

Matlab Problem:

1. In order to illustrate the law of large numbers and the Central Limit Theorem generate 64 random numbers uniformly distributed in the interval $[0,1]$ and find their sample average. Repeat the process 100 times (this means that you generated 6,400 numbers in batches of 64 each, and thus generated 100 sample averages).
 - a. Plot a histogram of the sample means (the 100 sample means you generated) and compare to the Gaussian probability density function you found in problem 1(b) below. (Plot both on the same graph).
 - b. What fraction of the 100 sample averages fall within 15% of the true mean of 0.5? Compare the result to that obtained in problem 1(c) below.
2. Generate 500 pairs of Gaussian random variables $\{X_i\}$ and $\{Y_i\}$ using the Matlab function *randn* using zero mean and unit variance. Transform the (X,Y) pairs that you generated to generate two new pairs (Z,W) of variables $\{Z_i\}$ and $\{W_i\}$ as follows: $Z_i = aX_i - bY_i$ and $W_i = aX_i + bY_i$; where $a = \sqrt{0.5(1+r)}$ and $b = \sqrt{0.5(1-r)}$ where r is a correlation coefficient. Use three cases for values of $r = 0.7, 0.9, -0.9$.
 - a. Plot a scatter plot of Z versus W and show that they are indeed correlated corresponding to the value of the correlation coefficient r . Compare to the scatter plot of X and Y .
 - b. Plot histograms of Z and W and show that they are indeed Gaussian.
 - c. Find the correlation coefficient from the data you generated and compare to the value of r . Compare to the correlation coefficient of X and Y .

Matlab Commands:

cov(x,y) = covariance of the two vectors x and y : the result is a 2×2 matrix whose diagonal elements are the variances of x and y , and the off-diagonal is the covariance of x and y .

corrcoef(x,y) = correlation coefficients of the x and y in matrix form as in the *cov* case.

Regular Problems:

1. Let $\{X_i\}$ for $i=1,2,3,\dots,n$ be independent random variables uniformly distributed in the interval $[0,1]$. We derive the following sample average Y (which we use as an estimate of the true mean of X_i which we know is 0.5) as follows:

$$Y = (X_1 + X_2 + \dots + X_n)/n$$

- a. Find the mean and variance of Y . Compute for $n=64$.

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- b. Use the Central Limit Theorem to write the approximate probability density function of $\bar{Y}$. Compute for $n=64$.
 - c. For $n=64$ find the probability that $P(|\bar{Y}-0.5| < 0.075)$ (this is a 15% error margin in our estimate of the mean which in this case we know is 0.5).
 - d. What should be the number of samples n if we wish the error margin to be 5% of the mean with the same confidence level we arrived at in c.
2. Let X and Y be correlated random variables with means $m_X=2$ and $m_Y=5$, and variances $\text{Var}(X)=9$ and $\text{Var}(Y)=4$, respectively, and covariance $C_{XY}=5$. Two new random variables are defined as follows:

$$U = aX + bY + m_1, \quad V = cX + dY + m_2$$

- a. For values of $a=2$, $b=1.5$, $c=2$, $d=-2.5$, $m_1=m_2=0$, find the means, variances, covariance, and correlation coefficient of U and V .
- b. Let $c=0$, find values of a , b , d , m_1 , and m_2 , so that U and V will have zero means, variances of 1, and covariance of 0.
- c. If X and Y are jointly Gaussian, write the joint probability density function of U and V you obtained in part b.