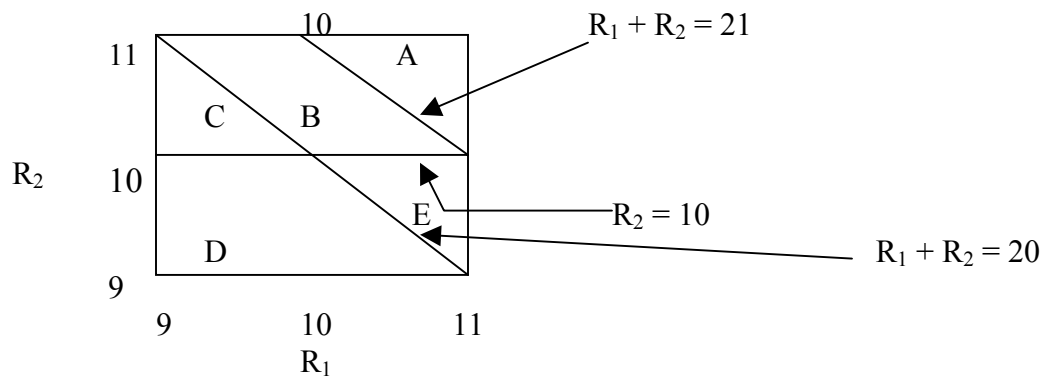


2-4. (a) List of the 8 events:

$$F_1 \cap F_2 \cap F_3, F_1 \cap F_2 \cap \bar{F}_3, F_1 \cap \bar{F}_2 \cap F_3, F_1 \cap \bar{F}_2 \cap \bar{F}_3, \\ \bar{F}_1 \cap F_2 \cap F_3, \bar{F}_1 \cap F_2 \cap \bar{F}_3, \bar{F}_1 \cap \bar{F}_2 \cap F_3, \bar{F}_1 \cap \bar{F}_2 \cap \bar{F}_3$$

(b) $F_1 \cap F_2 \cap F_3, F_1 \cap F_2 \cap \bar{F}_3, F_1 \cap \bar{F}_2 \cap F_3, F_1 \cap \bar{F}_2 \cap \bar{F}_3, \\ \bar{F}_1 \cap F_2 \cap F_3 \\ \Rightarrow F = F_1 \cup (F_2 \cap F_3)$

2-6. For simplicity we shall ignore outcomes falling exactly on the lines, hence we use $>$ or $<$ only rather than $\geq$ or $\leq$.



(a) $A = \{R_1 + R_2 > 21\}$
 $B = \{(20 < R_1 + R_2 < 21) \cap (R_2 > 10)\}$
 $C = \{(R_1 + R_2 < 20) \cap (R_2 > 10)\}$
 $D = \{(R_1 + R_2 < 20) \cap (R_2 < 10)\}$
 $E = \{(R_1 + R_2 > 20) \cap (R_2 < 10)\}$

(b) $(C \cup D) = \{R_1 + R_2 < 20\}$
 $(B \cup E) = \{20 < R_1 + R_2 < 21\}$

(c) $P(A) = \frac{\text{Area bound by } A}{\text{Area of square}} = \frac{\frac{1 \times 1}{2}}{2 \times 2} = \frac{1}{8}$

$$P(C \cup D) = \frac{\frac{2 \times 2}{2}}{2 \times 2} = \frac{1}{2}$$

(d) $\{R_1 + R_2 < 21\} = \bar{A}$ (see first line in a.)
 $\{(R_1 + R_2 < 21) \cap (R_2 > 10)\} = B \cup C$ (see 2nd & 3rd line in a.)

2-7. $A = \{0 \leq \xi \leq 10\}, B = \{9 \leq \xi \leq 19\}, C = \{19 < \xi \leq 20\}, D = \{8 \leq \xi \leq 16\}$

$$(a) \quad A \cup B = \{0 \leq \xi \leq 19\} \quad P(A \cup B) = \frac{19}{20} = 0.95$$

$$(b) \quad (A \cup B \cup C) = \{0 \leq \xi \leq 20\} = S \quad P(A \cup B \cup C) = 1$$

$$(c) \quad (A \cap B) = \{8 \leq \xi \leq 10\} \quad P(A \cap B) = \frac{2}{20} = 0.1$$

$$(d) \quad (A \cup B) \cap D = D \quad P(D) = \frac{8}{20} = 0.4$$

$$(e) \quad (A \cup D) = \{8 \leq \xi \leq 10\} \quad P(A \cup D) = \frac{2}{20} = 0.1$$

$$(f) \quad (A \cup B) \cap (D \cup C) = \{8 \leq \xi \leq 19\} \quad P\{(A \cup B) \cap (D \cup C)\} = \frac{11}{20} = 0.55$$

$$(g) \quad (A \cap B) \cup C = \{8 \leq \xi \leq 10 \text{ or } 19 \leq \xi \leq 20\}$$

$$P\{(A \cap B) \cup C\} = \frac{1+2}{20} = 0.15$$

$$(h) \quad \bar{A} = \{10 < \xi \leq 20\} \quad P(\bar{A}) = \frac{10}{20} = \frac{1}{2}$$

$$(i) \quad \overline{(A \cup B)} = \{19 < \xi \leq 20\} = C \quad P\{\overline{(A \cup B)}\} = \frac{1}{20} = 0.05$$

$$(j) \quad \overline{(A \cap B)} = \{\xi < 8 \text{ (or) } 10 < \xi \leq 20\}$$

$$P\{\overline{(A \cap B)}\} = \frac{8+10}{20} = 0.9 = 1 - P(A \cap B)$$

2-8. Total number of components = 325

Resistors (A) = 200, Capacitors (B) = 100, Inductors (C) = 25

High tolerance (D) = 210, Low tolerance (E) = 115

$$P(A) = \frac{200}{325} = 0.6154$$

$$P(B) = \frac{100}{325} = 0.3077$$

$$P(C) = \frac{25}{325} = 0.0769$$

$$P(D) = \frac{210}{325} = 0.6462$$

$$P(E) = \frac{115}{325} = 0.3538 = 1 - P(D)$$

$$P(A) + P(B) + P(C) = 1$$

$$P(A \cap D) = \frac{120}{325} = 0.3692 \quad \text{Component is a resistor of high tolerance}$$

$$P(A \cap E) = \frac{80}{325} = 0.2462 \quad \text{Component is a resistor of low tolerance}$$

$$P(A) = P(A \cap D) + P(A \cap E)$$

$$P(B \cap D) = \frac{75}{325} = 0.2308 = \text{Capacitor of high tolerance}$$

$$P(B \cap E) = \frac{25}{325} = 0.0769 = \text{Capacitor of low tolerance}$$

$$P(B) = P(B \cap D) + P(B \cap E)$$

$$P(C \cap D) = \frac{15}{325} = 0.0461 = \text{Inductor of high tolerance}$$

$$P(C \cap E) = \frac{10}{325} = 0.0308 = \text{Inductor of low tolerance}$$

$$P(C) = P(C \cap D) + P(C \cap E)$$

$$P(A \cup B) = P(A) + P(B) = 0.9231 = \frac{300}{325} = \text{Resistor or Capacitor}$$

$$P\{(A \cup B) \cap D\} = P(A \cap D) + P(B \cap D) = 0.6000 = \frac{195}{325} =$$

Either resistor or capacitor of high tolerance

$$2-9. \quad P(F_1 \cap F_2 \cap F_3) = 0.002$$

$$P\{(F_2 \cap F_3) \cap \bar{F}_1\} = 0.018$$

$$P\{(\bar{F}_2 \cup \bar{F}_3) \cap F_1\} = 0.008$$

Let $F_2 \cap F_3 = A$

We have $P(A \cap F_1) = 0.002$

$$P(A \cap \bar{F}_1) = 0.018$$

$$A = (A \cap F_1) \cup (A \cap \bar{F}_1) \Rightarrow P(A) = P(A \cap F_1) + P(A \cap \bar{F}_1) = 0.02$$

Since A is given as a union of mutually exclusive events that include the entire space S.

$$\therefore P(F_2 \cap F_3) = P(A) = 0.02 \quad (\text{from the definition of } A).$$

$$\bar{A} = \overline{(F_2 \cap F_3)} = \bar{F}_2 \cup \bar{F}_3 \quad \text{by De Morgan's Law.}$$

Hence we have $P\{(\bar{F}_2 \cup \bar{F}_3) \cap F_1\} = 0.008$ but this is the same as $P(\bar{A} \cap F_1) = 0.008$

We also have from $P(F_1 \cap F_2 \cap F_3) = 0.002 = P(A \cap F_1)$

Hence, since A and $\bar{A}$ are mutually exclusive and span the entire space, we have

$$F_1 = (F_1 \cap A) \cup (F_1 \cap \bar{A}) \quad P(F_1) = P(F_1 \cap A) + P(F_1 \cap \bar{A}) = 0.008 + 0.002 = 0.01$$

We cannot find $P(F_2)$ or $P(F_3)$ since no information is provided involving each of the events F_2 and F_3 separately. We can find P(F) since

$$P(F) = P(F_1 \cup A) = P(F_1) + P(A) - P(A \cap F_1) = 0.01 + 0.02 - 0.002 = 0.028$$