

$$\begin{aligned} P(\text{Failure}) &= (0.6)^0 \bullet (0.4)^5 + \binom{5}{2} (0.6)^1 (0.4)^4 = \\ &= (0.4)^5 + 5 \bullet 0.6(0.4)^4 = 0.08704 \end{aligned}$$

2-15. For each “word” 9 bits are sent — 8 data bits and one parity bit. An error occurs if 2, 4, 6, 8 of the 9 sent are reversed (thus preserving the even parity):

$$P(\text{error}) = (2 \text{ bits in error out of 9}) + P(4 \text{ bits in error}) + P(6 \text{ bits in error}) + P(8 \text{ bits in error})$$

$$\begin{aligned} &= \binom{9}{2} (0.02)^2 (0.98)^7 + \binom{9}{4} (0.02)^4 (0.98)^5 + \binom{9}{6} (0.02)^6 (0.98)^3 \\ &+ \binom{9}{8} (0.02)^8 (0.98) = 0.01252 \end{aligned}$$