

## Number of Trials to First Success

So far we performed a fixed number of trials and counted the number of successes. We now consider the case when we do not fix the number of trials, but we continue to repeat the experiment until we achieve the first success. In the notes on page 2-30, second paragraph, we considered the general case. In that case we found that the probability of the first success at the k-th trial has a geometric form. Here we shall look at a simple example to illustrate the approach and also relate it to the case of fixed number of trials.

Suppose there is an apple on a tree and you jump to try to get it. Suppose that the probability of success in each jump is  $p = 0.6$ . We shall denote a success in trial #i by  $S_i$ , and we assume that  $S_1$  and  $S_2$  are independent and have probability  $p=0.6$  each. We shall denote the complement of the events  $S_i$  by  $F_i$ .

### What is probability of getting the apple in the first trial?

Let us denote this event by  $A_1$ . Clearly the answer to this question is 0.6, since we have

$$P\{A_1\} = P\{S_1\} = 0.6.$$

### What is the probability of getting the apple in the second trial?

Let us denote this event by  $A_2$ . Obviously you do not try again unless the first trial is unsuccessful, which means that the event  $A_2$  can be written as the intersection of a failure during the first trial and a success during the second:

$$A_2 = F_1 \cap S_2$$

Since the events are independent, we obtain the probability:

$$P\{A_2\} = P\{F_1 \cap S_2\} = P\{F_1\}P\{S_2\} = (0.4)(0.6) = 0.24$$

We now may ask about the probability that you get the apple in at most two trials, which we shall denote by  $B$ . This event is the union of  $A_1$  and  $A_2$  and therefore its probability is the sum of their probabilities resulting in:

$$P\{B\} = P\{A_1 \cup A_2\} = P\{A_1\} + P\{A_2\} = 0.6 + 0.24 = 0.84$$

We can also obtain this probability as the complement of the events of failure during the first two trials:

$$B = (\overline{F_1 \cap F_2})$$

We therefore have:

$$P(B) = 1 - P(F_1 \cap F_2) = 1 - P\{F_1\}P\{F_2\} = 1 - (0.4)(0.4) = 0.84$$

The sample space for this problem is shown in Figure A-1.

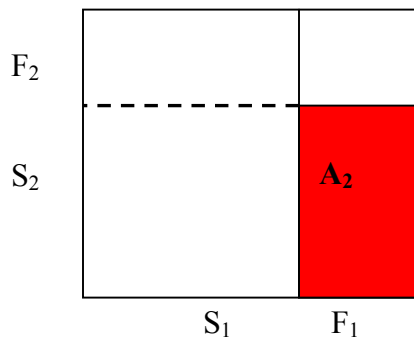


**Figure A-1 Successes and Failures in Trials 1 and 2.**

### The relation to independent trials with $n=2$ :

We now relate it to the repeated trials case with  $n = 2$ . It is seen from the expression for  $B$  that it is the complement of the event of zero successes in two trials. Hence the complement  $B$  is the event that there are one or two successes in two trials for  $n=2$ :

$$P(B) = P(1 \text{ or } 2 \text{ successes in } 2 \text{ trials}) = 2 (0.6)(0.4) + (0.6)(0.6) = 0.84$$

Since we claimed that there was only one apple, how do we justify the statement that  $B$  means 1 or 2 apples in two jumps? The answer to the question is provided by the sample space in Figure A-1. If the first trial was a success, then it does not matter what happens at a second trial if there is one (that is why the outcomes of the second trials are shown via a dashed line in the figure). The large square on the left means that we get two apples if there were two, and the smaller rectangle on the left (above the dashed line) means that we do not get a second apple. However, both of these events are included in the event of the first success  $S_1$ . We see therefore, that the outcomes of the second trial are not relevant to the result as both are included in the event  $S_1$ .

We can further illustrate this by a tree diagram (no pun intended) in Figure A-2:

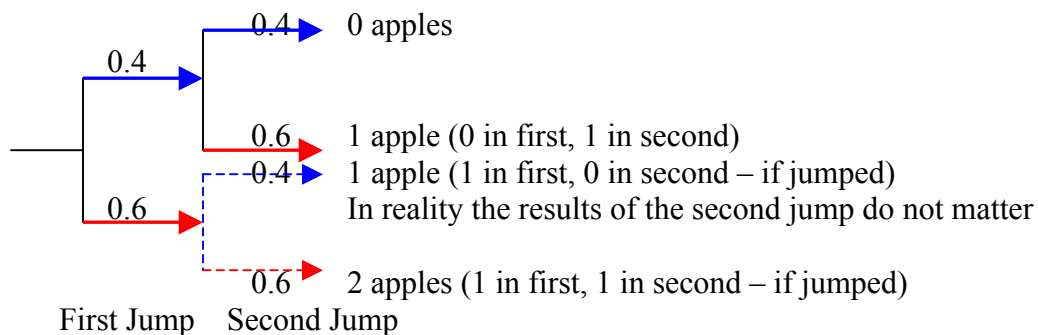


**Figure A-2 Tree diagram illustrating the two jumps**

In the figure successes are shown in red and failures in blue. Note that the second jump results (which need not occur, as the jump will not be taken) are shown in dashed lines. We may compute these probabilities and add them, but they will only contribute the value of 0.6 for the first success:

$$0.6(0.4+0.6) = 0.6$$

As we see we need only add 0.6 to the probability of the top branch, and not worry about adding the probability of each branch of the dashed event:

$$(0.6)(0.6) + (0.6)(0.4) = 0.6$$

We obtain the same answer if we were to use the repeated trials with  $n=2$ , as then we will be adding the dashed lines individually, which as seen from the figure yields the same answer.

### **Acknowledgement**

Thanks to Professor Plonus for suggesting the example and his contribution to this note.