

Matlab Problem:

Generate 5,000 random numbers $\{X_i\}$ with normal (Gaussian) distribution with mean zero and unit standard deviation. Generate another 5,000 numbers $\{Y_i\}$ with the same distribution. Now use the formula $Z_i = \sqrt{X_i^2 + Y_i^2}$ to obtain 5,000 new random variables $\{Z\}$. Draw a histogram of $\{Z\}$ using bin size of 0.1 and show that it is approximately a Rayleigh density. Plot the Rayleigh density obtained in class to illustrate this fact.

Regular Problems:

1. The random variables X and Y are uniformly distributed in the triangle described below (the density is zero outside the triangle):

$$f_{XY}(u,v) = 0.5, \quad \text{for } 0 < u, 0 < v, \text{ and } 0 < u + v < 2, \\ \text{and it is zero elsewhere}$$

- a. Find the marginal density functions of X and Y
 - b. Are X and Y dependent?
 - c. Find the joint distribution function of X and Y
- Find the expected value of the following functions $g(X,Y)$
- d. $g(X,Y) = XY$
 - e. $g(X,Y) = \exp\{-(X+Y)\}$
 - f. Find the correlation coefficient between X and Y .

2. The random variables X and Y have following joint density function:

$$f_{XY}(u,v) = 1.5 \exp(-3v), \quad \text{for } 0 < u < 2, \text{ and } 0 < v < \infty; \\ \text{and it is zero elsewhere.}$$

- a. Find the density function of their sum $Z = (X+Y)$
 - b. Find the mean and variance of their sum Z .
 - c. Find the expected value of $g(X,Y) = \exp(-XY)$
3. Let X and Y be independent random variables with means $m_X=2$ and $m_Y=5$, and variances $\text{Var}(X)=9$ and $\text{Var}(Y)=4$, respectively. Define a new random variable Z as follows:

$$Z = 2X + 1.5Y - 2.5$$

Find the mean and variance of Z . Find the covariance and correlation coefficient between: X and Z , Y and Z .