

Probability:

S = Sample space; $A, B, C, \dots$ = events; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; $P(S) = 1$;

Independent events: $P(A \cap B) = P(A)P(B)$; Conditional probability: $P(A|B) = P(A \cap B)/P(B)$;

Total probability: $P(A) = \sum P(A|B_i)P(B_i)$, B_i are disjoint and $\cup B_i = S$.

Bayes' rule: $P(B_j|A) = P(A|B_j)P(B_j)/P(A)$, where $P(A)$ is as in total probability

Random Variables:

Cumulative distribution function: $F_X(u) = P\{X \leq u\} = \int_{-\infty}^u f_X(v)dv$

Expectations: $E\{g(X)\} = \int_{-\infty}^{\infty} g(v) f_X(v)dv$ (for continuous R.V.);

$E\{g(X)\} = \sum g(x_i)P(X=x_i)$ (for discrete R.V.)

Properties: $E\{a g(X) + b h(X)\} = a E\{g(X)\} + b E\{h(X)\}$; if $g(X) \geq 0$, then $E\{g(X)\} \geq 0$

Mean: $m_X = E\{X\}$, Variance: $\text{Var}(X) = \sigma_X^2 = E\{[X - m_X]^2\} = E\{X^2\} - m_X^2$

Chebyshev's inequality: $P\{|X - m_X| \geq a\} \leq \sigma_X^2/a^2$

Special random variables:

Binomial (number of successes in n trials): $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k=0,1,2,\dots,n$

$m_X = np$, $\text{Var}(X) = np(1-p)$

Poisson (number of arrivals in T time units, where rate is λ): $P(X=k) = (\lambda T)^k \exp(-\lambda T)/k!$
for $k=0,1,2,\dots$, $m_X = \lambda T$, $\text{Var}(X) = \lambda T$

Geometric (number to first success): $P(X=k) = p(1-p)^{k-1}$ for $k=1,2,3,\dots$ $m_X = 1/p$

Exponential (lifetime, inter-arrival time): $f_X(v) = \lambda \exp(-\lambda v)$ for $v > 0$, $m_X = 1/\lambda$, $\sigma_X = 1/\lambda$

Gaussian: $f_X(v) = \frac{\exp\{-\frac{(v-m_X)^2}{2\sigma_X^2}\}}{\sqrt{2\pi}\sigma_X}$, $\Phi(u) = \int_{-\infty}^u \frac{\exp(-\frac{v^2}{2})}{\sqrt{2\pi}} dv$, cumulative unit Gaussian

distribution.

u	0.5	0.75	1.0	1.25	1.5	1.75	2	2.25	2.5	3
$\Phi(u)$	0.6915	0.7734	0.8413	0.8944	0.9332	0.9599	0.9798	0.9878	0.9946	0.9987

Transformation of random variables: $Y = g(X)$: $F_Y(u) = P\{g(X) \leq u\}$;

$Y = aX + b$, then $f_Y(v) = \frac{1}{|a|} f_X\left(\frac{v-b}{a}\right)$