Signature Sequence Adaptation for DS-CDMA With Multipath

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***Abstract*—Joint transmitter-receiver adaptation is studied for the reverse link of a direct sequence-code division multiple access system with short signature sequences. The signature for a particular user is computed at the receiver and transmitted back to the transmitter via a feedback channel. A reduced-rank transmitter adaptation scheme is presented in which the signature is constrained to lie in a lower dimensional subspace. This allows a tradeoff between system performance and the number of estimated parameters. Analytical and simulation results show that adaptation of relatively few transmitter coefficients can lead to significant performance improvements. Adaptive algorithms are derived for estimating the transmitter coefficients in the presence of multipath. We consider both *collective* optimization, in which the users adapt together to improve a global system performance criterion, and *individual* optimization, in which the signature for a particular user is adapted to optimize individual performance. Numerical results are presented, which show that both individual and collective joint transmitter-receiver adaptation can effectively preequalize the channel and avoid interference at high loads.**

***Index Terms*—Code division multiple access, interference avoidance, transmitter optimization.**

I. Introduction

THE USE OF SHORT, or repeated signature sequences in a direct sequence (DS)-code division multiple access (CDMA) system enables the use of adaptive techniques for suppression of multiple access interference (MAI) [1]. In addition, short codes also enable the possibility of selecting a user signature sequence to *avoid* interference [2]–[7], [20], [21]. Here, we present and evaluate the performance of adaptive transmitter-receiver algorithms for reverse link CDMA in the presence of multipath.

In general, adaptation of a user signature sequence in the presence of multipath serves two purposes: preequalization of the channel and interference avoidance. There has been increasing interest in preprocessing techniques for DS-CDMA systems, partly motivated by the possibility of shifting the bulk of processing from the mobile to the base station on the forward link. For example, "pre-Rake" and waveform design schemes have been proposed to compensate for the effect of the channel prior to transmission in [8]–[10]. Transmitter precoding for the forward link is presented in [11].

Additional performance enhancements may be possible by combining transmitter preprocessing with adaptive receivers. Joint optimization of a user signature sequence with a linear adaptive receiver was presented in [2]. No multipath is assumed in that work, so that the transmitter signature sequence is matched to the receiver filter. It is shown that continuous adaptation achieves single-user performance for the loads considered. Joint transmitter-receiver optimization for forward link CDMA in the presence of multipath, based on the minimum mean square error (MMSE) performance criterion, is described in [3]. Selection of an ensemble of signature sequences that minimize total interference power with matched filter (MF) receivers is considered in [5] and [6]. The design of signature waveforms to optimize bandwidth efficiency is studied in [12]. Earlier work on joint MMSE transmitter-receiver optimization for the multiple access channel with linear dispersive channels is presented in [13]. Other related work on siggnature optimization for CDMA is presented in [20]–[23]

A drawback associated with transmitter adaptation in general is the feedback bandwidth required, which increases with the number of transmitter coefficients to be estimated. We present a "reduced-rank" transmitter adaptation scheme, in which each signature sequence is constrained to lie in a lower dimensional subspace, spanned by some orthogonal basis. The weights for the basis are then selected to optimize the performance criterion, namely output signal-to-interference-plus noise ratio (SINR). Different orthogonal bases are assigned to different users. Selection of the subspace dimension allows a tradeoff between the number of parameters to be estimated and steady-state performance. A subspace dimension of one corresponds to conventional power control. Numerical results presented for a synchronous CDMA system show that relatively few combining coefficients can provide a substantial improvement in performance.

Algorithms for group transmitter-receiver adaptation in the presence of multipath are presented for different scenarios. Specifically, we distinguish between *individual* and *collective* adaptation. Individual adaptation refers to the scenario in which each user adapts to optimize his/her own performance without regard to the performance of other users in the system. This is motivated by a peer-to-peer network where a receiver may not have access to parameters (i.e., channels and receivers filters) for other users. Collective adaptation refers to the scenario in which each user adapts to optimize an overall system objective function (e.g., mean square error (MSE) summed over all users). This is more appropriate for the reverse link of a cellular

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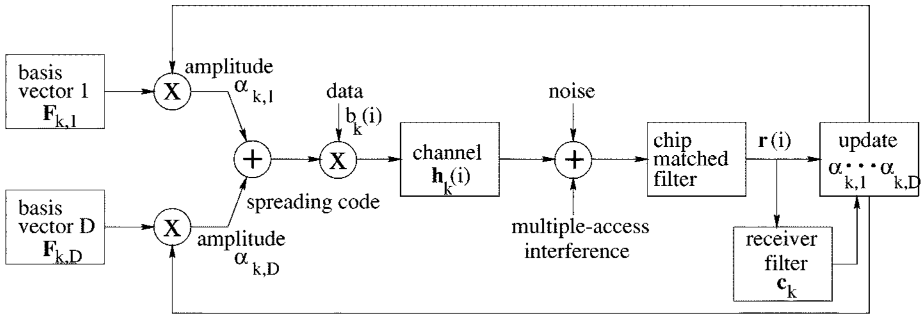

Fig. 1. Discrete-time baseband model for joint transmitter-receiver adaptation.

system. For ideal channels without multipath, it has been shown in [5] and [6] that group adaptation with individual, or local MSE cost functions also optimizes a collective, or global cost function (namely, sum MSE). Here, we show that individual and collective adaptation generally do not give the same performance with multipath, although numerical results show that individual adaptation performs nearly as well as collective adaptation for the cases examined.

Numerical results are presented, which illustrate the convergence performance of adaptive transmitter-receiver algorithms as a function of the estimation window size (i.e., number of received observations). Both single-user adaptation, in which the interferers do not adapt, and group adaptation, in which all users adapt, are considered. These results show that the individual algorithms converge more slowly than the collective algorithms. Also, adaptive receivers based on a least squares performance criterion generally perform worse than the nonadaptive Rake, or MF receiver, since the adaptive receiver introduces tracking error when the signatures are time-varying.

The reverse link CDMA model is presented in Section II. Algorithms for individual and collective adaptation are considered in Sections III and IV, respectively. Large system performance results for reduced-rank transmitter adaptation in the absence of multipath are presented in Section V. Numerical results illustrating the performance of the proposed schemes in the presence of multipath are presented in Section VI.

II. System Model

We assume a synchronous DS-CDMA system with K users and processing gain N. The received vector is given by

$$\mathbf{r}(i) = \sum_{k=1}^{K} A_k[\mathbf{H}_k^+(i)b_k(i)+\mathbf{H}_k^-(i-1)b_k(i-1)]\mathbf{p}_k+\mathbf{n}(i) \quad (1)$$

where $b_k(i)$ is the ith symbol transmitted with $E[|b_k(i)|^2] = 1, \mathbf{p}_k$ is the $N \times 1$ signature, and A_k is the amplitude, all for user k. The matrices $\mathbf{H}_k^{\pm}$ represent the channel for user k, to be described, and $\mathbf{n}(i)$ is the white Gaussian noise vector with covariance matrix $\sigma_n^2\mathbf{I}$.

The spread symbols for user k are passed through the discrete-time channel with impulse response given by the $N \times 1$ vector

$$\mathbf{h}_k = [h_{k,1} \quad h_{k,2} \quad \cdots \quad h_{k,L_k} \quad 0 \quad \cdots \quad 0]^T \quad (2)$$

where L_k represents the number of paths, assumed to be spaced at the chip duration $T_c = T/N$, where T is the symbol duration and $L_k < N$. The channel matrices in (1) are then

$$\begin{aligned}\mathbf{H}_k^+ &= \left[\mathbf{h}_k \quad \mathbf{h}_k^{+1} \quad \mathbf{h}_k^{+2} \quad \cdots \quad \mathbf{h}_k^{+(N-1)}\right]\\ \mathbf{H}_k^- &= \left[\mathbf{h}_k^{-N} \quad \mathbf{h}_k^{-(N-1)} \quad \cdots \quad \mathbf{h}_k^{-2} \quad \mathbf{h}_k^{-1}\right]\end{aligned} \quad (3)$$

where $\mathbf{h}_k^{\pm n}$ is $\mathbf{h}_k$ shifted down $(+)$ or up $(-)$ by n positions, and the vacant positions are filled with zeros. $\mathbf{H}_k^+$ represents the contribution from symbols $b_k(i), k = 1, \ldots, K$, and $\mathbf{H}_k^-$ represents the intersymbol interference (ISI) from symbols $b_k(i-1), k = 1, \ldots, K$. Both channel matrices are $(N \times N)$, and $\mathbf{H}_k^-$ is sparse if $L_k \ll N$.

In what follows, we will assume that the channel delay spreads are small compared with the symbol duration and, hence, neglect ISI (i.e., $\mathbf{H}_k^- = \mathbf{0}$) in order to simplify the model. We, therefore, use $\mathbf{H}_k$ to denote the channel matrix for user k. The necessary conditions for the optimal signatures presented here can be extended to asynchronous CDMA with ISI by expanding the observation window for the received signal. This complicates the derivations while adding little insight to the synchronous case, so that only synchronous CDMA with negligible ISI is considered throughout the paper. Related work on group signature optimization for asynchronous CDMA with ideal channels is presented in [14]. It is shown there that when the signatures are selected optimally, the MMSE receiver is an MF, which spans only a single symbol interval.

Fig. 1 shows a block diagram of a single-user communications system with joint transmitter-receiver adaptation. The received signal from (1) is the input to a linear filter $\mathbf{c}_k$. We consider two types of filters: 1) a coherent Rake filter (i.e., maximum ratio combiner) or MF given by

$$\mathbf{c}_k = \frac{\mathbf{H}_k\mathbf{p}_k}{||\mathbf{H}_k\mathbf{p}_k||} \quad (4)$$

and 2) an MMSE filter given by

$$\mathbf{c}_k = \mathbf{R}^{-1}\mathbf{H}_k\mathbf{p}_k \quad (5)$$

where

$$\mathbf{R} = \sum_{k=1}^{K} \mathbf{H}_k\mathbf{p}_k\mathbf{p}_k^H\mathbf{H}_k^H + \sigma_n^2\mathbf{I} \quad (6)$$

is the received covariance matrix.

Transmitter optimization or *adaptation* for user k refers to selecting $\mathbf{p}_k$ to optimize a cost function such as MSE or SINR at the output of $\mathbf{c}_k$. More generally, a *group* of users may adapt to optimize a *collective* performance criterion, such as sum MSE over all users. We will consider "reduced-rank approximations" to the optimal $\mathbf{p}_k$ [15] by constraining $\mathbf{p}_k$ to lie in a D-dimensional space where $D < N$. That is

$$\mathbf{p}_k = \mathbf{F}_k \alpha_k \tag{7}$$

where $\mathbf{F}_k$ is an $N \times D$ matrix whose D columns are the basis vectors for the k_{th} user, and the elements of the $D \times 1$ vector α_k are the combining coefficients.

Varying the subspace dimension D allows a tradeoff between system performance and the number of adaptive parameters at the transmitter. Specifically, selecting $D = N$ is equivalent to optimizing $\mathbf{p}_k$ directly (i.e., taking $\mathbf{F}_k = \mathbf{I}$), assuming that the given $\mathbf{F}_k$ is full-rank. Selecting $D = 1$ corresponds to conventional power control. As D increases from one to N, the performance improves in a stationary environment, but more information is required at the transmitter, which further constrains the mobile speeds for which transmitter adaptation is beneficial.

We will assume that the set of basis vectors $\mathbf{F}_k$ are random, and that the random elements are selected from a binary distribution. For example, one possibility is to choose the elements of $\mathbf{F}_k$ independently, so that $\mathbf{p}_k$ is a linear combination of D independent random signatures. For purposes of computing the optimal combining vector, it is convenient for the columns of $\mathbf{F}_k$ to be orthogonal. This can be easily accomplished by selecting a single random signature sequence $\mathbf{F}_{k;1}$, the first column of $\mathbf{F}_k$, and generating successive columns of $\mathbf{F}_k$ by masking with different orthogonal sequences. For example, consider a system with $N = 4, K = 2$ and subspace dimension $D = 2$. We might choose

$$\mathbf{F}_1 = \begin{bmatrix} -0.5 & 0 \\ 0.5 & 0 \\ 0 & 0.5 \\ 0 & 0.5 \end{bmatrix} \quad \mathbf{F}_2 = \begin{bmatrix} -0.5 & 0 \\ -0.5 & 0 \\ 0 & 0.5 \\ 0 & -0.5 \end{bmatrix} \tag{8}$$

so that the columns of $\mathbf{F}_k, k = 1, 2$ are orthogonal. In this case, the assigned signature sequence for each user $(k = 1, 2)$ is divided into two sections, which are independently weighted by the components of α_k. For general N and D, where N/D is an integer, we can choose the columns of $\mathbf{F}_k$ to be nonoverlapping segments containing N/D elements of an assigned random sequence.

III. Individual Optimization

In this section and the next, we present algorithms for joint transmitter-receiver optimization. "Adaptive" means that the transmitter and receiver parameters can be estimated directly from the received data. In what follows, we will assume that the channel coefficients are known, since these can be estimated from a pilot signal.

We first consider individual optimization, in which each user attempts to optimize individual performance, subject to a constraint on the transmitted power, with no regard to the performance of the other users in the system. This is a suitable optimization model for a peer-to-peer network, in which the receiver typically does not possess information about the interferers.

A. Alternating Updates

We first optimize the signature sequence for user k, assuming that the receiver is fixed. Joint transmitter-receiver optimization is then achieved by alternating the transmitter update with an update for the receiver filter. Given the energy constraint $\|\mathbf{p}_k\|^2 \le \Pi_k$, the cost function is

$$J_k = E[|b_k(i) - d_k(i)|^2] + \lambda_k(\|\mathbf{F}_k\alpha_k\|^2 - \Pi_k) \tag{9}$$

where

$$d_k(i) = \mathbf{c}_k^H \left(\sum_{j=1}^{K} A_j(i) b_j(i) \mathbf{H}_j \mathbf{F}_j \alpha_j + \mathbf{n}(i) \right) \tag{10}$$

is the soft decision at the output of the k_{th} user receiver and λ_k is a Lagrange multiplier. Setting $\nabla_{\alpha_k} J_k(\alpha_k) = \mathbf{0}$ gives the necessary condition for optimality

$$\alpha_k = \left(|A_k|^2 \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{c}_k \mathbf{c}_k^H \mathbf{H}_k \mathbf{F}_k + \lambda_k \mathbf{F}_k^H \mathbf{F}_k \right)^{-1} A_k^* \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{c}_k \tag{11}$$

$$= \sqrt{\frac{\Pi_k}{\beta_k |A_k|^2}} A_k^* \left(\mathbf{F}_k^H \mathbf{F}_k \right)^{-1} \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{c}_k \tag{12}$$

where $\beta_k = \mathbf{c}_k^H \mathbf{H}_k \mathbf{F}_k (\mathbf{F}_k^H \mathbf{F}_k)^{-1} \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{c}_k$, the last equality follows from the matrix inversion lemma, and the Lagrange multiplier is selected to satisfy the energy constraint. The full-rank solution (i.e., $D = N$) is

$$\mathbf{p}_k = \beta_k' \mathbf{H}_k^H \mathbf{c}_k \tag{13}$$

where β_k' is a scalar. In this case, the reduced-rank solution $\mathbf{F}_k \alpha_k$ is simply the projection of the full-rank solution onto the space spanned by the columns of $\mathbf{F}_k$. Joint transmitter-receiver optimization can be accomplished by successively applying the preceding expression with the MMSE update for the receiver filter (5) until convergence of the cost function (desired user MSE) is achieved.

We observe that individual, alternating adaptation with the MF does not avoid interference. Specifically, for $\mathbf{H}_k = \mathbf{I}$, the transmitter update (12) becomes $\mathbf{p}_k = \mathbf{c}_k$. That is, the transmitter is matched to the receiver, which gives a fixed point for any initial receiver filter $\mathbf{c}_k$. In contrast, selecting $\mathbf{c}_k$ according to (5) and combining with (13) gives

$$\mathbf{p}_k = \nu \mathbf{R}^{-1} \mathbf{p}_k \tag{14}$$

where ν is a constant. To minimize the MSE, $\mathbf{p}_k$ should be the eigenvector of $\mathbf{R}$ corresponding to the *minimum* eigenvalue ν. (See, also, [5] and [6], where the same condition is derived.)

Substituting (12) into (9) yields

$$J_k = 1 + \sigma_n^2 \mathbf{c}_k^H \mathbf{c}_k - A_k \mathbf{c}_k^H \mathbf{H}_k \mathbf{p}_k + \mathbf{c}_k^H \left(\sum_{m \ne k} \mathbf{H}_m \mathbf{p}_m \mathbf{p}_m^H \mathbf{H}_m^H \right) \mathbf{c}_k - \lambda_k \Pi_k. \tag{15}$$

The term $\mathbf{c}_k^H (\sum_{m \ne k} \mathbf{H}_m \mathbf{p}_m \mathbf{p}_m^H \mathbf{H}_m^H) \mathbf{c}_k$ is MAI, which is independent of $\mathbf{p}_k$. Hence, we conclude that with fixed receiver filters, individual transmitter adaptation does not avoid the multiuser interference, unlike collective adaptation to be described in Section IV.

B. Joint Optimization: MMSE Receiver

Rather than alternate between transmitter and receiver optimization, here we jointly optimize the signature sequence for user k with the MMSE filter (5). Let

$$\gamma_k = \frac{\left|\mathbf{c}_k^H \mathbf{H}_k \mathbf{F}_k \alpha_k\right|^2}{\mathbf{c}_k^H \mathbf{R}_k \mathbf{c}_k} \tag{16}$$

be the received SINR where $\mathbf{R}_k$ is the interference plus noise covariance matrix for user k given by

$$\mathbf{R}_k = \sum_{j \neq k}^{K} \mathbf{H}_j \mathbf{p}_j \mathbf{p}_j^H \mathbf{H}_j^H + \sigma_n^2 \mathbf{I}. \tag{17}$$

We wish to select α_k to maximize

$$J_k = \gamma_k + \lambda_k (\|\mathbf{F}_k \alpha_k\|^2 - \Pi_k). \tag{18}$$

It is easily shown that maximizing the SINR is equivalent to minimizing the MSE. That is, both criteria give the same optimized signature. Here, we choose SINR since the associated derivation is somewhat simpler. Applying the matrix inversion lemma to $\mathbf{R}^{-1}$ gives

$$\begin{aligned}\mathbf{R}^{-1} &= \left(\mathbf{R}_k + \mathbf{H}_k \mathbf{F}_k \alpha_k \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H\right)^{-1} \\ &= \mathbf{R}_k^{-1} - \frac{\mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{F}_k \alpha_k \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1}}{1 + \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{F}_k \alpha_k}\end{aligned} \tag{19}$$

and substituting (19) in (18) gives

$$J_k = \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{F}_k \alpha_k + \lambda_k (\|\mathbf{F}_k \alpha_k\|^2 - \Pi_k). \tag{20}$$

Maximizing with respect to α_k yields the following necessary condition:

$$\left\{\left(\mathbf{F}_k^H \mathbf{F}_k\right)^{-1} \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{F}_k\right\} \alpha_k = \nu \alpha_k. \tag{21}$$

That is, the optimal α_k is the eigenvector of $(\mathbf{F}_k^H \mathbf{F}_k)^{-1} \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{F}_k$ that maximizes the SINR γ_k. Substituting (21) into (20) reveals that the desired eigenvector corresponds to the maximum eigenvalue. The full-rank solution $(D = N)$ satisfies the necessary condition

$$\left\{\mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k\right\} \mathbf{p}_k = \nu \mathbf{p}_k. \tag{22}$$

In the absence of multipath, we have $\mathbf{H}_k = \mathbf{I}$, and (22) implies that $\mathbf{p}_k$ is the eigenvector corresponding to the maximum eigenvalue of $\mathbf{R}_k^{-1}$. The optimal signature sequence, therefore, lies in the subspace containing the least interference plus noise. In a single-user system with multipath, we have $\mathbf{R}_k = \sigma_n^2 \mathbf{I}$, so that $\mathbf{p}_k$ is chosen as the eigenvector corresponding to the maximum eigenvalue of $\mathbf{H}_k^H \mathbf{H}_k$. This amounts to aligning the user's transmissions along the strongest channel component. In general, in the presence of both multipath and multiuser interference, the condition (22) optimizes the tradeoff between exploiting the strongest channel components and avoiding interference.

We observe that a different, but equivalent, form for the necessary condition for optimality is obtained by substituting the MMSE receiver filter, given by (5), into the MMSE transmitter condition (12). Namely

$$\left\{\left(\mathbf{F}_k^H \mathbf{F}_k\right)^{-1} \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}^{-1} \mathbf{H}_k \mathbf{F}_k\right\} \alpha_k = \nu \alpha_k \tag{23}$$

where ν is again the maximum eigenvalue of the associated matrix. The condition (23) is the same as (22) except that $\mathbf{R}_k$ in (22) is replaced by $\mathbf{R}$. The equivalence between (22) and (23) can be established directly by applying the matrix inversion lemma (19)–(23). Since $\mathbf{R}$ depends on $\mathbf{p}_k$, the condition (22) is more convenient to solve in practice.

The condition (23) shows that when the signature is optimized in the presence of multipath, the MMSE receiver is generally *not* the same as the MF. Furthermore, when $\mathbf{H}_k = \mathbf{I}$, (23) states that the MF is the projection of the MMSE receiver onto the space spanned by the columns of $\mathbf{F}_k$.

The optimal signature $\mathbf{p}_k$ can be adaptively estimated from (22) by replacing $\mathbf{H}_k$ by the corresponding channel estimate $\hat{\mathbf{H}}_k$, and by estimating $\mathbf{R}_k$ as

$$\hat{\mathbf{R}}_k = \frac{1}{n}\left(\sum_{i=1}^{n} \mathbf{r}(i)\mathbf{r}(i)^H\right) - |A_k|^2 \hat{\mathbf{H}}_k \mathbf{p}_k \mathbf{p}_k^\dagger \hat{\mathbf{H}}_k^\dagger. \tag{24}$$

C. Joint Optimization: MF Receiver

Here, we consider joint optimization of the desired user spreading code with the MF receiver given by (4). Although joint optimization with the MMSE receiver should have better steady-state performance, joint optimization with the MF receiver is observed to provide superior transient performance in an adaptive mode. Taking the gradient of the cost function J_k in (18) with respect to α_k gives the necessary condition

$$\begin{aligned}&\left(\mathbf{F}_k^H \mathbf{F}_k\right)^{-1} \mathbf{F}_k^H \mathbf{H}_k^H \\ &\left[\left(\frac{2}{\alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{F}_k \alpha_k}\right)\mathbf{I} - \left(\frac{1}{\alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}_k \mathbf{H}_k \mathbf{F}_k \alpha_k}\right)\mathbf{R}_k\right] \\ &\qquad \mathbf{H}_k \mathbf{F}_k \alpha_k = \nu \alpha_k\end{aligned} \tag{25}$$

where $\nu = -\lambda_k / \gamma_k$. For $D = N$ this reduces to

$$\begin{aligned}&\mathbf{H}_k^H \left[\left(\frac{2}{\|\mathbf{H}_k \mathbf{p}_k\|^2}\right)\mathbf{I} - \left(\frac{1}{\mathbf{p}_k^H \mathbf{H}_k^H \mathbf{R}_k \mathbf{H}_k \mathbf{p}_k}\right)\mathbf{R}_k\right] \\ &\qquad \mathbf{H}_k \mathbf{p}_k = \nu \mathbf{p}_k\end{aligned} \tag{26}$$

and $\nu = \mathbf{p}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{p}_k$. The optimal α_k is, therefore, an eigenvector of a matrix

$$\mathbf{A} = \left(\mathbf{F}_k^H \mathbf{F}_k\right)^{-1} \mathbf{F}_k^H \mathbf{H}_k^H (a\mathbf{I} - b\mathbf{R}_k) \mathbf{H}_k \mathbf{F}_k \tag{27}$$

where the scalars a, b (and the associated eigenvalue) depend on α_k.

For a single-user system, $\mathbf{R}_k = \sigma_n^2 \mathbf{I}$, and $\mathbf{A} = a_k (\mathbf{F}_k^H \mathbf{F}_k)^{-1} \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{F}_k$ where a_k is a scale factor, and it is easily verified that the optimal choice for α_k is the eigenvector corresponding to the maximum eigenvalue of $\mathbf{A}$. If the channel is ideal, i.e., $\mathbf{H}_k = \mathbf{I}$, and there are interferers present, then $\mathbf{A} = (\mathbf{F}_k^H \mathbf{F}_k)^{-1} \mathbf{F}_k^H (2\mathbf{I} - a\mathbf{R}_k)\mathbf{F}_k$, and it is easily verified that the optimal α_k is the eigenvector corresponding to the minimum eigenvalue of $\mathbf{R}_k$ projected onto the space spanned by the columns of $\mathbf{F}_k$.

Solving (25) for general $\mathbf{H}_k$ and $\mathbf{R}_k$ does not appear to be straightforward. In addition, there can be multiple solutions, corresponding to local optima of the objective function J_k. An iterative algorithm for solving (25) is given as follows. 1) Select a random initialization for $\alpha_k^{(0)} = \alpha_k$. 2) Compute the matrix $\mathbf{A}$. 3) Select $\alpha_k^{(n+1)}$ as the eigenvector of $\mathbf{A}$ that maximizes the SINR γ_k. 4) Iterate steps 2) and 3) until convergence. 5) Repeat 1)–4) for different random initializations $\alpha_k^{(0)}$. We have observed that occasionally the iteration in step 4) does not converge. However, numerical results indicate that steps 1)–4) typically computes a $\mathbf{p}_k$, which achieves near single-user performance. We remark that the eigenvector in step 3) does not necessarily correspond to the maximum or minimum eigenvalue.

IV. Collective Optimization

We now consider collective optimization of signatures, in which a single user or group of users adapts to improve a global performance criterion, namely, MSE summed over all users. Collective optimization penalizes any additional interference to other users as a result of the change in signature sequence, in contrast to individual optimization. Here, we assume that the receiver for each user has knowledge of the channels and receivers for *all* users being adapted. Collective optimization is, therefore, appropriate for the reverse link of a DS-CDMA cellular system. That is, the signatures for all users are computed at the base station, and the transmitter coefficients (or coefficient updates) are fed back to the mobiles over the forward link.

A. Alternating Updates

We wish to minimize the sum MSE over all users subject to the energy constraints $\|\mathbf{p}_k\|^2 \le \Pi_k, k = 1, \ldots, K$. The cost function is, therefore

$$J = E[\|\mathbf{b}(i) - \mathbf{d}(i)\|^2] + \sum_{k=1}^{K} \lambda_k (\|\mathbf{F}_k \alpha_k\|^2 - \Pi_k) \quad (28)$$

where the decision vector $\mathbf{d}(i) = \mathbf{C}^H \mathbf{r}(i), \mathbf{C}$ is the $N \times K$ matrix of receiver filters, and the received vector $\mathbf{r}(i)$ is given by (1). Minimizing with respect to α_k gives

$$\alpha_k = \Big(|A_k|^2 \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{C}\mathbf{C}^H \mathbf{H}_k \mathbf{F}_k + \lambda_k \mathbf{F}_k^H \mathbf{F}_k \Big)^{-1} A_k^* \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{c}_k. \quad (29)$$

Joint optimization of the transmitter and receiver can be accomplished by iterating the preceding expression with an update for the receiver until convergence of the cost function given by (28) is achieved. For each transmitter update, the Lagrange multiplier must be selected via a numerical search to satisfy the energy constraint. As with individual optimization, convergence is guaranteed with the MMSE receiver, but is more difficult to establish with the MF receiver, since the MF receiver does not necessarily reduce the MSE at each iteration.

Substituting the optimality condition (29) into the cost function (28) yields

$$J = K + \sigma_n^2 \mathrm{trace}(\mathbf{C}^H \mathbf{C}) - \sum_{k=1}^{K} A_k \mathbf{c}_k^H \mathbf{H}_k \mathbf{F}_k \alpha_k - \sum_{k=1}^{K} \lambda_k \Pi_k = \sum_{k=1}^{K} (\xi_k - \lambda_k \Pi_k) \quad (30)$$

where

$$\xi_k = 1 + \sigma_n^2 \|\mathbf{c}_k\|^2 - A_k \mathbf{c}_k^H \mathbf{H}_k \mathbf{F}_k \alpha_k \quad (31)$$

is the MSE for user k with receiver filter $\mathbf{c}_k$, and α_k is given by (29).

We emphasize that collective transmitter optimization, represented by (29), requires knowledge of the receivers and channels for *all* demodulated users. In contrast, individual transmitter adaptation, represented by (12), requires knowledge of the receiver and channel for only the desired user.

B. Joint Optimization: MMSE Receiver

The sum MSE cost function with a power constraint, assuming the MMSE receiver filter (5) for each user, can be written as

$$\begin{aligned} J &= E[\|\mathbf{b}(i) - \mathbf{d}(i)\|^2] + \sum_{k=1}^{K} \lambda_k (\|\mathbf{F}_k \alpha_k\|^2 - \Pi_k) \\ &= K + \sigma_n^2 \sum_{k=1}^{K} \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}^{-2} \mathbf{H}_k \mathbf{F}_k \alpha_k \\ &\quad - 2 \sum_{k=1}^{K} \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}^{-1} \mathbf{H}_k \mathbf{F}_k \alpha_k \\ &\quad + \sum_{k=1}^{K} \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}^{-1} \tilde{\mathbf{P}}_k \tilde{\mathbf{P}}_k^H \mathbf{R}^{-1} \mathbf{H}_k \mathbf{F}_k \alpha_k \\ &\quad + \sum_{k=1}^{K} \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{C}_k \mathbf{C}_k^H \mathbf{H}_k \mathbf{F}_k \alpha_k \\ &\quad + \sum_{k=1}^{K} \lambda_k (\|\mathbf{F}_k \alpha_k\|^2 - \Pi_k) \end{aligned} \quad (32)$$

where the $N \times (K-1)$ matrix $\mathbf{C}_k$ is the receiver matrix $\mathbf{C}$ without column k, and the columns of the $N \times (K-1)$ matrix $\tilde{\mathbf{P}}_k$ are the effective signatures (i.e., channel matrix times transmitted signature) of all users except user k. Minimizing this cost function with respect to α_k gives

$$\mathbf{Q}_k \alpha_k = \nu \alpha_k \quad (33)$$

where

$$\mathbf{Q}_k = \left(\mathbf{F}_k^H \mathbf{F}_k\right)^{-1} \mathbf{F}_k^H \mathbf{H}_k^H \left\{ \mathbf{R}_k^{-1} - (\eta_k + 1)^2 \mathbf{C}_k \mathbf{C}_k^H \right\} \mathbf{H}_k \mathbf{F}_k \quad (34)$$

and

$$\eta_k = \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}_k^{-1} \mathbf{H}_k \mathbf{F}_k \alpha_k. \quad (35)$$

Of course, this condition can also be derived by combining the receiver condition (5) with the transmitter condition (29). We remark that in contrast with individual optimization, $\mathbf{R}_k$ cannot generally be replaced by $\mathbf{R}$.

Substituting (33) into the sum MSE part of the cost function given by (32) gives

$$\begin{aligned} E[\|\mathbf{b}(i)-\mathbf{d}(i)\|^2] &= -\frac{\eta_k}{(1+\eta_k)} + \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{C}_k \mathbf{C}_k^H \mathbf{H}_k \mathbf{F}_k \alpha_k + \Xi_k \\ &= -\frac{1}{(1+\eta_k)^2}\left\{\eta_k^2 + \nu\|\alpha_k\|^2\right\} + \Xi_k \end{aligned} \quad (36)$$

where

$$\Xi_k = K + \sum_{l\neq k} \mathbf{c}_l^H \mathbf{R}_k \mathbf{c}_l - \sum_{l\neq k}\left(\mathbf{c}_l^H \mathbf{H}_l \mathbf{p}_l + \mathbf{p}_l^H \mathbf{H}_l^H \mathbf{c}_l\right) \quad (37)$$

does not depend on α_k. We, therefore, conclude that the sum MSE is minimized by selecting α_k as the eigenvector of $\mathbf{Q}_k$ corresponding the maximum eigenvalue.

The condition (33) would be straightforward to solve numerically except that $\mathbf{Q}_k$ depends on α_k through η_k. Consequently, the numerical results in the next section were generated by iterating (5) and (29), instead of trying to solve (33)–(35) directly.

Of course, the condition (33)–(34) is not equivalent to the analogous condition for single-user optimization (12). Consider group adaptation, in which all users jointly optimize their signature sequences with an MMSE receiver. We define a *fixed point* for collective or individual optimization as a set of signatures $\mathbf{p}_k = \mathbf{F}_k\alpha_k, k = 1,\ldots,K$, which satisfy the corresponding conditions for optimality (33)–(34) or (12), respectively, where the columns of $\mathbf{C}_k$ in (34) are the MMSE receivers for users $m \neq k$.

Theorem: As $\sigma_n^2 \to 0$, the set of fixed points for collective optimization converge to the set of fixed points for individual optimization.

The proof is given in the Appendix I. The theorem states that at high signal-to-noise ratios (SNRs), individual optimization can provide a solution which is close to the global optimum in the sum MSE sense. We remark that for any $\sigma_n^2 > 0$ and arbitrary $\mathbf{H}_k$'s, the fixed points for collective optimization are not generally the same as those for individual optimization. However, for the ideal channel $\mathbf{H}_k = \mathbf{I}$, we show in the Appendix that the individual condition (14) is indeed equivalent to (33)–(34) with MMSE receivers. This is to be expected, since it is shown in [16] that with ideal channels, individual optimization minimizes the sum MSE cost function.

C. Joint Optimization: MF Receiver

The sum MSE cost function with a power constraint, assuming the receiver filter for each user is a MF, as in (4), can be written as

$$\begin{aligned} J &= E[\|\mathbf{b}(i)-\mathbf{d}(i)\|^2] + \sum_{k=1}^{K}\lambda_k(\|\mathbf{F}_k\alpha_k\|^2 - \Pi_k) \\ &= -2\|\mathbf{H}_k\mathbf{p}_k\| + \|\mathbf{H}_k\mathbf{p}_k\|^2 + \frac{\alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}_k \mathbf{H}_k \mathbf{F}_k \alpha_k}{\|\mathbf{H}_k\mathbf{P}_k\|^2} \\ &\quad + \alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{C}_k \mathbf{C}_k^H \mathbf{H}_k \mathbf{F}_k \alpha_k + \lambda_k\|\mathbf{F}_k\alpha_k\|^2 + \Xi_k \end{aligned} \quad (38)$$

where Ξ_k does not depend on α_k. Minimizing with respect to α_k and rearranging gives

$$\left(\mathbf{F}_k^H\mathbf{F}_k\right)^{-1}\mathbf{M}\alpha_k = \nu\alpha_k \quad (39)$$

where

$$\begin{aligned} \mathbf{M} = \mathbf{F}_k^H\mathbf{H}_k^H \Bigg\{ &\left(1 - \frac{1}{\|\mathbf{H}_k\mathbf{p}_k\|} - \frac{\mathbf{p}_k^H \mathbf{H}_k^H \mathbf{R}_k \mathbf{H}_k \mathbf{p}_k}{\|\mathbf{H}_k\mathbf{P}_k\|^4}\right)\mathbf{I} \\ &+ \frac{\mathbf{R}_k}{\|\mathbf{H}_k\mathbf{p}_k\|^2} + \mathbf{C}_k\mathbf{C}_k^H \Bigg\} \mathbf{H}_k\mathbf{F}_k \end{aligned} \quad (40)$$

We remark that combining (4) and (29) also yields the condition (39), but with a *different* matrix $\mathbf{M}$ from that in (40). Namely

$$\mathbf{M} = \mathbf{F}_k^H\mathbf{H}_k^H\left\{\left(1 - \frac{1}{\|\mathbf{H}_k\mathbf{p}_k\|}\right)\mathbf{I} + \mathbf{C}_k\mathbf{C}_k^H\right\}\mathbf{H}_k\mathbf{F}_k. \quad (41)$$

That is, combining (4) and (29) is *not* equivalent to minimizing the sum MSE cost function (38), and generally leads to worse performance. Mathematically, the difference arises from the cross-product term $\mathbf{c}_k^H\mathbf{R}_k\mathbf{c}_k = (\alpha_k^H \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{R}_k \mathbf{H}_k \mathbf{F}_k \alpha_k)/(\|\mathbf{H}_k\mathbf{P}_k\|^2)$ in (38), which is treated differently by alternating and joint optimization.

Solving (39) is complicated by the fact that $\mathbf{M}$ depends on α_k. An iterative numerical method can again be applied, as was discussed for individual optimization. We also remark that as in the individual case, it is unknown which eigenvalue is associated with the solution eigenvector α_k.

D. Iterative Algorithms for Group Optimization

The necessary conditions for alternating transmitter-receiver optimization (5) and (29), and the joint optimization condition (33)–(34) lead to two different iterative algorithms for finding fixed points for group optimization. Suppose that the users are initially assigned some arbitrary set of signatures $\mathbf{p}_1,\ldots,\mathbf{p}_K$. Alternating transmitter-receiver optimization implies that the receivers are updated according to (5) with fixed transmitters, followed by the signature update (29) with fixed receivers, and so forth. This is illustrated in Fig. 2. In contrast, the condition (33)–(34) can be applied successively across users, which is referred to as "user-by-user" optimization in Fig. 2. Note that the sum MSE must converge in either case, since it cannot increase after an update.

Of course, alternating transmitter-receiver and user-by-user optimization can also be carried out by iterating the individual necessary conditions (5) and (12), or (23) across users. In that case, convergence to a fixed point is more difficult to establish. A user-by-user optimization algorithm without multipath, based on individual updates, has been presented and analyzed in [6], [16], [17] where it is referred to as *interference avoidance*. (See, also, [7], which considers interference avoidance in the context of a single-user dispersive channel.) In [17], a modified version of interference avoidance is presented which is guaranteed to converge to a solution which minimizes the sum MSE. In the presence of multipath, the condition (22) implies that interference avoidance must be traded off against the benefit of exploiting the channel eigenvectors with largest eigenvalues.

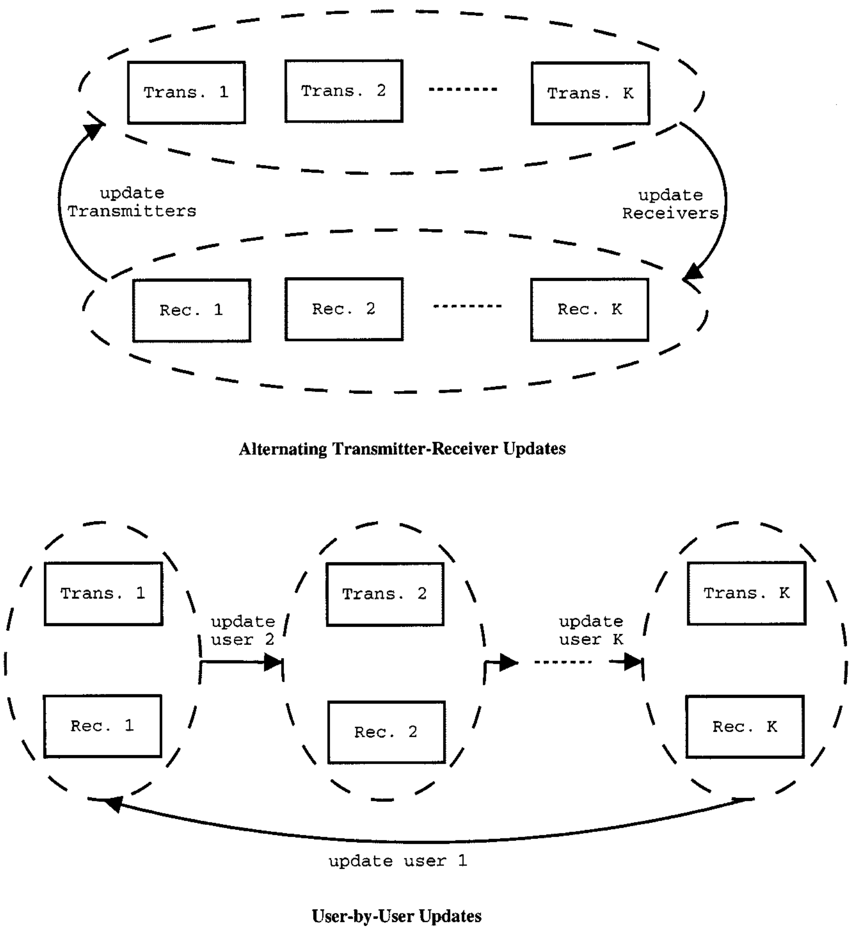

Fig. 2. Alternating transmitter-receiver versus user-by-user adaptation.

In general, characterizing the performance of fixed-points as a function of the user channels, as well as establishing the convergence of group optimization with individual cost functions to a fixed point in the presence of multipath, remain open problems. Our experimental results, including the numerical results in Section VI, indicate that the joint transmitter-receiver optimization algorithms presented here generally converge to the optimal solution, in the sense of obtaining (near) single-user performance for $K < N$.

V. Large System Performance Without Multipath

In this section, we analyze the performance of reduced-rank transmitter optimization assuming first single-user adaptation in the presence of nonadaptive interference, and then group adaptation. To simplify the analysis, we assume ideal channels without multipath and MF receivers. Although (14) states that the full-rank MMSE receiver reduces to the MF (see, also, [14]), the MMSE receiver gives better performance with a reduced-rank signature. Our approach is to evaluate the large system performance with randomly assigned signatures as the dimension D, number of users K, and processing gain N all tend to infinity with fixed normalized dimension $\bar{D} = D/K$, and normalized load $\bar{K} = K/N$. (See, also, [18] which presents a large system analysis of reduced-rank receivers.)

A. Single-User Optimization

Consider a CDMA system with K users in which a single user k adapts his signature to minimize MSE with an MF receiver. The MSE for user k is given by

$$\begin{aligned}\xi_k &= E\left(|b_k(i) - \mathbf{p}_k^H[\mathbf{PAb}(i) + \mathbf{n}(i)]|^2\right)\\ &= [1 - (A_k + A_k^*)|A_k|^2\|\mathbf{p}_k\|^2 + \|\mathbf{p}_k\|^4] + \sigma_n^2\|\mathbf{p}_k\|^2\\ &\quad + \alpha_k^H\left(\mathbf{F}_k^H\mathbf{P}_k\mathbf{A}_k\mathbf{A}_k^H\mathbf{P}_k^H\mathbf{F}_k\right)\alpha_k \end{aligned} \tag{42}$$

where $\mathbf{P}_k$ is the $N \times (K-1)$ matrix obtained from $\mathbf{P}$ by removing the $k_{\rm th}$ column (i.e., the $k_{\rm th}$ user's signature), and $\mathbf{A}_k$ is the $(K-1) \times (K-1)$ diagonal matrix containing the amplitudes of all users except for user k. If $A_k = \|\mathbf{p}_k\|^2 = 1$ for each k, then (42) reduces to

$$\xi_k = \sigma_n^2 + \alpha_k^H\left(\mathbf{F}_k^H\mathbf{P}_k\mathbf{P}_k^H\mathbf{F}_k\right)\alpha_k. \tag{43}$$

This expression is minimized by choosing α_k to be the eigenvector corresponding to the minimum eigenvalue of $\mathbf{F}_k^H\mathbf{P}_k\mathbf{P}_k^H\mathbf{F}_k$, which we denote as $\lambda_{\min}$. The associated SINR is

$$\gamma_k = \frac{1}{\sigma_n^2 + \lambda_{\min}}. \tag{44}$$

Now suppose that the signatures assigned to the users consist of independent identically distributed (i.i.d.) binary random elements. Suppose also that the columns of $\mathbf{F}_k$ consist of

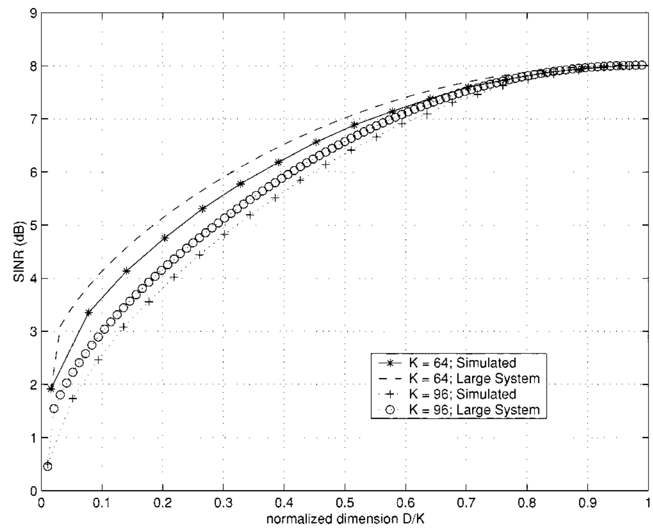

Fig. 3. Average SINR (dB) versus normalized dimension $\bar{D}$ assuming only one user adapts. $N = 128$ and $\mathrm{SNR} = 8$ dB.

nonoverlapping, equal-length segments of the random signature assigned to user k, as illustrated in (8). Since the length of each nonzero segment is N/D, and $\mathbf{F}_k^H \mathbf{F}_k = \mathbf{I}$, the nonzero elements of $\mathbf{F}_k$ are $\pm\sqrt{D/N}$. It is easily verified that the elements of $\mathbf{F}_k^H \mathbf{P}_k$ are i.i.d. with mean zero and variance $1/N$.

We now consider the large system limit $N \to \infty, K \to \infty$, and $D \to \infty$ with $D/K = \bar{D}$ and $K/N = \bar{K}$, where $\bar{D}$ and $\bar{K}$ are the normalized dimension and load, respectively. It is known that the empirical distribution of eigenvalues of the matrix $(\mathbf{F}_k^H \mathbf{P}_k \mathbf{P}_k^H \mathbf{F}_k)$ converges to a deterministic distribution [19]. The minimum eigenvalue converges to

$$\lambda_{\min}^{\infty} = (1 - \sqrt{\bar{D}})^2 (\bar{K}) \tag{45}$$

so that the large system SINR converges to $\gamma_k^{\infty} = 1/(\sigma_n^2 + \lambda_{\min}^{\infty})$.

Fig. 3 shows plots of SINR at the output of the receiver filter versus normalized dimension $\bar{D}$ for normalized loads $\bar{K} = 1/2$ and $3/4$. Simulated results corresponding to $N = 128$ are compared with the large system results. The background SNR $1/\sigma_n^2 = 8$ dB. The simulated results are averaged over random binary signature sequences. These curves show that for the cases considered, a normalized dimension $\bar{D} = 1/2$ results in between one and two decibels degradation relative to full-rank performance while reducing the number of parameters to be estimated by a half. These results also show close agreement between the large system and simulated results.

B. Group Optimization

We now analyze the performance of reduced-rank transmitter-receiver optimization when all users optimize signatures. All users are assumed to transmit with power equal to one, so that the sum MSE is

$$\begin{aligned} \xi &= K\sigma_n^2 + \sum_{k=1}^{K} \mathbf{p}_k^H \mathbf{P}_k \mathbf{P}_k^H \mathbf{p}_k \\ &= K\sigma_n^2 + \sum_{k=1}^{K} \mathbf{p}_k^H \left(\mathbf{P}_{1:k-1}\mathbf{P}_{1:k-1}^H + \mathbf{P}_{k+1:K}\mathbf{P}_{k+1:K}^H\right)\mathbf{p}_k \end{aligned} \tag{46}$$

where $\mathbf{P}_{1:k-1}$ is the $N \times (k-1)$ matrix containing the signatures of users $1, \ldots, (k-1)$, and $\mathbf{P}_{k+1:K}$ is the $N \times (K-k)$ matrix containing the signatures of users $(k+1), \ldots, K$. From symmetry, it is apparent that the two terms in the summation in (46) will contribute the same interference in the large system limit, so that we define

$$\tilde{\xi} = K\sigma_n^2 + 2\sum_{k=1}^{K-1} \mathbf{p}_k^H \mathbf{P}_{k+1:K}\mathbf{P}_{k+1:K}^H \mathbf{p}_k \tag{47}$$

and note that $E[\tilde{\xi}] = E[\xi]$ when the signatures are i.i.d.

We now consider a successive optimization scheme in which users are added successively in decreasing order, $k = K, \ldots, 1$, and each user optimizes his signature based on the users present. The users do not change their signatures as more users are added, so that this scheme is suboptimal. The performance of this method, which we can analyze, then lower bounds the optimal performance. To facilitate the large system analysis, we will also make the approximation that the elements of the signatures of the users added before a particular user are i.i.d.

We first observe that for a given D, the matrix $(\mathbf{F}_k^H \mathbf{P}_{k+1:K}\mathbf{P}_{k+1:K}^H \mathbf{F}_k)$ has rank less than D for $k = K - D + 1, \ldots, K$. Consequently, for these users we can choose α_k so that $\alpha_k^H(\mathbf{F}_k^H \mathbf{P}_{k+1:K}\mathbf{P}_{k+1:K}^H \mathbf{F}_k)\alpha_k = 0$, and

$$\tilde{\xi} = K\sigma_n^2 + 2\sum_{k=1}^{K-D} \alpha_k^H \left(\mathbf{F}_k^H \mathbf{P}_{k+1:K}\mathbf{P}_{k+1:K}^H \mathbf{F}_k\right)\alpha_k. \tag{48}$$

Each user $k \le K - D$ added to the system minimizes the corresponding contribution to the sum MSE in (48). This is equivalent to individual optimization of α_k given that only users $k+1, \ldots, K$ are present. α_k is, therefore, the eigenvector corresponding to the minimum eigenvalue of the matrix $\mathbf{F}_k^H \mathbf{P}_{k+1:K}\mathbf{P}_{k+1:K}^H \mathbf{F}_k$.

Given the preceding optimization procedure, we can write the sum MSE cost function as

$$\tilde{\xi} = K\sigma_n^2 + 2\sum_{k=1}^{K-D} \lambda_{\min}\left(\mathbf{F}_k^H \mathbf{P}_{k+1:K}\mathbf{P}_{k+1:K}^H \mathbf{F}_k\right) \tag{49}$$

where $\lambda_{k,\min}(\mathbf{V})$ is the minimum eigenvalue of the matrix $\mathbf{V}$.

Our objective is to evaluate the large system limit of $\tilde{\xi}$ as $(D, K, N) \to \infty$. We assume that the columns of $\mathbf{F}_k$ are nonoverlapping segments of a random i.i.d. signature so that $\mathbf{F}_k^H \mathbf{F}_k = \mathbf{I}$. Each element of $\mathbf{F}_k^H \mathbf{P}_{k+1:K}$ is, therefore, a randomly weighted sum of elements in a segment of $\mathbf{p}_m, k+1 \le m \le K$. The segments corresponding to different elements are nonoverlapping. In general, these segments are correlated; however, in what follows, we will assume that the elements of $\mathbf{F}_k^H \mathbf{P}_{k+1:K}$ become i.i.d. as $(D, K, N) \to \infty$. Additional numerical experiments have shown that the expression for SINR, which follows from this assumption, accurately predicts the performance of the successive optimization scheme.

With the preceding assumption, we can again apply the results on the asymptotic eigenvalue distribution of a class of random matrices presented in [19]. Namely

$$\lambda_{k,\min} \xrightarrow[(D,K,N)\to\infty]{} \left(1 - \sqrt{\frac{\bar{D}}{1-x}}\right)^2 (\bar{K} - x) \tag{50}$$

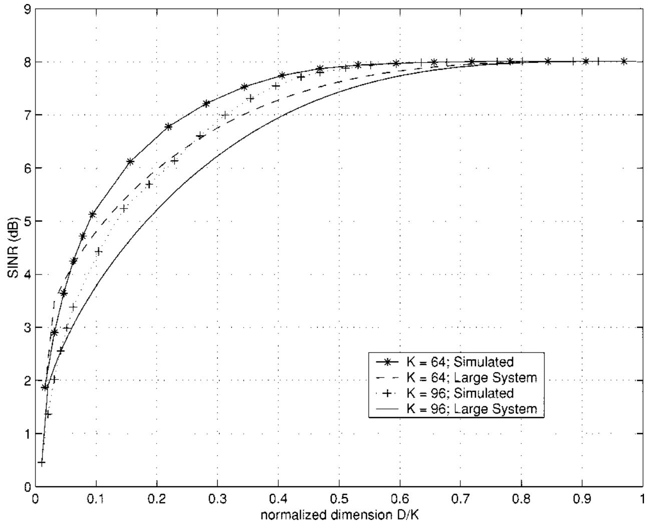

Fig. 4. Average SINR (dB) versus normalized dimension $\bar{D}$ assuming all users adapt. $N = 128$ and $\mathrm{SNR} = 8$ dB.

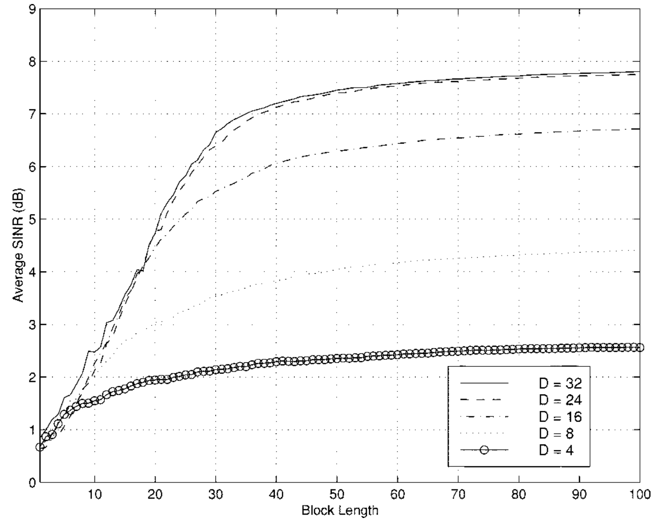

Fig. 5. Average SINR versus blocklength for single-user adaptation with nonadaptive, random interferers, and ideal channels. $N = 32$, $K = 24$, and $\mathrm{SNR} = 8$dB.

where $k/K \to x$, so that

$$\begin{aligned}
&\lim_{(D,K,N)\to\infty} \frac{1}{K}\tilde{\xi} \\
&\quad\approx \sigma_n^2 + \lim_{(D,K,N)\to\infty} 2\frac{1}{K}\sum_{k=1}^{K-D}\left(1-\sqrt{\frac{\bar{D}}{1-k/K}}\right)^2 \\
&\qquad\times \bar{K}(1-k/K) \\
&\quad= \sigma_n^2 + 2\bar{K}\int_0^{1-\bar{D}}\left(1-\sqrt{\frac{\bar{D}}{1-x}}\right)^2(1-x)\,dx \\
&\quad= \sigma_n^2 + \bar{K}\left(1-\frac{\bar{D}^2}{3}+2\bar{D}-\frac{8}{3}\sqrt{\bar{D}}\right). \qquad (51)
\end{aligned}$$

We define the averaged SINR over all users as

$$\begin{aligned}
\gamma &= \frac{K}{K\sigma_n^2 + \sum_{k=1}^{K}\sum_{l\neq k}\left|\mathbf{p}_k^H\mathbf{p}_l\right|^2} \\
&= \frac{K}{K\sigma_n^2 + \sum_{k=1}^{K}\mathbf{p}_k^H\mathbf{P}_k\mathbf{P}_k^H\mathbf{p}_k} \\
&= \frac{K}{\xi}. \qquad (52)
\end{aligned}$$

The large system SINR for any particular user optimized over the signatures is then lower bounded as

$$\begin{aligned}
\gamma^\infty &\geq \lim_{(D,K,N)\to\infty}\frac{1}{\xi/K} \\
&\approx \frac{1}{\sigma_n^2 + \bar{K}\left(1-\frac{\bar{D}^2}{3}+2\bar{D}-\frac{8}{3}\sqrt{\bar{D}}\right)}. \qquad (53)
\end{aligned}$$

Fig. 4 shows averaged SINR versus normalized dimension $\bar{D}$ for two different loads $\bar{K} = 1/2$ and $3/4$. The background SNR $1/\sigma_n^2 = 8$ dB. Simulated curves are shown for group optimization with $N = 128$ averaged over random signatures. These are compared with the large system expression (53). The large system SINR is always within 1 dB of the simulated results, and accurately predicts the $\bar{D}$ for which full-rank performance is nearly achieved. Comparing these results with the single-user results in Fig. 3 shows that group adaptation gives a 1 to 2 dB improvement in received SINR relative to single-user adaptation in the presence of random interferers.

VI. Numerical Results

In this section, we present numerical results, which illustrate the relative performance of the algorithms presented in the preceding sections. We first show convergence results for adaptive transmitter-receiver algorithms. Fig. 5 shows received SINR versus blocklength (number of received vectors) for single-user adaptation with different transmitter ranks D and MF receivers. As derived in Section V, the vector of combining coefficients α_k for the desired user is the eigenvector corresponding to the smallest eigenvalue of $\mathbf{F}_k^H\mathbf{R}_k\mathbf{F}_k$, where $\mathbf{R}_k$ is replaced by $\hat{\mathbf{R}}_k$ given by (24), the signatures for the interferers are randomly assigned, and $\mathbf{F}_k^H\mathbf{F}_k = \mathbf{I}$. These results are for an ideal channel ($\mathbf{H}_k = \mathbf{I}$) with $K = 24, N = 32$, background SNR $1/\sigma_n^2 = 8$ dB, and are averaged over the signatures of the interferers. As the blocklength becomes large, the performance corresponds to the large system results shown in Fig. 3. These curves show that a substantial improvement in performance is obtained with a blocklength $n = N$.

Fig. 6 shows averaged symbol error rate versus blocklength for joint transmitter-receiver adaptation when all users adapt in the presence of multipath. In this case, $K = 12, N = 16$, and the background $\mathrm{SNR} = 10$ dB. For each user, the number of paths $L_k = 3$. The channel coefficients (elements of $\mathbf{h}_k, k = 1,\ldots,K$) are independent complex Gaussian random variables, and the channel vectors $\mathbf{h}_k$ are normalized so that the average power is unity, i.e., $E[||\mathbf{h}_k||^2] = 1$. For these results and the following results, the power constraint $||\mathbf{p}_k|| = 1$ applies to all users $k = 1,\ldots,K$.

Results are shown for both adaptive LS and MF receivers. Specifically, the LS receiver filter for user k minimizes $\sum_{i=1}^{n} w^{n-i}||\mathbf{b}_k(i) - \mathbf{c}_k^H\mathbf{r}(i)||^2$ at each iteration n, and is given by

$$\mathbf{c}_k(n) = \hat{\mathbf{R}}^{-1}(n)\hat{\mathbf{p}}_k(n) \qquad (54)$$

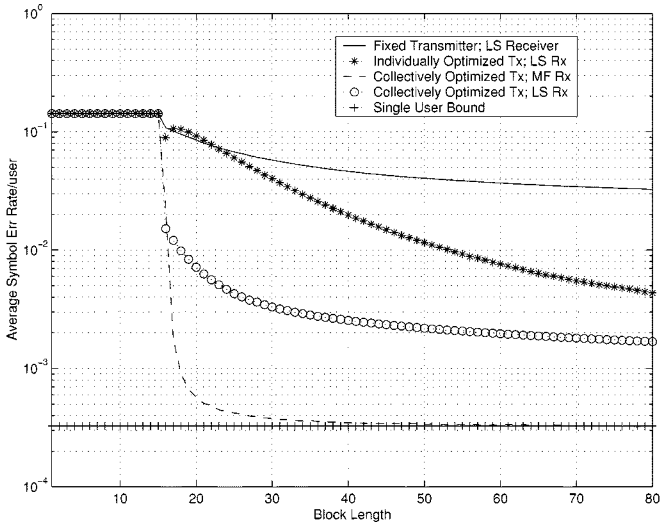

Fig. 6. Average symbol error rate versus blocklength (number of training samples) with group adaptation. $N = 16, K = 12$, and $\mathrm{SNR} = 10$dB.

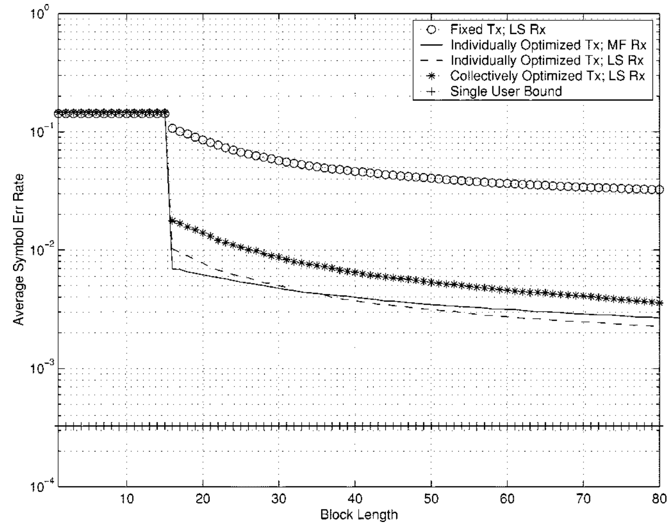

Fig. 7. Average symbol error rate versus blocklength (number of training samples) for single-user adaptation with nonadaptive interference. $N = 16, K = 12$, and $\mathrm{SNR} = 10$dB.

where $\hat{\mathbf{R}}(n) = \sum_{i=1}^{n} w^{n-i}\mathbf{r}(i)\mathbf{r}^H(i)$ is the sample covariance matrix, $\hat{\mathbf{p}}_k(n) = \sum_{i=1}^{n} w^{n-i} b_k^*(i)\mathbf{r}(i)$ is the k_{th} user steering vector, and w is an exponential weight. The MF assumes knowledge of the desired user's channel, whereas the LS receiver requires a training sequence. These results are for alternating optimization, in which the optimal transmitter is computed for the given receiver, and is instantaneously transmitted back to the transmitter at each iteration.

The average error rate shown in Fig. 6 is computed by assuming that the residual interference plus noise at the output of the filter is Gaussian. The error rate is averaged over all users, and over many runs with different random initializations for the signatures $\mathbf{p}_k$ and channel vectors $\mathbf{h}_k, k = 1, \ldots, K$. An initial blocklength equal to the filter dimension is accumulated to avoid illconditioning of the sample covariance matrix. The single-user bound shown corresponds to the performance of a single-user system with an MMSE receiver.

Fig. 6 shows that collective adaptation converges significantly faster than individual adaptation. Collective adaptation approaches the single-user bound as the blocklength increases. Individual adaptation is unable to achieve this steady-state performance, although it offers a substantial improvement in performance relative to receiver adaptation alone. We observe that collective transmitter adaptation performs better with the MF receiver than with the LS receiver. This is because the LS receiver has difficulty tracking the time-varying MMSE solution (due to the time-varying signatures), whereas the MF is instantaneously tracked. Given the channels for all users, the jointly optimal transmitters and MF receivers can be computed iteratively offline, so that for the MF receiver, the abscissa in Fig. 6 represents the number of iterations, rather than the number of training samples received. Results for individual adaptation with the tracking MF are not shown, since only a modest performance gain is obtained relative to a nonadaptive transmitter, as explained in Section III.

Fig. 7 shows convergence plots for single-user adaptation with multipath. The system parameters are the same as for Fig. 6. We observe that individual adaptation with the MF, based on (25), converges slightly faster than adaptation with the LS receiver initially, but has slightly worse steady-state performance. We also observe that individual adaptation performs better than collective adaptation. This is because collective optimization penalizes interference to other users, which compromises the performance of the desired user. For these results, the steering vector for the LS receiver with signature adaptation is $\mathbf{p}_k$, instead of $\hat{\mathbf{p}}_k$. This change was observed to improve convergence. Also, the single-user performance curve corresponds to the jointly optimized transmitter-receiver pair with $K = 1$.

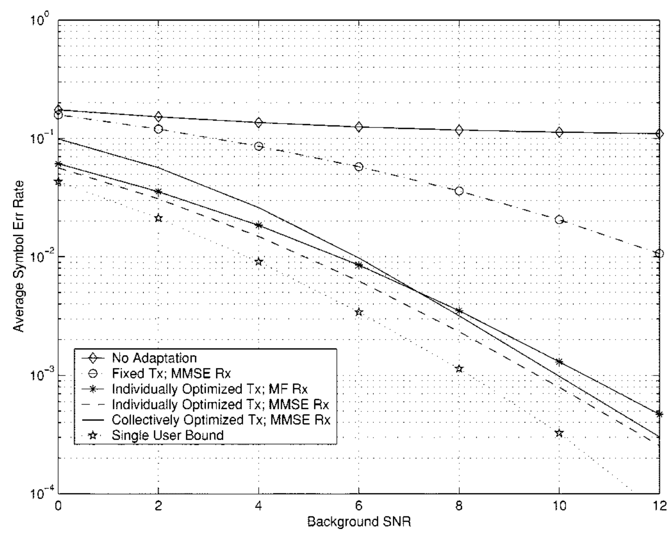

Fig. 8. Average symbol error rate versus background SNR for a single user with optimized signature in the presence of nonadaptive interference. $N = 16$ and $K = 12$.

Fig. 8 shows asymptotic bit error rate (BER) versus SNR for the different single-user optimization schemes considered. "Asymptotic" means that the block size tends to infinity so that the jointly optimal MMSE transmitter-receiver combination is assumed. At an error rate of 10^{-3}, individual optimization with the MMSE receiver is approximately 1.5 dB away from the

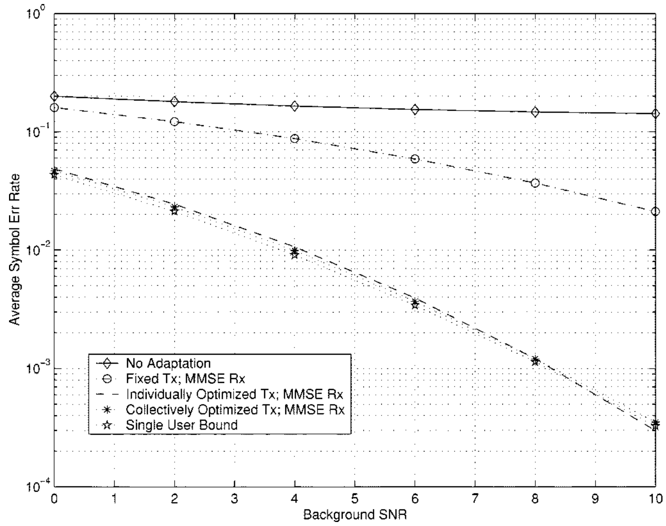

Fig. 9. Average symbol error rate versus background SNR with group optimization of signatures. $N = 16$ and $K = 12$.

single-user bound. A substantial performance gain is obtained relative to receiver adaptation alone. As expected, the curve corresponding to collective optimization with the MMSE receiver converges to the individual optimization curve as the background SNR increases. Collective optimization with the MF receiver performs slightly better than collective optimization with the MMSE receiver at low SNRs, and performs only slightly worse than the MMSE receiver at high SNRs. Fig. 9 shows asymptotic BER versus SNR for the different schemes considered when all users adapt. All adaptive transmitter-receiver algorithms perform close to the single-user bound.

VII. Conclusion

Methods for reduced-rank joint transmitter-receiver optimization for reverse link DS-CDMA have been presented. By varying the number of combining coefficients, or degrees of freedom at the transmitter, steady-state performance can be traded off against the amount of data needed to obtain accurate estimates of the transmitter coefficients. Our results indicate that adapting relatively few transmitter coefficients can lead to a substantial improvement in performance, relative to one-dimensional power control with an MMSE receiver.

Algorithms for both individual and collective optimization of transmitter signatures were presented in the presence of multipath. In general, when all users optimize signatures, the resulting fixed points are different, although numerical results show that for the cases considered, individual optimization performs nearly as well as collective optimization. The convergence of adaptive transmitter-receiver algorithms were also illustrated numerically. Our results show that group adaptation with individual or local cost functions converges more slowly than group adaptation with a collective, or global, cost function. Also, an adaptive receiver may not perform as well as an MF when the transmitted signatures are time-varying. A challenging topic for further study is the convergence and tracking properties of jointly adaptive transmitter-receiver algorithms in cellular and peer-to-peer environments.

Appendix I
Proof of Theorem

We need to show that as $\sigma_n^2 \to 0$, the condition (33)–(34) reduces to (23). Applying the matrix inversion lemma to the MMSE filter for user k gives

$$\mathbf{c}_l = \mathbf{R}^{-1}\mathbf{H}_l\mathbf{p}_l = \mathbf{R}_k^{-1}\mathbf{H}_l\mathbf{p}_l - \frac{\eta_{k,l}\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{p}_k}{1+\eta_k} \tag{55}$$

where $\eta_{k,l} = \mathbf{p}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-1}\mathbf{H}_l\mathbf{p}_l$ and $\eta_k = \eta_{k,k}$. We can, therefore, write

$$\mathbf{c}_l\mathbf{c}_l^H = \mathbf{R}_k^{-1}\mathbf{H}_l\mathbf{p}_l\mathbf{p}_l^H\mathbf{H}_l^H\mathbf{R}_k^{-1} - \frac{\eta_{l,k}}{1+\eta_k}\mathbf{R}_k^{-1}\mathbf{H}_l\mathbf{p}_l\mathbf{p}_k^H \times \mathbf{H}_k^H\mathbf{R}_k^{-1} - \frac{\eta_{k,l}}{1+\eta_k}\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{p}_k\mathbf{p}_l^H\mathbf{H}_l^H\mathbf{R}_k^{-1} + \frac{|\eta_{k,l}|^2}{(1+\eta_k)^2}\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{p}_k\mathbf{p}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-1} \tag{56}$$

and

$$\mathbf{C}_k\mathbf{C}_k^H = \sum_{l\neq k}\mathbf{c}_l\mathbf{c}_l^H = \mathbf{R}_k^{-1}\tilde{\mathbf{P}}_k\tilde{\mathbf{P}}_k^H\mathbf{R}_k^{-1} - \frac{1}{1+\eta_k}\mathbf{R}_k^{-1}\tilde{\mathbf{P}}_k\tilde{\mathbf{P}}_k^H\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{p}_k\mathbf{p}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-1} - \frac{1}{1+\eta_k}\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{p}_k\mathbf{p}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-1}\tilde{\mathbf{P}}_k\tilde{\mathbf{P}}_k^H\mathbf{R}_k^{-1} + \frac{\mathbf{p}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-1}\tilde{\mathbf{P}}_k\tilde{\mathbf{P}}_k^H\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{p}_k}{(1+\eta_k)^2}\mathbf{R}_k^{-1}\mathbf{H}_k\mathbf{p}_k\mathbf{p}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-1} \tag{57}$$

Combining this expression with (33)–(34) and rearranging gives

$$\mathbf{Q}_k\mathbf{p}_k = \left(\mathbf{F}_k^H\mathbf{F}_k\right)^{-1}\mathbf{F}_k^H\mathbf{H}_k^H \left\{\left[1-\frac{1+\sigma_n^2\delta_k}{1+\eta_k^2}\right]\mathbf{R}_k^{-1} + \frac{\sigma_n^2}{1+\eta_k}\mathbf{R}_k^{-2}\right\}\mathbf{H}_k^H\mathbf{F}_k\alpha_k \tag{58}$$

where $\delta_k = \mathbf{p}_k^H\mathbf{H}_k^H\mathbf{R}_k^{-2}\mathbf{H}_k\mathbf{p}_k$, and the fact that $\tilde{\mathbf{P}}_k\tilde{\mathbf{P}}_k^H = \mathbf{R}_k - \sigma_n^2\mathbf{I}$ has been used.

As $\sigma_n^2 \to 0$, this condition becomes equivalent to (23). We remark that letting $\sigma_n^2 = 0$ causes technical problems in that $\mathbf{R}_k$ is likely to become singular. However, the limit of $\mathbf{Q}_k\mathbf{p}_k$ as $\sigma_n^2 \to 0$ is still well-defined.

Suppose now that $\mathbf{H}_k = \mathbf{I}$. Writing the covariance matrix as $\mathbf{R}_k = \mathbf{V}\Lambda\mathbf{V}^H$, where the columns of $\mathbf{V}$ are the normalized eigenvectors and Λ is the diagonal matrix of eigenvalues, the full-rank condition (33) becomes

$$\mathbf{Q}_k\mathbf{p}_k = \mathbf{V}\left\{\left(1-\frac{1+\sigma_n^2\delta_k}{1+\eta_k^2}\right)\Lambda^{-1} + \frac{\sigma_n^2}{1+\eta_k}\Lambda^{-2}\right\}\mathbf{V}^H\mathbf{p}_k = \nu_k\mathbf{p}_k \tag{59}$$

This implies that $\mathbf{p}_k$ must be an eigenvector of $\mathbf{R}_k$. It remains to show that to minimize the sum MSE, this eigenvector must correspond to the minimum eigenvalue.

Suppose that $\mathbf{p}_k$ is the eigenvector $\mathbf{v}_l$ associated with eigenvalue λ_l. Then from (59) we have that

$$\nu_k = \left(1-\frac{1+\sigma_n^2\delta_k}{1+\eta_k^2}\right)\frac{1}{\lambda_l} + \frac{\sigma_n^2}{1+\eta_k}\frac{1}{\lambda_l^2}. \tag{60}$$

Now $\eta_k = \mathbf{p}_k^H \mathbf{R}_k^{-1} \mathbf{p}_k = 1/\lambda_l$ and $\delta_k = \mathbf{p}_k^H \mathbf{R}_k^{-2} \mathbf{p}_k = 1/\lambda_l^2$, so that this simplifies to

$$\nu_k = \frac{1}{\lambda_l(1+\lambda_l)} + \frac{2}{1+\lambda_l}. \tag{61}$$

Clearly, ν_k is a decreasing function of λ_l so that choosing the minimum λ_l maximizes ν_k, which in turn minimizes the sum MSE.

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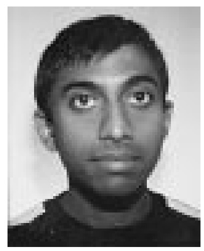

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