Minimum Mean-Squared Error Multiuser Decision-Feedback Detectors for DS-CDMA

Graeme Woodward, *Member, IEEE*, Rapeepat Ratasuk, *Member, IEEE*, Michael L. Honig, *Fellow, IEEE*, and Predrag B. Rapajic, *Senior Member, IEEE*

***Abstract*—Multiuser decision-feedback detectors (DFDs) for direct-sequence code-division multiple access, based on the minimum mean-squared error (MMSE) performance criterion, are described. Both successive and parallel feedback (interference cancellation) with hard decisions are considered. An iterative DFD is presented, which consists of cascaded DFDs, each performing successive cancellation. The two-stage DFD achieves the single-user bound in the absence of error propagation, and performs significantly better than an MMSE DFD with parallel feedback. The filter structures are generalized to include finite impulse response feedforward and feedback matrix filters, which account for asynchronous users and intersymbol interference. The effect of error propagation is illustrated through simulation. Both uncoded and coded performance results are presented. Although error propagation can significantly degrade performance, the DFDs still offer a significant performance gain relative to linear MMSE detection.**

***Index Terms*—Code-division multiple access (CDMA), decision feedback, interference cancellation, interference suppression, multiuser detection.**

I. Introduction

MULTIUSER detection has been proposed as a way to increase the spectral efficiency of code-division multiple-access (CDMA) systems. The information theoretic tradeoff between power efficiency and spectral efficiency for the synchronous multiple-access channel with additive Gaussian noise (AGN) has been quantified with different types of linear and nonlinear multiuser detectors in [1]–[3]. This work has shown that given sufficient E_b/N_0, at very high loads (the ratio of users to processing gain close to one), the spectral efficiency of nonlinear multiuser detection is significantly higher than that of linear detection.

Paper approved by L. Vandendorpe, the Editor for Transmission Systems of the IEEE Communications Society. Manuscript received September 15, 2000; revised April 15, 2001 and June 15, 2002. This work was supported in part by the National Science Foundation under Grant NCR-9628365. This paper was presented in part at the International Conference on Communications, Vancouver, BC, Canada, June 1999.

G. Woodward was with Southern-Poro Communications, Annandale, N.S.W. 2038, Australia. He is now with Bell Labs, Lucent Technologies, North Ryde, N.S.W. 2113, Australia (e-mail: graemew@lucent.com).

R. Ratasuk was with the Department of Electrical and Computer Engineering, Northwestern University, Evanston, IL 60208 USA. He is now with Motorola Inc., Arlington Heights, IL 60004 USA (e-mail: ratasuk@motorola.com).

M. L. Honig is with the Department of Electrical and Computer Engineering, Northwestern University, Evanston, IL 60208 USA (e-mail: mh@ece.nwu.edu).

P. B. Rapajic is with the Mobile Telecommunications Research Group, School of Electrical Engineering and Telecommunications, Department of Electrical and Computer Engineering, University of New South Wales, Sydney, N.S.W. 2052, Australia (e-mail: P.Rapajic@unsw.edu.au).

Digital Object Identifier 10.1109/TCOMM.2002.806550

Here, we consider a class of nonlinear minimum mean-square error (MMSE) multiuser decision-feedback detectors (DFDs), which are relatively simple, and can perform significantly better than a linear multiuser detector. When used with short or repeated spreading codes, the MMSE criterion leads to adaptive implementations which require only a training sequence for estimation of the filter parameters (see [3] and [4]).

Previous work on MMSE DFDs with successive cancellation has been presented in [5]–[9]. Here, we consider an MMSE DFD in which an arbitrary subset of users is canceled. This includes parallel feedback, in which all demodulated users except for the desired user are canceled. We show that the feedforward filter of a parallel (P)-DFD consists of the linear MMSE filter followed by an error *estimation* filter. The latter filter is analogous (but not equivalent) to the error whitening filter for a successive (S)-DFD. It is shown that the P-DFD satisfies the reverse link objectives in a cellular system, namely, cancellation of intracell interference and suppression of the remaining other-cell interference. Other related, but different, multiuser decision-feedback cancelers have been presented in [10], and [11].

Multistage, or iterative, DFDs are also presented, in which symbol estimates at a given stage are used to refine the symbol estimates at the succeeding stage. In addition to the iterative P-DFD, we propose an iterative successive (IS)-DFD in which the first stage is an S-DFD, and the second stage consists of the P-DFD structure, but the users are detected successively in reverse order relative to the first stage. This structure is motivated by the observation that successive feedback and decoding mitigates the deleterious effects of error propagation. Our numerical results show that the two-stage IS-DFD offers uniformly better performance (over all users) relative to the P-DFD.

Simulation results are shown, which compare the performance of S-, P-, and IS-DFDs with the linear MMSE receiver, with and without coding. These results show that with coding, the S-DFD can offer significantly better performance relative to linear MMSE detection for some users (i.e., those ordered toward last). The IS-DFD is shown to improve performance significantly for all users. The P-DFD provides relatively uniform performance over the user population, but the average gain in performance relative to the linear MMSE detector is modest for relatively small systems. This is due to error propagation, which significantly degrades performance at moderate error rates. Our results show that the performance gain of the DFDs relative to the linear MMSE receiver increases with system size.

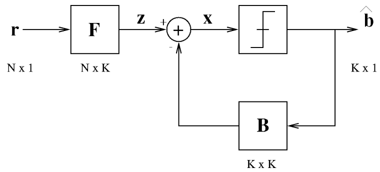

Fig. 1. Block diagram of multiuser decision-feedback receiver.

In the next section, we derive MMSE DFDs for synchronous CDMA. In Section II-C, we present iterative versions of these receivers. Numerical results showing the performance with coding are presented in Section III. Finally, we extend the S- and P-DFDs to asynchronous CDMA with multipath in Section IV. For the latter case, we assume finite-length feedforward and feedback (matrix) filters.

II. MMSE Decision Feedback Detector

Fig. 1 shows a block diagram of the MMSE DFD. For now, we assume a quasi-synchronous baseband CDMA model in which the received vector of N samples during the ith symbol interval is

$$\boldsymbol{r}(i) = \boldsymbol{P}\boldsymbol{b}(i) + \boldsymbol{n}(i) \tag{1}$$

where

$$\boldsymbol{P} = [\boldsymbol{p}_1 \ldots \boldsymbol{p}_K] \tag{2}$$

is the $N \times K$ matrix of spreading codes observed by the receiver, N is the processing gain, K is the number of users, and $\boldsymbol{p}_k$ is the spreading code for user k, scaled such that $\|\boldsymbol{p}_k\| = A_k$, the amplitude for user k. The vector $\boldsymbol{b}(i)$ contains the ith unit-variance symbols across users, and $\boldsymbol{n}(i)$ is the corresponding noise vector. We assume that the noise is white with covariance matrix $\sigma^2\boldsymbol{I}$, although the following results are easily generalized to account for colored noise. The receiver input covariance matrix is

$$\boldsymbol{R} \triangleq E[\boldsymbol{r}(i)\boldsymbol{r}^H(i)] = \boldsymbol{P}\boldsymbol{P}^H + \sigma^2\boldsymbol{I}. \tag{3}$$

We remark that defining the signatures as *received* signatures accounts for both multipath associated with a small delay spread, and for chip asynchronism. The extension to symbol- and chip-asynchronous CDMA, where the multipath channels may have a large delay spread, is discussed in Section IV.

The input to the decision device corresponding to the ith symbol is

$$\boldsymbol{x}(i) = \boldsymbol{F}^H\boldsymbol{r}(i) - B^H\hat{\boldsymbol{b}}(i) \tag{4}$$

where $\boldsymbol{F}$ is the $N \times K$ feedforward matrix (filter), and $\hat{\boldsymbol{b}}$ is the $K \times 1$ vector of estimated symbols, which are fed back through the $K \times K$ feedback filter $\boldsymbol{B}$. We assume that the spreading code for each user is repeated from symbol to symbol, which enables adaptive estimation of $\boldsymbol{F}$ and $\boldsymbol{B}$. For the S-DFD, the feedback matrix $\boldsymbol{B}$ is strictly upper triangular, whereas for the P-DFD, $\boldsymbol{B}$ is generally full except for zeros along the diagonal. To derive the optimal filters, we will assume perfect feedback, i.e., $\hat{\boldsymbol{b}} = \boldsymbol{b}$.

Let

$$\boldsymbol{e}_{\text{dfd}}(i) \triangleq \boldsymbol{b}(i) - \boldsymbol{x}(i) \tag{5}$$

be the error at the DFD output. The error covariance matrix is then

$$\boldsymbol{\mathcal{E}}_{\text{dfd}} \triangleq E[\boldsymbol{e}_{\text{dfd}}\boldsymbol{e}_{\text{dfd}}^H] \tag{6}$$

and the MSE for user k is

$$[\boldsymbol{\mathcal{E}}_{\text{dfd}}]_{kk} = E\left[|b_k(i) - \boldsymbol{F}_k^H\boldsymbol{r}(i) + \boldsymbol{B}_k^H\boldsymbol{b}(i)|^2\right]. \tag{7}$$

We divide the users into two sets

$$\mathtt{D} = \{j : \hat{b}_j \text{ is fed back}\} \tag{8}$$
$$\mathtt{U} = \{j : j \notin \mathtt{D}\} \tag{9}$$

i.e., "detected" and "undetected" users. In general, these two sets depend on the particular user being detected. We also define the $N \times |\mathtt{D}|$ matrix of spreading sequences for the detected users as $\boldsymbol{P}_D$, and similarly, $\boldsymbol{P}_U$ contains the signatures for $k \in \mathtt{U}$.

Selecting $\boldsymbol{F}_k$ and $\boldsymbol{B}_k$ to minimize $[\boldsymbol{\mathcal{E}}_{\text{dfd}}]_{kk}$ in (7) gives

$$\boldsymbol{F}_k = \boldsymbol{R}_U^{-1}\boldsymbol{p}_k, \qquad \boldsymbol{B}_k = \boldsymbol{P}_D^H\boldsymbol{F}_k \tag{10}$$

where

$$\boldsymbol{R}_U \triangleq \boldsymbol{P}_U\boldsymbol{P}_U^H + \sigma^2\boldsymbol{I} \tag{11}$$
$$= \boldsymbol{R} - \boldsymbol{P}_D\boldsymbol{P}_D^H \tag{12}$$

is the covariance matrix for the undetected users.

The feedforward filter for user k is, therefore, the linear MMSE filter assuming that only users in $\mathtt{U}$ are present. The resulting MMSE is

$$[\boldsymbol{\mathcal{E}}_{\text{dfd}}]_{kk} = 1 - \boldsymbol{p}_k^H\boldsymbol{R}_U^{-1}\boldsymbol{p}_k \tag{13}$$

which is the same form as the MMSE for a linear receiver, except that $\boldsymbol{R}$ is replaced by $\boldsymbol{R}_U$. In the absence of error propagation, interference from all users in set $\mathtt{D}$ is, therefore, eliminated, and user k is affected only by the users in set $\mathtt{U}$. That is, the MMSE DFD cancels interference from the users in set $\mathtt{D}$, while suppressing interference from users in $\mathtt{U}$ in an MMSE sense. This result is analogous to previous results for decision feedback, or data-aided, equalization [12], [13]. Namely, it is shown there that conditioned on perfect feedback, the feedback filter cancels the associated intersymbol interference (ISI), and the feedforward filter suppresses the remaining ISI.

An alternative interpretation for the multiuser DFD filters can be obtained by minimizing $\text{tr}[\boldsymbol{\mathcal{E}}_{\text{dfd}}]$ with respect to $\boldsymbol{F}$ and $\boldsymbol{B}$. This gives

$$\boldsymbol{F} = \boldsymbol{R}^{-1}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{B}). \tag{14}$$

The feedforward filter is, therefore, a concatenation of the linear MMSE filter, $\boldsymbol{F}_{\text{lin}} = \boldsymbol{R}^{-1}\boldsymbol{P}$ [14], [15], and a filter $(\boldsymbol{I}+\boldsymbol{B})$, where $\boldsymbol{B}$ is the feedback matrix, as shown in Fig. 2. Using (4), (5) and (14) the error covariance matrix can be expressed as a function of $\boldsymbol{B}$

$$\boldsymbol{\mathcal{E}}_{\text{dfd}} = (\boldsymbol{I} + \boldsymbol{B})^H\boldsymbol{\mathcal{E}}_{\text{lin}}(\boldsymbol{I} + \boldsymbol{B}) \tag{15}$$

where $\boldsymbol{\mathcal{E}}_{\text{lin}}$ is the error covariance matrix of the linear MMSE filter. That is, let $\boldsymbol{e}_{\text{lin}} = \boldsymbol{b}(i) - \boldsymbol{F}_{\text{lin}}^H\boldsymbol{r}(i)$ be the error at the output of the linear MMSE filter. Then $\boldsymbol{\mathcal{E}}_{\text{lin}} = E[\boldsymbol{e}_{\text{lin}}\boldsymbol{e}_{\text{lin}}^H] = \boldsymbol{I} - \boldsymbol{P}^H\boldsymbol{R}^{-1}\boldsymbol{P}$.

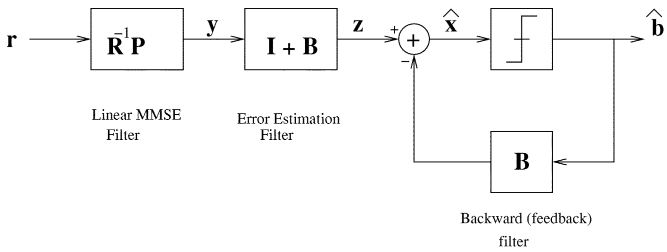

Fig. 2. Block diagram of optimal MMSE multiuser decision-feedback receiver.

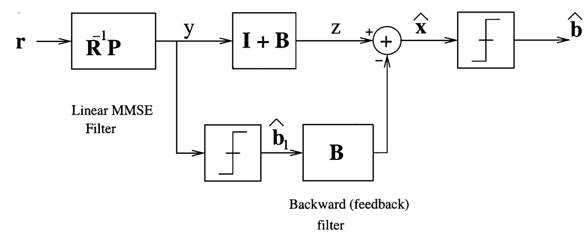

Fig. 3. Example of P-DFD.

The aim is to find the kth column of $\boldsymbol{B}$, denoted as $\boldsymbol{B}_k$, which minimizes $\mathrm{tr}[\boldsymbol{\mathcal{E}}_{\mathrm{dfd}}]$ (equivalently, $[\boldsymbol{\mathcal{E}}_{\mathrm{dfd}}]_{kk}$), for a given set $\mathtt{D}$. Let $\boldsymbol{B}_{k,D}$ be the vector containing only the elements of $\boldsymbol{B}_k$ with indices in $\mathtt{D}$. All other elements of $\boldsymbol{B}_k$ are zero. Minimizing (15) gives

$$\begin{aligned}\boldsymbol{B}_{k,D} =&(\boldsymbol{I}-\boldsymbol{P}_D^H\boldsymbol{R}^{-1}\boldsymbol{P}_D)^{-1}\boldsymbol{P}_D^H\boldsymbol{R}^{-1}\boldsymbol{p}_k \quad (16)\\ =&-[\boldsymbol{\mathcal{E}}_{\mathrm{lin}}]_{D,D}^{-1}[\boldsymbol{\mathcal{E}}_{\mathrm{lin}}]_{k,D} \quad (17)\end{aligned}$$

where $[\boldsymbol{\mathcal{E}}_{\mathrm{lin}}]_{k,D}$ is the k^{th} column of $\boldsymbol{\mathcal{E}}_{\mathrm{lin}}$ taking only rows in set $\mathtt{D}$, and $[\boldsymbol{\mathcal{E}}_{\mathrm{lin}}]_{D,D}$ is the matrix formed from only those rows and columns of $\boldsymbol{\mathcal{E}}_{\mathrm{lin}}$ in $\mathtt{D}$. We remark that the expression for $\boldsymbol{B}_k$ in (10) follows directly from (17) by applying the matrix inversion lemma [16, Sec. 13.2].

The expression (17) can be interpreted as the MMSE estimation filter for the error $\boldsymbol{e}_{\mathrm{lin},k}$ (kth component of $\boldsymbol{e}_{\mathrm{lin}}$) given $\boldsymbol{e}_{\mathrm{lin},D}$ (i.e., $\boldsymbol{e}_{\mathrm{lin},m}$ for $m \in \mathtt{D}$). That is, combining (4) and (14) with (5) gives

$$\boldsymbol{e}_{\mathrm{dfd}} = (\boldsymbol{I}+\boldsymbol{B})^H\boldsymbol{e}_{\mathrm{lin}} \quad (18)$$

so that

$$\boldsymbol{e}_{\mathrm{dfd},k} = \boldsymbol{e}_{\mathrm{lin},k} + \boldsymbol{B}_{k,D}^H\boldsymbol{e}_{\mathrm{lin},D}. \quad (19)$$

Selecting $\boldsymbol{B}_{k,D}$ to minimize $E[|\boldsymbol{e}_{\mathrm{dfd},k}|^2]$ gives (16). Note that the orthogonality principle implies

$$E[\boldsymbol{e}_{\mathrm{dfd},k}\boldsymbol{e}_{\mathrm{lin},m}^*] = 0 \quad (20)$$

where $m \in \mathtt{D}$.

A. Successive Decision Feedback

For the S-DFD, we have for user k

$$\mathtt{D} = \{1,\ldots,k-1\}, \qquad \mathtt{U} = \{k,\ldots,K\}. \quad (21)$$

From (20) and (19), it is easily shown that

$$E[\boldsymbol{e}_{\mathrm{dfd},k}\boldsymbol{e}_{\mathrm{dfd},m}^*] = 0, \qquad k \neq m \quad (22)$$

which from (18) implies that $\boldsymbol{\mathcal{E}}_{\mathrm{dfd}}$ is *diagonal*. That is, $\boldsymbol{I}+\boldsymbol{B}$ can be interpreted as an error whitening filter, which has been observed in [5]. In this case, $\boldsymbol{B}$ can be computed via a Cholesky decomposition.

B. Parallel Decision Feedback

For the P-DFD in a single isolated cell, we have for user k

$$\mathtt{U} = \{k\}, \qquad \mathtt{D} = \{1,\ldots,k-1,k+1,\ldots,K\}. \quad (23)$$

The initial symbol estimates for feedback can be obtained from the output of the linear MMSE component of the feedforward filter as shown in Fig. 3. In this case, we have

$$\boldsymbol{F_k} = \boldsymbol{R}_U^{-1}\boldsymbol{p}_k = \frac{\boldsymbol{p}_k}{|A_k|^2+\sigma^2}. \quad (24)$$

and combining (14) with (24) gives

$$\boldsymbol{I}+\boldsymbol{B} = (\boldsymbol{P}^H\boldsymbol{P}+\sigma^2\boldsymbol{I})(|\boldsymbol{A}|^2+\sigma^2\boldsymbol{I})^{-1} \quad (25)$$

where $\boldsymbol{A}$ is the diagonal matrix of received amplitudes. That is, the feedforward filter is a bank of scaled matched filters, and the off-diagonal components of the feedback matrix are the scaled sequence cross correlations, so that ideal cancellation is achieved with correct feedback estimates. The MMSE P-DFD in a single isolated cell is, therefore, equivalent to the (scaled) conventional parallel interference canceler (IC) [17], [18].

The resulting MMSE is obtained by substituting (12) into (13), which gives

$$[\boldsymbol{\mathcal{E}}_{p-\mathrm{dfd}}]_{kk} = 1-\boldsymbol{p_k}^H(\boldsymbol{p_k}\boldsymbol{p_k}^H+\sigma^2\boldsymbol{I})^{-1}\boldsymbol{p_k} = \frac{\sigma^2}{A_k^2+\sigma^2} \quad (26)$$

which is the single-user bound. (Note that for the conventional IC, the MMSE is σ^2 with ideal feedback, which is slightly higher than for the MMSE P-DFD.)

With multiple cells, we have for user k

$$\begin{aligned}\mathtt{D} =& \{\text{intracell users except } k\}\\ \mathtt{U} =& \{k, \text{other cell users}\} \quad (27)\end{aligned}$$

and from (10), it is apparent that the feedforward filter uses the available degrees of freedom to suppress *only* other-cell interference. This is the main advantage of the MMSE P-DFD relative to conventional IC schemes for which the feedforward matrix is a bank of matched filters.

From (20) and (19), it can be shown that the error covariance matrix for the P-DFD is given by

$$\boldsymbol{\mathcal{E}}_{p\text{-dfd}} = (\boldsymbol{I}-\boldsymbol{B})\boldsymbol{D}_p \quad (28)$$

where $\boldsymbol{D}_p$ is the diagonal matrix with $[\boldsymbol{D}_p]_{k,k} = E[|\boldsymbol{e}_{p-\mathrm{dfd},k}|^2]$. In contrast to the S-DFD, the error covariance matrix for the P-DFD is generally a full matrix, and the matrix $\boldsymbol{I}+\boldsymbol{B}$ no longer has the interpretation of an error whitening filter.

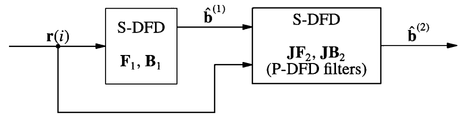

Fig. 4. Two-stage DFD with successive decoding at each stage. The filters in the second stage are permuted P-DFD filters, so that the users are decoded in reverse order, relative to the first stage.

C. Iterative Decision Feedback

In this section, we present iterative DFDs based on both parallel and successive feedback with hard decisions. An iterative P-DFD with hard-decision feedback is defined by the recursion

$$\boldsymbol{x}^{(m+1)}(i) = \boldsymbol{F}^H \boldsymbol{r}(i) - \boldsymbol{B}^H \hat{\boldsymbol{b}}^{(m)}(i) \tag{29}$$

where $\boldsymbol{F}$ and $\boldsymbol{B}$ are the P-DFD filters, and $\hat{\boldsymbol{b}}^{(m)}$ is the vector of tentative decisions from the preceding iteration. For the uncoded P-DFD with hard decisions and binary signaling, we have

$$\hat{\boldsymbol{b}}^{(1)}(i) = \text{sgn}\,\{\boldsymbol{F}_{\text{lin}}^H \boldsymbol{r}(i)\} \tag{30}$$

$$\hat{\boldsymbol{b}}^{(m)}(i) = \text{sgn}\{\boldsymbol{x}^{(m)}(i)\}, \qquad m > 1. \tag{31}$$

Related work on iterative MMSE parallel decision feedback with soft cancellation is presented in [19] and [20].

In general, the effects of error propagation can be mitigated by using successive cancellation and demodulation rather than parallel cancellation. The S-DFD is optimal, in the sense that it achieves the sum capacity of the synchronous CDMA channel with additive white Gaussian noise (AWGN) [7]. However, a disadvantage of the S-DFD relative to the P-DFD for some applications is that it generally does not give uniform performance over the users.

To equalize the performance over the users with successive demodulation, we consider the two-stage IS-DFD shown in Fig. 4. The first stage is an S-DFD with filters $\boldsymbol{F}_1$ and $\boldsymbol{B}_1$. The tentative decisions are passed to the second stage, which consists of a P-DFD with filters $\boldsymbol{F}_2$ and $\boldsymbol{B}_2$. The users in the second stage are decoded *successively*, and in reverse order, relative to the first stage.

The output of the second stage of the IS-DFD is given by

$$x_j^{(2)}(i) = [\boldsymbol{JF}]_j^H \boldsymbol{r}(i) - [\boldsymbol{JB}]_j^H \hat{\boldsymbol{b}}_j^{(2)}(i) \tag{32}$$

where x_j is the jth component of the soft output vector $\boldsymbol{x}$, $\boldsymbol{J}$ is a square permutation matrix with ones along the reverse diagonal and zeros elsewhere, $\boldsymbol{M}_j$ denotes the jth column of the matrix $\boldsymbol{M}$

$$[\hat{\boldsymbol{b}}_j^{(2)}]_\ell(i) = \begin{cases} \hat{b}_j^{(2)}(i), & \text{for } \ell > j \\ \hat{b}_j^{(1)}(i), & \text{for } \ell < j \end{cases} \tag{33}$$

and for uncoded binary signaling

$$\hat{b}_j^{(m)}(i) = \text{sgn}\{x_j^{(m)}(i)\}. \tag{34}$$

Note that a two-stage DFD reaches the single-user bound in the absence of error propagation. Of course, additional stages can be added where the order of the users is reversed from stage to stage. (Other metrics can also be used to reorder the users.) The P-DFD filters $\boldsymbol{F}$ and $\boldsymbol{B}$ remain the same for all stages $m \geq 2$.

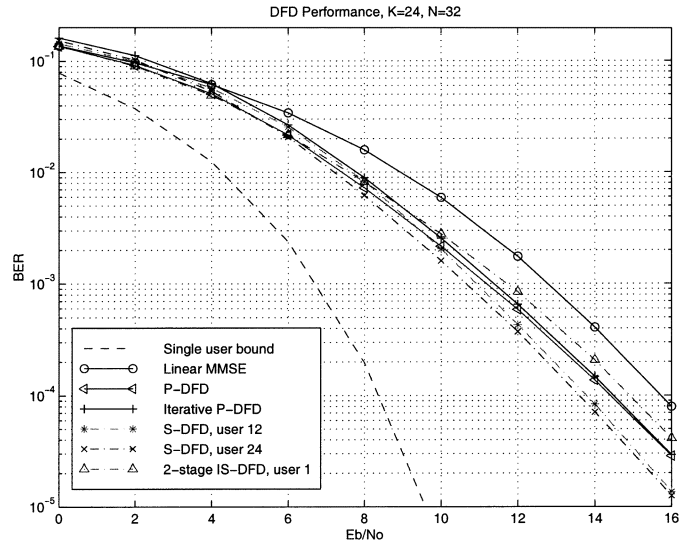

Fig. 5. Performance comparison of different receivers for synchronous CDMA with 24 users and $N = 32$.

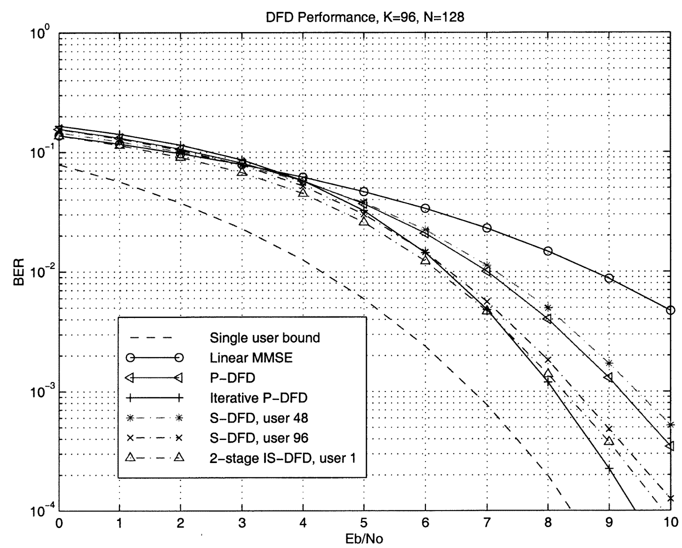

Fig. 6. Performance comparison of different receivers for synchronous CDMA with 96 users and $N = 128$.

D. Numerical Comparison

Figs. 5 and 6 compare the performance (bit error rate (BER) versus E_b/N_0) of the linear MMSE, P-DFD, S-DFD, iterative P-DFD with two additional iterations, and two-stage IS-DFD receivers for a synchronous CDMA system with $(K, N) =$ (24,32), and $(K, N) =$ (96,128), respectively. Because of the high load, the error rate for the matched filter is interference limited, so that the BER remains high independent of E_b/N_0, and is not shown. The results are averaged over randomly assigned spreading sequences (and channels), so that the performance for the linear MMSE and P-DFD receivers is invariant over the users. Two curves are shown for the S-DFD, corresponding to the performance obtained by user $K/2$ and the last user. The performance for the first S-DFD user corresponds to the results for the linear MMSE receiver. For the IS-DFD, results are plotted for the first user. The last user has the same performance as the last S-DFD user.

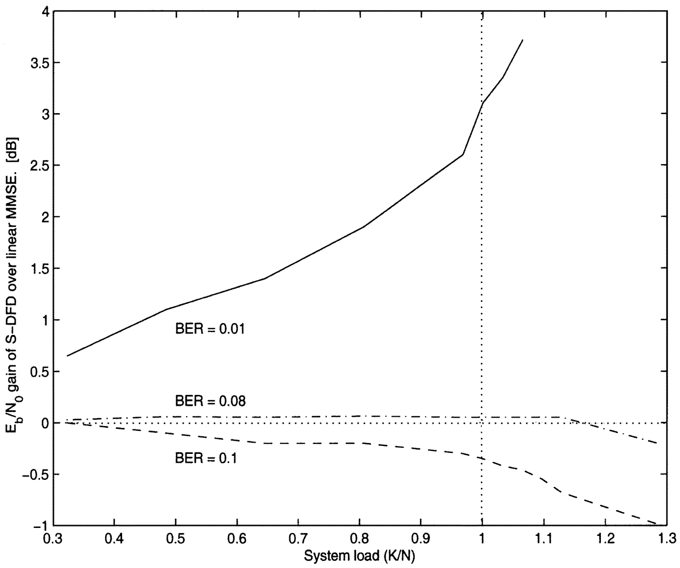

Fig. 7. Gain of the last detected S-DFD user relative to linear MMSE user with target BER as a parameter.

Comparing Figs. 5 and 6 shows that error propagation has a much more deleterious effect on the small system ($N = 32$) than on the large system ($N = 128$). In contrast, for fixed load K/N, the performance of the linear MMSE receiver is an insensitive function of the system size. These results are consistent with the large system analysis reported in [21]. Figs. 5 and 6 show that the iterative P-DFD offers a significant performance improvement relative to the P-DFD for the large system, but performs slightly *worse* than the P-DFD for the small system. For the IS-DFD, user 1 potentially benefits the most from IC, since that user is demodulated last in the second stage. Fig. 6 shows that user 1 does receive the best performance in the large system. However, error propagation in the small system causes the performance for user 1 to be significantly *worse* than that of the last user. Still, the error rate curve for user $K/2 = 12$ (not shown) is essentially the same as that for the last user K, so that performance starts to degrade only for the users demodulated near last.

For the small system, almost all of the gain offered by the S-DFD relative to the linear MMSE receiver is realized for the twelfth user. Namely, at a BER of 10^{-3} the S-DFD shows a 2-dB gain for the twelfth user, and only 0.1-dB additional gain for the last user. For the large system, the IS-DFD offers nearly uniform performance over the user population. Namely, the curves corresponding to the IS-DFD users lie between the IS-DFD curve for the first user and the S-DFD curve for the last user.

These results show that for the small system, the P-DFD offers only a modest performance gain relative to the linear MMSE receiver (approximately 1 dB for BERs between 10^{-3} and 10^{-5}). The gap between the P-DFD and single-user performance, which is due to error propagation, is very large (approximately 6 dB at a BER of 10^{-3}). In contrast, for the large system, this gap closes to approximately 2.5 dB for the P-DFD, and to less than 1.5 dB for the iterative P-DFD.

Fig. 7 shows the gain achieved by the last user detected by the S-DFD, relative to the linear MMSE receiver versus load (K/N). The results assume a synchronous system with $N = 31$ and random sequences. Plots are shown for target BERs of 10^{-1} and 10^{-2}. For a target BER of 10^{-2} the gain increases significantly with load. For the target BER of 10^{-1}, error propagation causes the DFD to perform worse than the linear MMSE detector. Fig. 5 shows that the S-DFD performs better than the linear MMSE detector when the BER is less than approximately 0.08. Fig. 7 includes a plot corresponding to this target BER. The gain is close to zero for loads up to 1.0, indicating that the first user in the DFD must have a BER less than 0.08 to achieve any overall gain.

III. Performance With Coding

We now present simulation results showing the performance of DFDs with coding. Decoding is illustrated in Fig. 8. For the P-DFD, the initial bit estimates are obtained by decoding the outputs $\mathbf{z}(i)$ from the linear MMSE receiver. These estimates are then reencoded and used to cancel interference. The DFD soft outputs $\mathbf{d}(i)$ are then passed through the Viterbi decoder to arrive at the final bit estimates. For the S-DFD, users are decoded successively and reencoded for feedback cancellation. Iterative P- and S-DFD receivers can be defined in analogy with the uncoded receivers. Significant improvements may be obtained from iterative techniques with soft cancellation methods and error control coding [19], [20], [22]–[24].

Fig. 9 shows performance versus user index with a rate 3/4, constraint length 7 convolutional code at $E_b/N_0 = 10$ dB. The results assume a synchronous CDMA system with 20 users and AWGN. The bandwidth expansion is 32, which means that for the coded system, the number of chips per bit is 24. The results are averaged over randomly selected sequences.

Fig. 9 shows that coding actually degrades the performance of the linear and P-DFD receivers. This is due to the reduction in spreading gain (number of chips per bit), which increases the interference power at the output of the linear MMSE filter. (This suggests the use of a higher rate code.) For the S-DFD, the users decoded near last benefit from reliable cancellation of the prior users, and hence, experience a significant improvement relative to all other receivers shown.

In order to exploit successive decoding and cancellation, it is generally beneficial to introduce power disparities in the user population and decode the users in order of decreasing power. This is illustrated in Fig. 10, which shows the performance of DFDs in a system with low- and high-rate users. The high-rate users transmit at twice the rate of the low-rate users, which is achieved by varying the spreading gain. To exploit the benefit offered by feedback cancellation, the high-rate users transmit with twice the power of the low-rate users, and are decoded first. For simplicity, we also assume that the high-rate users are uncoded, and the low-rate users use a rate 1/2, constraint length 7 convolutional code so that the symbol rate is the same for all users. The symbol energy for the high-rate users is, therefore, four times that for the low-rate users.

The results in Fig. 10 assume four low- and high-rate users (total of eight) and a spreading gain of 16. Results are shown for the following receivers: linear MMSE, S-, P-, and IS-DFDs, and iterative P-DFD with three iterations. The S-DFD provides the low-rate users with significantly better performance than

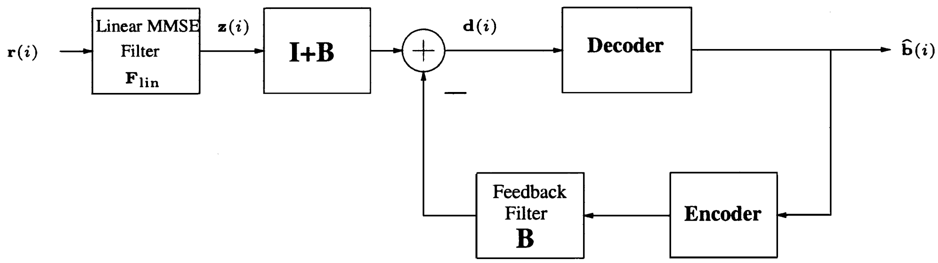

Fig. 8. Combined DFD with Viterbi algorithm for soft decoding.

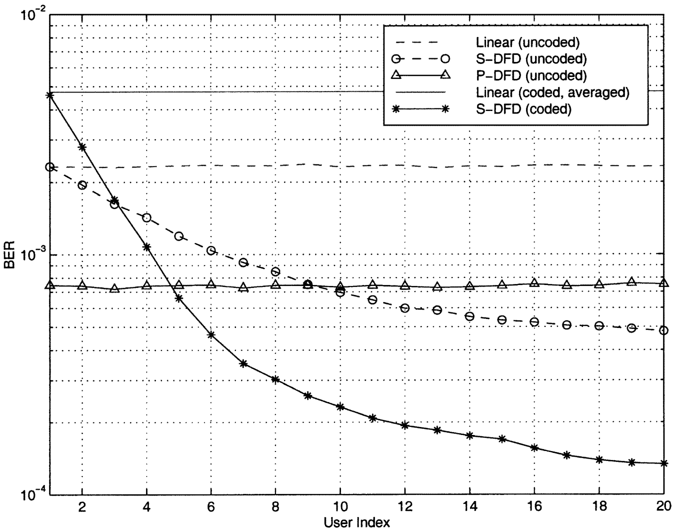

Fig. 9. Receiver performance with rate 3/4 convolutional code.

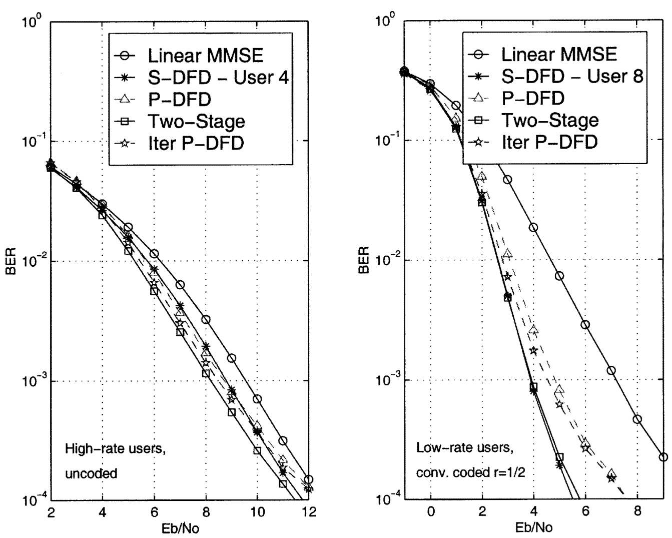

Fig. 10. Receiver performance with low- and high-rate users.

the linear and P-DFD receivers. Specifically, at an error rate of 10^{-3}, the S-DFD gives a 3-dB gain relative to the linear MMSE receiver, and provides nearly a 1-dB gain relative to the P-DFD.

The IS-DFD gives essentially the same performance for all low-rate users as that seen by the last decoded user with the S-DFD. It also offers more than 0.5-dB gain relative to the last decoded high-rate S-DFD user. For the P-DFD and the IS-DFD, the performance is relatively uniform over users in each class. The iterative P-DFD performs only marginally better than the P-DFD for both the high- and low-rate users. This is consistent with the uncoded results for the small system shown in the preceding section.

IV. Asynchronous Users With Multipath

Here, we show how the derivation for the MMSE DFD in the preceding sections can be extended to account for both asynchronous users and ISI due to multipath. For simplicity, we assume that the users are chip synchronous and symbol asynchronous, so that the transmitted symbols are offset by nT_c where $-T/2 < nT_c < T/2$ and T_c is the chip duration. (The MMSE DFD with chip-asynchronous users can be implemented with fractional-chip sampling. The following discussion still applies where the vector of received samples is appropriately modified.) We remark that this section is related to the work presented in [9] on the multiple-input/multiple-output (MIMO) decision-feedback equalizer (DFE). Our approach is somewhat different from the approach taken in [9], since we assume that an arbitrary subset of symbols is fed back for cancellation.

In the absence of multipath, with asynchronous users, the received vector at time i can be written as

$$\boldsymbol{r}(i) = [\,\boldsymbol{P}^{-} \quad \boldsymbol{P}^{0} \quad \boldsymbol{P}^{+}\,] \begin{bmatrix} \boldsymbol{b}(i-1) \\ \boldsymbol{b}(i) \\ \boldsymbol{b}(i+1) \end{bmatrix} + \boldsymbol{n}(i) \qquad (35)$$

where the kth column of $\boldsymbol{P}^{-}$ is the shifted and truncated signature sequence associated with $b_k(i-1)$, and $\boldsymbol{P}^{0}$ and $\boldsymbol{P}^{+}$ are similarly defined. When multipath is present, we can write

$$\boldsymbol{r}(i) = \sum_{m=-L_H}^{L_H} \boldsymbol{H}(m)\boldsymbol{b}(i-m) + \boldsymbol{n}(i) \qquad (36)$$

where the lth column of $\boldsymbol{H}(m)$ represents the multipath component corresponding to $b_l(i-m)$, and the channel impulse response for all users is assumed to span no more than $2L_H+1$ symbols centered around the desired symbol at time i. The matrix $\boldsymbol{H}(m)$ accounts for both the spreading and the channel impulse response.

In general, for asynchronous CDMA (with or without multipath), the MMSE DFD filters are infinite impulse response matrix filters with transfer functions $\boldsymbol{F}(z)$ and $\boldsymbol{B}(z)$. For the S-DFD, it has been shown that $\boldsymbol{F}(z)$ and $\boldsymbol{B}(z)$ can be obtained via a matrix spectral factorization [5]. Here, we consider the case where $\boldsymbol{F}(z)$ and $\boldsymbol{B}(z)$ are finite impulse response filters, as would likely be the case in an adaptive implementation. Furthermore, as before, we consider feeding back an arbitrary set of estimated bits (both across users and in time) to estimate a particular bit $b_k(i)$.

Our objective is to select the feedforward and feedback matrix impulse responses $\{\boldsymbol{F}(n)\}$ and $\{\boldsymbol{B}(n)\}$ to minimize the MSE given by (37) at the bottom of the page, where L_F and L_B determine the lengths of the feedforward and feedback impulse responses, respectively. The solution is obtained by forming the vector of stacked received vectors

$$\bar{\boldsymbol{r}}(i) = \begin{bmatrix} \boldsymbol{r}(i+L_F) \\ \boldsymbol{r}(i+L_F-1) \\ \vdots \\ \boldsymbol{r}(i-L_F+1) \\ \boldsymbol{r}(i-L_F) \end{bmatrix} = \bar{\boldsymbol{P}}\bar{\boldsymbol{b}}(i) + \bar{\boldsymbol{n}}(i) \quad (38)$$

where

$$\bar{\boldsymbol{b}}^H = [\boldsymbol{b}^H(i+L_F+L_H)\ldots \boldsymbol{b}^H(i-L_F-L_H)] \quad (39)$$
$$\bar{\boldsymbol{n}}^H = [\boldsymbol{n}^H(i+L_F)\ldots \boldsymbol{n}^H(i-L_F)] \quad (40)$$

and (41) at the bottom of the page is the $[(2L_F+1)N]\times[(2L_F+2L_H+1)K]$ matrix of "effective" spreading codes.

Selecting the feedforward and feedback filters to minimize the MSE in (37) is, therefore, equivalent to

$$\min_{\mathcal{F}_k,\mathcal{B}_k} \mathcal{E}_k = \min_{\mathcal{F}_k,\mathcal{B}_k} \mathrm{E}\left(\left|b_k(i) - \mathcal{F}_k^H\bar{\boldsymbol{r}}(i) + \mathcal{B}_k^H\bar{\boldsymbol{b}}(i)\right|^2\right), \quad k=1,\ldots,K \quad (42)$$

where

$$\mathcal{F}_k^H = [\,\boldsymbol{F}_k^H(-L_F) \;\; \cdots \;\; \boldsymbol{F}_k^H(L_F)\,] \quad (43)$$
$$\mathcal{B}_k^H = [\,\boldsymbol{B}_k^H(-L_B) \;\; \cdots \;\; \boldsymbol{B}_k^H(L_B)\,] \quad (44)$$

and we have again assumed correct feedback estimates.

This optimization has exactly the same form as that considered for synchronous CDMA, where the received vector $\boldsymbol{r}$ and vector of symbols for feedback are replaced by $\bar{\boldsymbol{r}}$ and $\bar{\boldsymbol{b}}$, respectively. To solve (42), we express the stacked received vector in terms of detected and undetected users as

$$\bar{\boldsymbol{r}}(i) = \bar{\boldsymbol{P}}_D\bar{\boldsymbol{b}}_D(i) + \bar{\boldsymbol{P}}_U\bar{\boldsymbol{b}}_U(i) + \bar{\boldsymbol{n}}(i) \quad (45)$$

where D again represents the set of indices corresponding to symbols which are fed back for a particular user k, and U is the complementary set. The matrices $\bar{\boldsymbol{P}}_D$ and $\bar{\boldsymbol{P}}_U$ consist of the columns of $\bar{\boldsymbol{P}}$ associated with $\bar{\boldsymbol{b}}_D$ and $\bar{\boldsymbol{b}}_U$, respectively.

Solving for the optimal feedforward and feedback filters gives

$$\mathcal{F}_k = \bar{\boldsymbol{R}}_U^{-1}\bar{\boldsymbol{p}}_k \quad (46)$$
$$\mathcal{B}_k = \bar{\boldsymbol{P}}_D^H\mathcal{F}_k \quad (47)$$

where $\bar{\boldsymbol{R}}_U = \bar{\boldsymbol{P}}_U\bar{\boldsymbol{P}}_U^H + \sigma^2\boldsymbol{I}$. That is, the feedforward filter is the optimal linear filter in the absence of interference due to the symbols in $\boldsymbol{b}_D(i)$, and the feedback filter $\mathcal{B}_k$ cancels the interference due to $\boldsymbol{b}_D$.

As in the synchronous CDMA case, we can interpret the feedback filter $\mathcal{B}_k$ as an error estimation filter. Namely, let $\boldsymbol{e}_{\mathrm{lin}}(i) = \boldsymbol{b}(i) - \mathcal{F}_{\mathrm{lin}}^H\bar{\boldsymbol{r}}(i)$ denote the K vector of errors at the output of the linear MMSE filter. That is, the kth column of $\mathcal{F}_{\mathrm{lin}}$ is $\mathcal{F}_{\mathrm{lin},k} = \bar{\boldsymbol{R}}^{-1}\bar{\boldsymbol{p}}_k$ where $\bar{\boldsymbol{p}}_k = E[b_k^*(i)\bar{\boldsymbol{r}}(i)]$, which contains the shifted spreading code for user k padded with zeros. Let $\boldsymbol{B}_{k,D}(m)$ denote the vector containing components from the kth column of $\boldsymbol{B}(m)$ in D. Then, in analogy with (19), the DFD error for user k is given by

$$\boldsymbol{e}_{\mathrm{dfd},k}(i) = \boldsymbol{e}_{\mathrm{lin},k}(i) - \sum_{m=-L_B}^{L_B} \boldsymbol{B}_{k,D}^H(m)\boldsymbol{e}_{\mathrm{lin},D}(i-m). \quad (48)$$

Minimizing $E[|\boldsymbol{e}_{\mathrm{dfd},k}|^2]$ is, therefore, equivalent to finding the sequence $\{\boldsymbol{B}_{k,D}(m)\}$, $m=-L_B,\ldots,L_B$, which minimizes the mean-squared estimation error for $\boldsymbol{e}_{\mathrm{lin},k}(i)$ given $\boldsymbol{e}_{\mathrm{lin},D}(i-L_B),\ldots,\boldsymbol{e}_{\mathrm{lin},D}(i+L_B)$. This latter interpretation is useful for the least-squares adaptive DFD presented in [4].

V. Conclusion

Multiuser DFDs based on the MMSE performance criterion have been presented. Because of the short code assumption, remodulation is not needed. A P-DFD and a two-stage IS-DFD were presented, both of which achieve the single-user bound for a single isolated cell in the absence of error propagation. Extensions of the DFD structures to asynchronous CDMA with multipath were also presented.

Numerical results show that for small systems, without coding the DFDs offer a modest, but significant performance improvement relative to the linear MMSE receiver at high loads. Error propagation causes a very large penalty in E_b/N_0 relative to the single-user bound. The P-DFD with randomly assigned spreading codes also achieves a more uniform perfor-

$$\mathcal{E} = \mathrm{E}\left(\left\|\boldsymbol{b}(i) - \sum_{n=-L_F}^{L_F}\boldsymbol{F}^H(n)\boldsymbol{r}(i+n) + \sum_{n=-L_B-1}^{L_B+1}\boldsymbol{B}^H(n)\hat{\boldsymbol{b}}(i+n)\right\|^2\right) \quad (37)$$

$$\bar{\boldsymbol{P}} = \begin{bmatrix} \boldsymbol{H}(-L_F) & \ldots & \boldsymbol{H}(L_F) & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}(-L_F) & \ldots & \boldsymbol{H}(L_F) & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \vdots & & & & & & \vdots \\ \boldsymbol{0} & \ldots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}(-L_F) & \ldots & \boldsymbol{H}(L_F) \end{bmatrix} \quad (41)$$

mance distribution over the user population relative to the linear MMSE receiver. For a fixed load (K/N), the performance improvement offered by the DFDs relative to the linear MMSE receiver increases significantly with system size.

Iterative DFDs using hard decisions were also presented. The IS-DFD mitigates the effects of error propagation and offers a significant performance improvement relative to the (single-stage) P- and S-DFDs. An example with mixed-rate users demonstrates that the S-DFD with coding can offer a substantial improvement relative to the linear MMSE receiver. As with linear MMSE receivers, the MMSE DFD coefficients can be estimated given training sequences for each user (see [4] and [25]). The combination of these adaptive techniques with iterative soft cancellation is currently being studied [24].

Acknowledgment

G. Woodward acknowledges the support of Prof. B. Vucetic at the University of Sydney, Australia.

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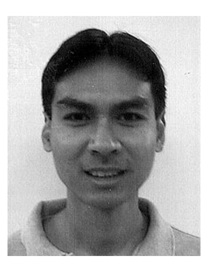
Graeme Woodward (S'94–M'99) received the B.Sc., B.E. (with honors and University Medal), and Ph.D. degrees from The University of Sydney, Sydney, Australia, in 1990, 1992, and 1999, respectively.

From 1991 to 1994, he was with Alcatel Australia in digital switching systems. During his Ph.D. studies, he was a visiting researcher at Yokohama National University, Yokohama, Japan, during 1996, and at Northwestern University, Evanston, IL, during 1998. During 1999–2000, he was with Southern-Poro Communications, Sydney, Australia, in channel modeling, 3G cellular systems, and multiuser detection for CDMA, in collaboration with The University of Oulu, Finland. In 2001, he joined Bell Laboratories Research, Lucent Technologies, Sydney, Australia, researching 3G systems, multiuser detection, interference suppression, and adaptive digital filter algorithms for equalization.



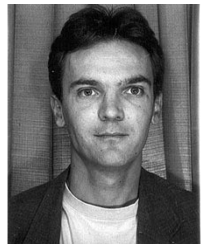
Rapeepat Ratasuk (S'95–M'02) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Northwestern University, Evanston, IL, in 1994, 1996, and 2000, respectively.

Since 1999, he has been with Motorola, Inc., Arlington Heights, IL, as a Systems Engineer in the Network Advanced Technologies Group. His current research interests are in the design and performance of next-generation wireless communication systems.

Michael L. Honig (S'80–M'81–SM'92–F'97) received the B.S. degree in electrical engineering from Stanford University, Stanford, CA, in 1977, and the M.S. and Ph.D. degrees in electrical engineering form the University of California, Berkeley, in 1978 and 1981, respectively.

He subsequently joined Bell Laboratories, Holmdel, NJ, where he worked on local area networks and voiceband data transmission. In 1983, he joined the Systems Principles Research Division at Bellcore, where he worked on digital subscriber lines and wireless communications. He was a Visiting Lecturer at Princeton University, Princeton, NJ, during fall 1993. Since fall 1994, he has been with Northwestern University, Evanston, IL, where he is a Professor with the Electrical and Computer Engineering Department.

Dr. Honig was an Associate Editor for the IEEE TRANSACTIONS ON INFORMATION THEORY and the IEEE TRANSACTIONS ON COMMUNICATIONS, and was a Guest Editor for the *European Transactions on Telecommunications* and *Wireless Personal Communications*. He was also a member of the Digital Signal Processing Technical Committee for the IEEE Signal Processing Society. He is a member of the Board of Governors for the IEEE Information Theory Society.



Predrag B. Rapajic (M'89–SM'99) received the B.E. degree from the University of Banja Luka, Yugoslavia, in 1982, and the M.E. degree from The University of Belgrade, Belgrade, Yugoslavia, in 1988. He received the Ph.D. degree from The University of Sydney, Sydney, Australia in 1994.

In 1996, he was appointed Head of the Communications Group, Motorola Australian Research Center. Since 2000, he has been a Senior Lecturer at The University of New South Wales, Sydney, Australia. His research interests include: adaptive multiuser detection, equalization, error control coding, mobile communication systems and multiuser information theory.