Asymptotic Analysis of LMMSE Multiuser Receivers for Multi-Signature Multicarrier CDMA in Rayleigh Fading

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***Abstract*—This paper considers a multicarrier (MC) code-division multiple-access system where each user employs multiple signatures. The receiver is linear and minimizes the mean square error of the data estimate. Both multiple-user and single-user systems are considered, as well as single and multiple signatures per user. In each case, an asymptotic analysis is used to derive the output signal-to-interference-plus-noise ratio (SINR) as a function of the system loading, the noise power, and the fading properties of the channel. Asymptotic in this case means that the number of independent subcarriers and number of signatures per user each tends to infinity with fixed ratio. The associated bit-error rate (BER) is evaluated for binary phase-shift keying symbols. Simulations show that the asymptotic SINRs and BERs derived in each case are accurate for realistic finite systems.**

***Index Terms*—Fading channels, large-system analysis, multicarrier code-division multiple access (MC-CDMA), multicarrier modulation, multi-signature.**

I. Introduction

BROADBAND wireless networks require transmission schemes, which are resilient to both fading and frequency selectivity, while allowing communication at high data rates and high mobility rates. Multicarrier code-division multiple access (MC-CDMA) is well suited to meet such requirements. This is mainly due to its implicit frequency diversity gain over alternative schemes such as orthogonal frequency-division multiplexing (OFDM), and its flexibility allowing multiple signatures per user, thus achieving high data rates.

We consider the uplink of a multiuser, multi-signature MC-CDMA system where each user spreads multiple data bits across multiple subcarriers using a set of frequency-domain signatures, as in [1] and [2]. A cyclic prefix may be inserted to counter the intersymbol interference (ISI) due to multipath. We consider the receiver structure, whereby symbol detection is performed via a set of frequency-domain linear minimum mean-square error (LMMSE) filters. We shall show that the asymptotic signal-to-interference-plus-noise ratio (SINR) can be derived as a function of the system load, the noise power, received powers, and the fading properties of the channel. Equally importantly, the average asymptotic bit-error rate (BER) can be predicted from the analytic results we derive in this paper. Here, asymptotic means that the number of signatures K, and the number of subcarriers N, both tend to infinity, while their ratio is held fixed at a value we call the *system load*.

There has been a large body of related work on MC-CDMA, including a wide range of system design issues such as channel estimation [3], frequency-offset correction [4], and transmitter and receiver optimization [5]. In this paper, we focus attention on the analysis of SINR performance. Recent related results in this area include a lower bound on the BER of MC-CDMA for a single user, presented in [6]. BER analysis has also been performed for a RAKE receiver [7], a maximum ratio combining (MRC) receiver [8], and for the LMMSE receiver in fully loaded systems [9]. A pairwise error probability (PEP) analysis of the LMMSE receiver has also been presented in [10]. These results were derived either using the characteristic function method, or, in most cases, by resorting to a Gaussian approximation for the inter-signature interference.

This paper considers an asymptotic (large-system) analysis approach. This approach has led in the past to important results for single-signature direct-sequence (DS)-CDMA systems with random signatures in additive white Gaussian noise (AWGN) and flat-fading channels (e.g., [11]–[15]). In that case, as the number of users and the processing gain tend to infinity, with fixed ratio, the random SINR expression converges to a deterministic quantity [11]. Also, large-system asymptotic results have been shown to be accurate for many realistic (finite) systems (e.g., see [16]), and hence, are a useful design tool. Asymptotic analysis of the MC-CDMA downlink has been considered in [17]. Here we consider asymptotic analysis of uplink multi-signature MC-CDMA with multiple signatures per user.

In this paper, we derive the asymptotic output SINR of an LMMSE receiver for MC-CDMA with frequency-selective Rayleigh fading channels. Both multiple-user and single-user systems are considered, as well as single and multiple signatures per user. In each case, the SINR is determined by solving a set of fixed-point equations. Simulation results show the

Paper approved by C. Tellambura, the Editor for Modulation and Signal Design of the IEEE Communications Society. Manuscript received August 28, 2002; revised October 7, 2003. This work was supported in part by the U.S. Army Research Office under DAAD19-99-1-0288, and in part by CSIRO Telecommunications and Industrial Physics, Australia. This paper was presented in part at the IEEE International Symposium on Spread Spectrum Techniques and Applications, Prague, Czech Republic, September 2002, and in part at the IEEE International Conference on Communications, Anchorage, AK, May 2003.

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Digital Object Identifier 10.1109/TCOMM.2004.829519

asymptotic analysis accurately predicts the average SINR for finite systems, such as those with only 16 subcarriers. Importantly, the computational cost of a Monte Carlo simulation of the MC-CDMA system far exceeds the cost of computing a numerical solution to our fixed-point equations.

For the multiuser, multi-signature model considered, the analysis depends on characterizing the asymptotic eigenvalue distribution (a.e.d.) of a sum of random matrices. This is accomplished through a free-matrix approximation and an application of the R-transform [18] (see also [19], which uses the R-transform in a different, but related, context.) The matrix approximation is shown to be accurate over a wide range of system parameters of interest.

The paper proceeds as follows. The system model used in our analysis is described in Section II, and the SINR expression for the LMMSE receiver is given in Section IV. Necessary theorems and definitions for the analysis are given in Section III. Following this, the fixed-point analytic expressions for the asymptotic SINR in each model are derived in Sections V and VI. Simulation results are presented in Section VIII, which compare the asymptotic results with the analogous results for finite-size systems.

II. System Model

A. Transmitter Model

An MC-CDMA symbol can be interpreted as an OFDM symbol with data symbols spread across all frequencies via spreading signatures (or spreading codes) in the frequency domain [1]. We consider the uplink of a multi-signature MC-CDMA system with J synchronous users. We use K to denote the total number of signatures used, K_j to denote the number of signatures assigned to user $j \in \mathcal{J}$, where $\mathcal{J} = \{1, \ldots, J\}$, and N to denote the spreading gain, which is also the number of subcarriers in the multicarrier system. We use the term *system load* for the ratio $\alpha = \sum_{j \in \mathcal{J}} K_j/N$, and per-user system load for the the ratio $\alpha_j = K_j/N$.

In matrix notation,[1] we denote the vector of K_j data symbols for the jth user as $\mathbf{b}_j = [b_{j,1}, b_{j,2}, \ldots, b_{j,K_j}]^T$. For ease of exposition, we will absorb the different transmit power levels of the users into the channel model, and hence, we may assume unit power, zero mean, independent and identically distributed (i.i.d.) data symbols without loss of generality, so that $\mathbf{E}[\mathbf{b}_j \mathbf{b}_j^\dagger] = \mathbf{I}_{K_j}$.

The matrix $\mathbf{S}_j$ is the jth user's $N \times K_j$ spreading matrix, where the kth column of $\mathbf{S}_j$ is the spreading signature $\mathbf{s}_{j,k}$. For purposes of analysis, we assume randomly assigned signatures, where each element of $\mathbf{S}$ is an i.i.d. circularly symmetric complex random variable, with zero mean and variance $1/N$. The asymptotic performance results do not depend on the particular distribution of the elements, and moreover, in this case, the asymptotic ($N \to \infty$) SINR of the MMSE receiver converges almost surely to a deterministic value.

[1]**Notation:** all vectors are defined as column vectors and designated with bold lower case; all matrices are given in bold upper case; $(\cdot)^T$ denotes transpose; $(\cdot)^\dagger$ denotes Hermitian (i.e., complex conjugate) transpose; $\mathrm{tr}[\cdot]$ denotes the matrix trace; $\mathrm{Tr}[\cdot]$ denotes the normalized matrix trace $\mathrm{tr}[\cdot]/N$; and $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. Expectation is denoted $\mathbf{E}[\cdot]$.

B. Channel Model

We assume that all users transmit their symbols synchronously, and any interference between successive MC-CDMA symbols (ISI) due to multipath is removed by the insertion of a cyclic prefix of length $G \geq M$. The signal-to-noise ratio (SNR) loss due to this prefix is not considered throughout this paper, as this loss goes to zero when we evaluate the asymptotic SINR in Sections V and VI. The received vector in the frequency domain is given by

$$\mathbf{z} = \sum_{j \in \mathcal{J}} \mathbf{H}_j \mathbf{S}_j \mathbf{b}_j + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is a $N \times 1$ vector of i.i.d. circularly symmetric complex Gaussian random variables, with zero mean and variance σ_n^2. The frequency-selective channel matrix for the jth user, $\mathbf{H}_j$, is diagonal with the nth diagonal element $H_j(n)$ given by the jth user's channel frequency response in the nth subcarrier. Note that the $\{H_j(n)\}_{n \in \{1,\ldots,N\}}$ are zero-mean circularly symmetric complex Gaussian random variables with variance p_j (i.e., Rayleigh fading). The average received SNR for signature k of user j is $\mathrm{SNR}_{j,k} = \mathbf{s}_{j,k}^\dagger \mathbf{s}_{j,k}\, p_j/\sigma_n^2$.

Note that in the asymptotic analysis of Sections V and VI, we assume infinite frequency diversity. That is, as $N \to \infty$, we assume that the bandwidth of the signal increases, as opposed to keeping the bandwidth constant and narrowing the subcarrier spacing. This is an important assumption, since in the latter case, the distribution of the channel matrix would depend on the coherence bandwidth of the channel, and it is possible that the SINR would not converge in the almost-sure sense asymptotically. This assumption implies that as $N \to \infty$, the empirical distribution of the N subchannels, which form a correlated random process, almost surely converges in distribution to the first-order probability distribution.

In this paper, we take a number of asymptotic limits, all of which let N and $K_j \to \infty$ with $K_j/N \to \alpha_j$ for each $j \in \mathcal{J}$, and will be denoted simply by "lim." Also, due to the assumption of infinite frequency diversity, the a.e.d. of the matrix $\mathbf{P}_j \triangleq \mathbf{H}_j \mathbf{H}_j^\dagger$ converges to the exponential distribution with mean p_j, but with the following caveat. In order to establish the convergence of various measures considered, we need to assume that the a.e.d. of $\mathbf{P}_j$ has compact support, and that there are no channel nulls, (i.e., $\mathbf{H}^{-1}$ exists).[2] So technically, we must assume a truncated exponential distribution, where the truncation value can be arbitrarily large. It can be verified that the associated truncation error vanishes as the truncation value tends to infinity.

III. Definitions, Identities, and Preliminary Theorems

Definition 1: For an $N \times N$ Hermitian matrix $\mathbf{A}_N$ with eigenvalues $\lambda_1 \ldots \lambda_N$, the empirical distribution function (e.d.f.) of the eigenvalues is defined as [20]

$$F_{\mathbf{A}}^N(\lambda) = \frac{1}{N} \cdot \text{cardinality}\{\lambda_i : \lambda_i \leq \lambda\}. \tag{2}$$

[2]If channel nulls exist, these subcarriers can be ignored by simply removing the appropriate rows and columns from $\mathbf{H}$ and $\mathbf{S}$. Therefore, this assumption does not limit the generality of the final result.

Under certain conditions, as $N \to \infty$ the e.d.f. for certain random matrices converges to a fixed distribution, $F_{\mathbf{A}}(\lambda)$. It is often useful to express this distribution in terms of its Stieltjes transform.

Definition 2: The **Stieltjes Transform** of the distribution $F_{\mathbf{A}}$ is defined as [20]

$$G_{\mathbf{A}}(z) = \int \frac{1}{\lambda - z} \mathrm{d}F_{\mathbf{A}}(\lambda), \quad \text{for } z \in \mathbb{C}^+ \tag{3}$$

where $\mathbb{C}^+ = \{z \in \mathbb{C} \mid \mathrm{Im}[z] > 0\}$.

Theorem 1: [21] Let $\mathbf{B}_N = \mathbf{A}_N + \mathbf{X}_N\mathbf{T}_N\mathbf{X}_N^\dagger$, where $\mathbf{X}_N$ is an $N \times K$ matrix of i.i.d. complex random variables with zero mean and variance $1/N$, and assume that $\lim_{N\to\infty} K/N = \alpha$. Let $\mathbf{T}_N = \mathrm{diag}(\tau_1^N \ldots \tau_K^N)$, $\tau_i^N \in \mathbb{R}$, and the e.d.f. of $\{\tau_1^N \ldots \tau_K^N\}$ almost surely converges vaguely to a cumulative distribution function (c.d.f.) $F_{\mathbf{T}}(\lambda)$ as $N \to \infty$. Let $\mathbf{A}_N$ be Hermitian $N \times N$ for which the e.d.f. of its eigenvalues, $F_{\mathbf{A}}^N(\lambda)$, converges vaguely to the nonrandom (possibly defective) d.f. $F_{\mathbf{A}}(\lambda)$ almost surely. Let $\mathbf{A}_N$, $\mathbf{X}_N$, and $\mathbf{T}_N$ be independent. Then, almost surely, $F_{\mathbf{B}_N}^N(\lambda)$, the e.d.f. of the eigenvalues of $\mathbf{B}_N$, converges vaguely, as $N \to \infty$, to a (nonrandom) d.f. $F_{\mathbf{B}}(\lambda)$, whose Stieltjes transform $G_{\mathbf{B}}(z)$ satisfies

$$G_{\mathbf{B}}(z) = G_{\mathbf{A}}\left(z - \alpha \int \frac{\lambda}{1 + \lambda G_{\mathbf{B}}(z)} \mathrm{d}F_{\mathbf{T}}(\lambda)\right). \tag{4}$$

Theorem 2: [22] Let $\mathbf{C}_N = \mathbf{X}_N\mathbf{X}_N^H\mathbf{D}_N$, where $\mathbf{X}_N$ is as defined in *Theorem 1*, and $\mathbf{D}_N$ is $N \times N$ random Hermitian nonnegative definite, with $F_{\mathbf{D}}^N(\lambda)$ converging almost surely in distribution to a c.d.f. on $[0, \infty)$ as $N \to \infty$. Then, almost surely, $F_{\mathbf{C}}^N(\lambda)$ converges in distribution as $N \to \infty$ to a (nonrandom) d.f. $F_{\mathbf{C}}(\lambda)$ whose Stieltjes transform satisfies

$$G_{\mathbf{C}}(z) = \int \frac{1}{\lambda(\alpha - 1 - zG_{\mathbf{C}}(z)) - z} \mathrm{d}F_{\mathbf{D}}(\lambda). \tag{5}$$

Lemma 1: For the definitions and conditions of *Theorem 1*, except where the X_{ij}^N have variance σ_X^2/N for all i, j, and N

$$G_{\mathbf{B}}(z) = G_{\mathbf{A}}\left(z - \alpha \int \frac{\lambda}{\sigma_X^{-2} + \lambda G_{\mathbf{B}}(z)} \mathrm{d}F_{\mathbf{T}}(\lambda)\right). \tag{6}$$

Proof: The proof consists of considering $\mathbf{B}'_N = \mathbf{B}_N/\sigma_X^2$, using *Theorem 1*, and the identity $G_{\mathbf{B}}(z) = G_{\mathbf{B}'}(z/\sigma_X^2)/\sigma_X^2$.

Lemma 2: Define

$$\mathbf{B}_N^J = \mathbf{A}_N + \sum_{j\in\mathcal{J}} \mathbf{X}_{N,j}\mathbf{T}_{N,j}\mathbf{X}_{N,j}^\dagger \tag{7}$$

where $\mathbf{X}_{N,j}$ is a $K_j \times N$ matrix, as defined in *Theorem 1*, except that each element has variance p_j, and the $\mathbf{A}_N$, $\mathbf{X}_{N,j}$, and $\mathbf{T}_{N,j}$ are mutually independent across j. Define $\alpha_j = K_j/N$. Then

$$G_{\mathbf{B}^J}(z) = G_{\mathbf{A}}\left(z - \sum_{j\in\mathcal{J}} \alpha_j \int \frac{\lambda}{p_j^{-1} + \lambda G_{\mathbf{B}^J}(z)} \mathrm{d}F_{\mathbf{T}_j}(\lambda)\right). \tag{8}$$

Proof: The proof is by induction. The statement is true for $J = 1$, according to *Lemma 1*. Now, assume that (8) is true for some $J - 1$. Then, according to *Lemma 1*

$$\begin{aligned} G_{\mathbf{B}^J}(z) &= G_{\mathbf{B}^{J-1}}\left(z - \alpha_J \int \frac{\lambda}{p_J^{-1} + \lambda G_{\mathbf{B}^J}(z)} \mathrm{d}F_{\mathbf{T}_J}(\lambda)\right) \\ &= G_{\mathbf{A}}\left(z - \sum_{j\in\mathcal{J}} \alpha_j \int \frac{\lambda}{p_j^{-1} + \lambda G_{\mathbf{B}^J}(z)} \mathrm{d}F_{\mathbf{T}_j}(\lambda)\right). \end{aligned}$$

In what follows, we will need to evaluate the Stieltjes transform associated with the sum of random matrices. This is accomplished with the aid of the R-transform, and its associated additivity property.

Definition 3: The **R-Transform** of a distribution in terms of its Stieltjes transform $G_{\mathbf{B}}(z)$ is defined as [18]

$$R_{\mathbf{B}}(t) = \frac{1}{t} + G_{\mathbf{B}}^{-1}(t) \tag{9}$$

where $G_{\mathbf{B}}^{-1}$ is the inverse function (with respect to composition). We also have

$$G_{\mathbf{B}}(z) = \frac{1}{-z + R_{\mathbf{B}} \circ G_{\mathbf{B}}(z)}. \tag{10}$$

We will also refer to $R_{\mathbf{B}}$ as the R-transform of $\mathbf{B}$.

The following important property of the R-transform is based on the theory of free probability. The notion of freeness in free probability theory is analogous to independence in classical probability theory. For more precise definitions and related properties, see [23].

Identity 1: [18] If $\mathbf{C}_N = \mathbf{A}_N + \mathbf{B}_N$, and $\mathbf{A}_N$ and $\mathbf{B}_N$ form a free family in the large matrix limit, then $R_{\mathbf{C}}(t) = R_{\mathbf{A}}(t) + R_{\mathbf{B}}(t)$.

The computation of the distribution of the sum of free random matrices is often referred to as the *additive free convolution* of the component distributions.

IV. Asymptotic LMMSE SINR

We consider a standard linear MMSE multiuser receiver, where the estimate of the symbol carried on the kth signature of the jth user is given by

$$\widehat{b}_{j,k} = \mathbf{s}_{j,k}^\dagger\mathbf{H}_j^\dagger\mathbf{R}^{-1}\mathbf{z}$$

where

$$\mathbf{R} = \sigma_n^2\mathbf{I}_N + \sum_{u\in\mathcal{J}} \mathbf{H}_u\mathbf{S}_u\mathbf{S}_u^\dagger\mathbf{H}_u^\dagger. \tag{11}$$

The SINR for the kth signature of user j is

$$\mathrm{SINR}_{j,k} \triangleq \gamma_{j,k} = \mathbf{s}_{j,k}^\dagger\mathbf{H}_j^\dagger\mathbf{R}_{(j,k)-}^{-1}\mathbf{H}_j\mathbf{s}_{j,k} \tag{12}$$

where $\mathbf{R}_{(j,k)-}$ is $\mathbf{R}$ with the (j,k)th signature removed.

Due to [14, Lemma 1], and the fact that the asymptotic distributions of $\mathbf{R}$ and $\mathbf{R}_{(j,k)-}$ are the same, we may state

$$\gamma_j^\infty \triangleq \lim \mathrm{SINR}_{j,k} \tag{13}$$

$$= \lim \mathrm{Tr}\left[\mathbf{H}_j^\dagger \mathbf{R}_{(j,k)-}^{-1}\mathbf{H}_j\right] \tag{14}$$
$$= \lim \mathrm{Tr}\left[\mathbf{P}_j \mathbf{R}^{-1}\right] \tag{15}$$

where $\mathrm{Tr}[\cdot]$ denotes the normalized trace $(1/N)\mathrm{tr}[\cdot]$. We observe that the SINR for each signature of user j is the same asymptotically. Therefore, we have omitted the index k from γ_j^∞ in (13) and all analysis which follows.

In order to establish the convergence of (15), we first consider the convergence of the e.d.f. of $\mathbf{RP}_j^{-1}$. The positive moments $\lim \mathrm{Tr}[(\mathbf{RP}_j^{-1})^k]$, $k > 0$, can be shown to converge under the truncated distribution assumption for $\mathbf{P}_j$ [24]. This implies the almost-sure convergence of the e.d.f. of $\mathbf{RP}_j^{-1}$. Moreover, this implies that the expected value of any bounded continuous function of the eigenvalues of $\mathbf{RP}_j^{-1}$ converges almost surely [25, Th. 4.4.1]. In particular, the function $1/\lambda$, on the truncated support, is bounded and continuous, which gives the almost-sure convergence of the e.d.f. of $\mathbf{P}_j\mathbf{R}^{-1}$. Finally, this implies that the mean of the e.d.f. of $\mathbf{P}_j\mathbf{R}^{-1}$, (i.e., γ_j^∞) converges almost surely.

V. Asymptotic Analysis: Single-User Case

In this section, we assume that a single user transmits at any one time. For notational simplicity, in this section, we often drop the user subscript j. Here we derive the asymptotic SINR using techniques similar to those used for DS-CDMA in [11], [12], and [14]. We also give a SINR expression for fixed channels. In Section VI, we consider the more general multiuser multi-signature case.

Note that (15) can be written as

$$\gamma^\infty = \lim \mathrm{Tr}\left[\overline{\mathbf{R}}^{-1}\right] \tag{16}$$

where $\overline{\mathbf{R}} = \mathbf{SS}^\dagger + \sigma_n^2\mathbf{P}^{-1}$. Since $\overline{\mathbf{R}}$ is Hermitian, it is possible to find a unitary matrix $\mathbf{V}$ of eigenvectors and a corresponding diagonal matrix $\mathbf{\Lambda}$ of eigenvalues, such that $\overline{\mathbf{R}} = \mathbf{V\Lambda V}^\dagger$, and therefore, (16) can be rewritten as

$$\gamma^\infty = \lim \mathrm{Tr}\left[\Lambda^{-1}\right] = \lim \frac{1}{N}\sum_{i=1}^{N}\frac{1}{\lambda_i} = \int \frac{1}{\lambda}\mathrm{d}F_{\overline{\mathbf{R}}}(\lambda)$$
$$= \lim_{z\to 0} G_{\overline{\mathbf{R}}}(z) \tag{17}$$

where λ_i is the ith eigenvalue of $\overline{\mathbf{R}}$, and $G_{\overline{\mathbf{R}}}(z)$ and $F_{\overline{\mathbf{R}}}(\lambda)$ are as defined in (3) and (2), respectively. Note that the last expression is written as a limit since, strictly speaking, the Stieltjes transform is not defined on the real axis.

Applying *Theorem 1* to the matrix $\overline{\mathbf{R}}$, and evaluating at $z = 0$ gives the following expression for $G_{\overline{\mathbf{R}}}(0)$, or equivalently, γ^∞ as defined in (17):

$$\gamma^\infty = G_{\mathbf{B}}\left(-\frac{\alpha}{1+\gamma^\infty}\right) \tag{18}$$

where $\mathbf{B} = \sigma_n^2\mathbf{P}^{-1}$.

To complete the solution, we seek the a.e.d. of $\mathbf{B} = \sigma_n^2\mathbf{P}^{-1}$. Despite the correlation between adjacent subcarrier gains, the e.d.f. of the eigenvalues of $\mathbf{B}$ converges in distribution to the c.d.f. of a diagonal element of $\mathbf{B}$, due to the assumption of infinite frequency diversity. Since the subcarrier gains $\{H(n)\}_{n\in\{1,\dots,N\}}$, are complex Gaussian with variance p_1^2, the powers $\{|H(n)|^2\}_{n\in\{1,\dots,N\}}$ are exponentially distributed with c.d.f. $F_{\mathbf{P}}(\lambda) = 1 - \exp(-\lambda/p_1)$. Hence, the a.e.d. of $\mathbf{B}$ is given by

$$F_{\mathbf{B}}(\lambda) = \exp\left(-\frac{\sigma_n^2}{p_1\lambda}\right), \quad \text{for } 0 < \lambda < \infty. \tag{19}$$

Using (3) and (19), we obtain

$$G_{\mathbf{B}}(z) = \frac{\sigma_n^2}{p_1}\int_0^\infty \frac{1}{\lambda^2(\lambda - z)}\exp\left(-\frac{\sigma_n^2}{p_1\lambda}\right)\mathrm{d}\lambda$$
$$= -\frac{1}{z} - \frac{\kappa}{z^2}\exp\left(-\frac{\kappa}{z}\right)\mathrm{Ei}\left(-\frac{\kappa}{z}\right) \tag{20}$$

where $\kappa = \sigma_n^2/p_1$ and $\mathrm{Ei}(\cdot)$ is the exponential integral with $n = 1$, given by

$$\mathrm{Ei}(x) = \int_1^\infty e^{-xt}t^{-1}\mathrm{d}t. \tag{21}$$

The main result of this section is now obtained in the form of a fixed-point equation for the average asymptotic SINR for single-user multi-signature MC-CDMA. Combining (20) and (18) gives

$$\boxed{\gamma^\infty = \overline{\gamma}^\infty - \kappa\,(\overline{\gamma}^\infty)^2\exp(\kappa\,\overline{\gamma}^\infty)\mathrm{Ei}(\kappa\,\overline{\gamma}^\infty)} \tag{22}$$

where $\overline{\gamma}^\infty = (1+\gamma^\infty)/\alpha$. This equation predicts the asymptotic SINR for MC-CDMA as a function of only the system load, α, the noise power, and the variance of the subcarrier gains. This equation has no closed-form solution, and hence, we must solve it numerically.

Note that for finite frequency diversity, the asymptotic LMMSE SINR does not converge unless the limit is conditioned on the channel. Consider the case of a known channel with L equal-width coherence bands. Each subcarrier within coherence band ℓ is assumed to have the same channel coefficient, h_ℓ. Letting $\rho_\ell = \sigma_n^2/|h_\ell|^2$ leads to

$$\gamma_L^\infty = \frac{1}{L}\sum_{\ell=1}^{L}\frac{1}{\rho_\ell + \frac{\alpha}{1+\gamma_L^\infty}}.$$

Here γ_L^∞ denotes the approximate asymptotic SINR with L coherence bands.

VI. Asymptotic Analysis: Multiuser Case

We now assume that multiple users transmit simultaneously. While this is a generalization of the model considered in the previous section, the analysis is not a straightforward extension.

A. Single Signature per User

Consider the case where $K_j = 1$ for all j. We will evaluate the SINR for each user as $(N, J) \to \infty$ with fixed J/N. Moreover, assume that there are only P_{tot} possible received powers $\{p(i)\}_{i\in\{1,\dots,P_{\mathrm{tot}}\}}$, and that the proportion of users within the ith power class remains fixed at q_i. (This restriction is relaxed later.) In this section, we will assume uncorrelated subchannel gains. Since (13) converges almost surely, it must converge to its expectation. If we average (14) over the distribution of the jth channel in the limit, then we obtain

$$\gamma_j^\infty = p(i_j)\lim \mathrm{Tr}\left[\mathbf{R}_{(j,1)-}^{-1}\right] \tag{23}$$

since $\mathbf{R}_{(j,1)-}^{-1}$ and $\mathbf{P}_j$ are independent, and where i_j is the power class for user j. Since the a.e.d. of $\mathbf{R}_{(j,1)-}$ is the same as the a.e.d. of $\mathbf{R}$, we can state

$$\gamma_j^\infty = p(i_j)G_{\mathbf{R}}(z=0). \tag{24}$$

Now, to determine $G_{\mathbf{R}}(z)$, we use *Lemma 2*. Note that in this scenario, $\mathbf{R}$ can be written as

$$\mathbf{R} = \sigma_n^2\mathbf{I}_N + \sum_{i=1}^{P_{\rm tot}} \widehat{\mathbf{S}}_i\widehat{\mathbf{S}}_i^\dagger \tag{25}$$

where $\widehat{\mathbf{S}}_i$ contains the channel-modified signatures $\mathbf{H}_j\mathbf{s}_{j,1}$ of all the users in the ith power class. Note that each element of $\widehat{\mathbf{S}}_i$ is the product of a random signature element and a random complex Gaussian channel value, and hence, the elements of $\widehat{\mathbf{S}}_i$ have zero mean, variance $p(i)/N$, and are i.i.d. Therefore, $\widehat{\mathbf{S}}_i$ meets the assumptions on the variable $\mathbf{X}_{N,j}$ in the statement of *Lemma 2*, and we can apply *Lemma 2* to (25), with $\mathbf{T}_{N,j} = \mathbf{I}_N$, to obtain

$$G_{\mathbf{R}}(z) = G_{\sigma_n^2\mathbf{I}}\left(z - \sum_{i=1}^{P_{\rm tot}} \frac{q_i}{p_i^{-1} + G_{\mathbf{R}}(z)}\right).$$

Since $G_{\sigma_n^2\mathbf{I}}(z) = (\sigma_n^2 - z)^{-1}$, we find

$$\frac{1}{G_{\mathbf{R}}(0)} = \sigma_n^2 + \sum_{i=1}^{P_{\rm tot}} \frac{q_i}{p_i^{-1} + G_{\mathbf{R}}(0)}$$

which is simply extended to any density of user powers $f_P(p)$

$$\frac{1}{G_{\mathbf{R}}(0)} = \sigma_n^2 + \alpha\int_0^\infty \frac{f_P(p)}{p^{-1} + G_{\mathbf{R}}(0)}\mathrm{d}p$$

where $f_P(p)\mathrm{d}p$ is the proportion of users with average power p. This must be solved numerically.

In summary, the following equations give the asymptotic SINR for single-signature multiuser MC-CDMA:

$$\gamma_j^\infty = p_jG_{\mathbf{R}}(0) \tag{26}$$

$$\frac{1}{G_{\mathbf{R}}(0)} = \sigma_n^2 + \alpha\int_0^\infty \frac{f_P(p)}{p^{-1} + G_{\mathbf{R}}(0)}\mathrm{d}p. \tag{27}$$

Note that this is identical to the so-called Tse–Hanly formula [11]. This shows that the asymptotic MMSE SINR of MC-CDMA with infinite frequency diversity in a frequency-selective channel is the same as the asymptotic MMSE SINR of DS-CDMA in an AWGN channel.

B. Multiple Signatures per User

We now assume multiple users with multiple signatures, and evaluate the SINR as N and $K_j \to \infty$ with fixed K_j/N. The number of users J is held fixed in this case. The main difficulty in this section comes from the fact that the channel for each user is the same across all assigned signatures, but is different from and independent of the other users' channels.

Consider the asymptotic SINR in the multiuser case, given by (15). If we were now to attempt an asymptotic analysis of the SINR, directly following the technique used for DS-CDMA in [11] (where $\mathbf{P}_j = \mathbf{I}$), and for single-user multi-signature MC-CDMA in Section V, the next step would be to attempt to determine the Stieltjes transform of the a.e.d. of $\mathbf{P}_j\mathbf{R}^{-1}$. However, this is not straightforward, so that we now seek an alternate approach.

Consider the following identity [26, Ch.4]:

$$\begin{aligned} 1 &= \mathrm{Tr}\left[\mathbf{R}^{-1}\mathbf{R}\right] \\ &= \mathrm{Tr}\left[\mathbf{R}^{-1}\left(\sum_{j\in\mathcal{J}}\mathbf{H}_j\mathbf{S}_j\mathbf{S}_j^\dagger\mathbf{H}_j^\dagger + \sigma_n^2\mathbf{I}\right)\right] \\ &= \sigma_n^2\mathrm{Tr}\left[\mathbf{R}^{-1}\right] + \mathrm{Tr}\left[\mathbf{R}^{-1}\sum_{j\in\mathcal{J}}\mathbf{H}_j\mathbf{S}_j\mathbf{S}_j^\dagger\mathbf{H}_j^\dagger\right]. \end{aligned} \tag{28}$$

Examining the last term on the right-hand side of (28) gives

$$\begin{aligned} &\mathrm{Tr}\left[\mathbf{R}^{-1}\sum_{j\in\mathcal{J}}\mathbf{H}_j\mathbf{S}_j\mathbf{S}_j^\dagger\mathbf{H}_j^\dagger\right] \\ &\quad= \sum_{j\in\mathcal{J}}\sum_{k=1}^{K_j}\mathrm{Tr}\left[\mathbf{R}^{-1}\mathbf{H}_j\mathbf{s}_{j,k}\mathbf{s}_{j,k}^\dagger\mathbf{H}_j^\dagger\right] \\ &\quad= \frac{1}{N}\sum_{j\in\mathcal{J}}\sum_{k=1}^{K_j}\frac{\mathbf{s}_{j,k}^\dagger\mathbf{H}_j^\dagger\mathbf{R}_{(j,k)-}^{-1}\mathbf{H}_j\mathbf{s}_{j,k}}{1 + \mathbf{s}_{j,k}^\dagger\mathbf{H}_j^\dagger\mathbf{R}_{(j,k)-}^{-1}\mathbf{H}_j\mathbf{s}_{j,k}} \\ &\quad= \frac{1}{N}\sum_{j\in\mathcal{J}}\sum_{k=1}^{K_j}\frac{\gamma_{j,k}}{1 + \gamma_{j,k}}. \end{aligned} \tag{29}$$

Therefore, as $N, K_j \to \infty$ while $K_j/N \to \alpha_j$, each $\gamma_{j,k} \to \gamma_j^\infty$, and hence, (28) and (29) give

$$1 = \sigma_n^2G_{\mathbf{R}}(0) + \sum_{j\in\mathcal{J}}\alpha_j\frac{\gamma_j^\infty}{1 + \gamma_j^\infty}. \tag{30}$$

To compute $G_{\mathbf{R}}(z)$ for this multi-signature case, we use *Theorem 2* to first compute $G_{\mathbf{R}_j}(z)$, the Stieltjes transform of the a.e.d. of $\mathbf{R}_j = \mathbf{H}_j\mathbf{S}_j\mathbf{S}_j^\dagger\mathbf{H}_j^\dagger$. Note that $\mathbf{R}_j$ has the same eigenvalues as $\mathbf{S}_j\mathbf{S}_j^\dagger\mathbf{P}_j$. Since we assume infinite frequency diversity and the subcarrier values are complex Gaussian, the a.e.d. of $\mathbf{P}_j$ is an exponential distribution with mean equal to p_j, the average received power of user j. We therefore have

$$G_{\mathbf{R}_j}(z) = -\frac{1}{z}f\left(\frac{p_j(1-\alpha_j)}{z} + G_{\mathbf{R}_j}(z)\right) \tag{31}$$

where

$$f(x) = x^{-1}\exp(x^{-1})\ \mathrm{Ei}\ (x^{-1}) \tag{32}$$

and $\mathrm{Ei}(\cdot)$ is given by (21).

We would now like to use the a.e.d.s of the $\mathbf{R}_j$ matrices to find the a.e.d. of $\mathbf{R}$. If the $\mathbf{R}_j$'s were asymptotically free, we would be able to use the R-transform and *Identity 1* directly. It turns out, though, that they are not asymptotically free. To our knowledge, there are no existing methods for computing the a.e.d. of sums of nonfree matrices of this form.

We proceed to approximate the $\mathbf{R}_j$ by a set of equivalent asymptotically free matrices, as follows. From [23, Th. 4.3.5], we know that unitarily invariant random matrices with limit distributions having bounded support are asymptotically free. It is shown in [24] that $\mathbf{R}$ and each $\mathbf{R}_j, j \in \mathcal{J}$, has a limit distribution with bounded support. We will, therefore, approximate $\mathbf{R}$ by

$$\underline{\mathbf{R}} = \sigma_n^2 \mathbf{I}_N + \sum_{j \in \mathcal{J}} \underline{\mathbf{R}}_j \tag{33}$$

$$\underline{\mathbf{R}}_j = \mathbf{U}_j \mathbf{H}_j \mathbf{S}_j \mathbf{S}_j^\dagger \mathbf{H}_j^\dagger \mathbf{U}_j^\dagger \tag{34}$$

where $\{\mathbf{U}_j\}_{j \in \{1 \ldots J\}}$ is a set of independent random unitary $N \times N$ matrices. Note that $\underline{\mathbf{R}}_j$ has the same eigenvalues as $\mathbf{H}_j \mathbf{S}_j \mathbf{S}_j^\dagger \mathbf{H}_j^\dagger$, however, the important difference is that the $\underline{\mathbf{R}}_j$ and $\sigma_n^2 \mathbf{I}_N$ matrices form an asymptotically free family. This allows the use of the R-transform to compute the distribution of the sum given in (33), which contains a single constant matrix and J unitarily invariant matrices with limit distributions on a compact support.

The approximation of $\mathbf{R}$ by $\underline{\mathbf{R}}$ leads us to calculate $\beta_0 \triangleq \lim \mathrm{Tr}[\underline{\mathbf{R}}^{-1}]$ in place of $G_{\mathbf{R}}(0) = \lim \mathrm{Tr}[\mathbf{R}^{-1}]$ in (30). In Section VIII, we present numerical results which show that this approximation is very accurate over a wide range of parameters (SNRs and user loads α_j). Furthermore, the first few asymptotic moments of $\mathbf{R}$ can be computed using a similar technique to that presented in [27, App. B]. Comparing with the moments of $\underline{\mathbf{R}}$ shows that the first three moments are identical, and that the difference in fourth moments increases with load α_j and SNR, but is relatively small over a wide range of SNRs and loads. Because the output SINR can be accurately approximated by a reduced-rank expression that depends only on the first few positive moments of $\mathbf{R}$ [27], this moment analysis indicates that the asymptotic SINR obtained from using $\underline{\mathbf{R}}$ instead of $\mathbf{R}$ should be quite close to the actual asymptotic SINR.

We now use *Definition 3* and *Identity 1* to compute the Stieltjes transform of (33) as follows:

$$\begin{aligned} R_{\underline{\mathbf{R}}}(t) &= \sigma_n^2 + \sum_{j \in \mathcal{J}} R_{\underline{\mathbf{R}}_j}(t) \\ &= \sigma_n^2 + \frac{J}{t} + \sum_{j \in \mathcal{J}} G_{\underline{\mathbf{R}}_j}^{-1}(t) \\ \frac{1}{G_{\underline{\mathbf{R}}}(z)} &= -z + R_{\underline{\mathbf{R}}}(G_{\underline{\mathbf{R}}}(z)) \\ &= -z + \sigma_n^2 + \frac{J}{G_{\underline{\mathbf{R}}}(z)} + \sum_{j \in \mathcal{J}} G_{\underline{\mathbf{R}}_j}^{-1}(G_{\underline{\mathbf{R}}}(z)). \end{aligned}$$

In summary, the following $J+1$ equations must be solved to find the fixed point β_0, which can then be substituted into (30) to find the asymptotic SINR, where $G_{\mathbf{R}}(0) \approx G_{\underline{\mathbf{R}}}(0) = \beta_0$:

$$\boxed{\begin{aligned} \beta_0 &= \frac{1 - J}{\sigma_n^2 + \sum_{j \in \mathcal{J}} z_j} && (35) \\ \beta_0 &= -\frac{1}{z_j} f\left(\frac{p_j(1-\alpha_j)}{z_j} + \beta_0 \right), \quad \text{for } j \in \mathcal{J}. && (36) \end{aligned}}$$

In the general case, (30) is an equation with J unknowns. However, if all the α_j's are equal, (all users have the same number of signatures), then asymptotically the values

$$\frac{\gamma_{j,k}}{p_j} = \mathbf{s}_{j,k}^\dagger \left(\frac{\mathbf{H}_j^\dagger}{\sqrt{p_j}} \right) \mathbf{R}_{(j,k)-}^{-1} \left(\frac{\mathbf{H}_j}{\sqrt{p_j}} \right) \mathbf{s}_{j,k}$$

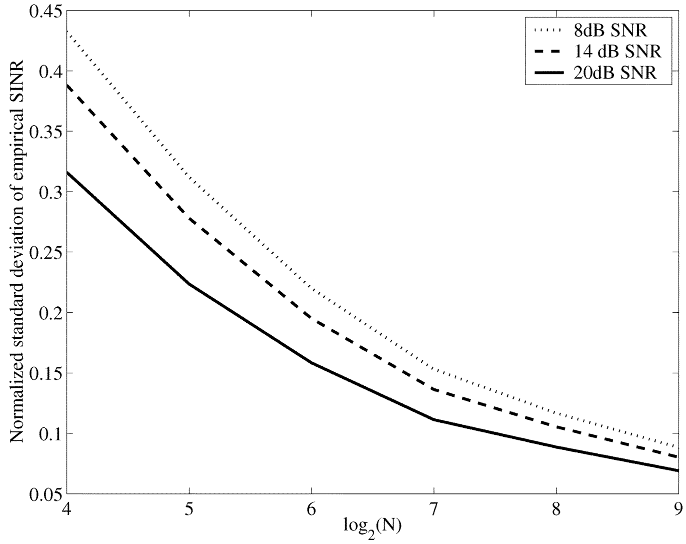

Fig. 1. Normalized standard deviation of single-user multiple-signature MC-CDMA empirical SINR for $\alpha = 0.5$.

are the same for all j and converge to the value $\gamma_{\mathrm{MU}}^\infty$. Hence

$$\boxed{1 = \sigma_n^2 G_{\mathbf{R}}(0) + \frac{\alpha}{J} \sum_{j \in \mathcal{J}} \frac{\gamma_{\mathrm{MU}}^\infty}{p_j^{-1} + \gamma_{\mathrm{MU}}^\infty}} \tag{37}$$

follows from (30), and the asymptotic SINR for each user is $\gamma_j^\infty = p_j \gamma_{\mathrm{MU}}^\infty$.

VII. Asymptotic BER

Provided that the asymptotic output multiple-access interference (MAI) plus noise is Gaussian (as suggested in [28] and [29] for DS-CDMA), the asymptotic BER for binary phase-shift keying (BPSK) symbols is given by the classical result [30, Ch.5]

$$P_b^\infty = Q\left(\sqrt{2\gamma^\infty}\right) \tag{38}$$

where $Q(x) = (2\pi)^{-1} \int_x^\infty \exp(-t^2/2) \mathrm{d}t$. This equation holds for the single-user and multiuser cases in Sections V and VI, and is supported by numerical results for systems with large N, shown in Section VIII.

VIII. Numerical Results

In what follows, we compare our asymptotic results to simulation results, corresponding to a MC-CDMA communication system with random spreading signatures, independent subcarrier complex gains, and binary data symbols.

A. Single-User, Multiple-Signature Simulations

Fig. 1 shows the empirical normalized standard deviation (normalized by the SINR) of the LMMSE output SINR as a function of N, in the case of a single user with $\alpha = 0.5$ for a range of per-signature received SNRs. These curves were found by averaging (12) over many random system realizations. From the figure, we observe that the SINR converges as N increases. Although not shown, we have also observed that any difference between the average empirical and asymptotic SINR values disappears rapidly as N increases. This effect has

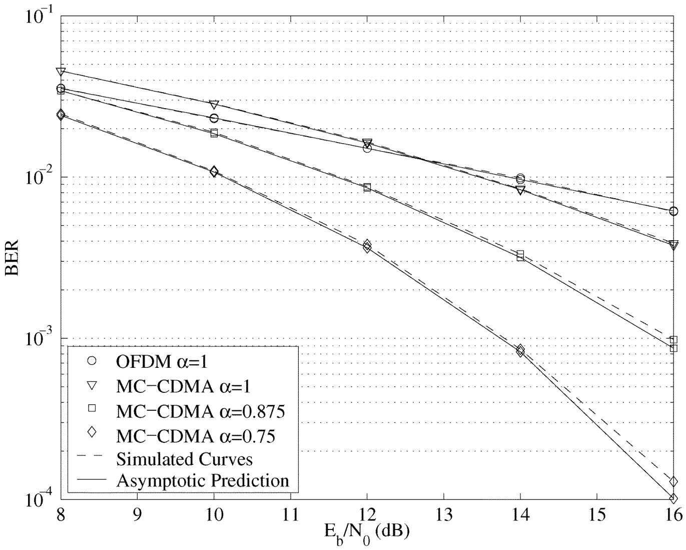

Fig. 2. Asymptotic and empirical BPSK BER for single-user multiple-signature MC-CDMA and OFDM with $N = 16$.

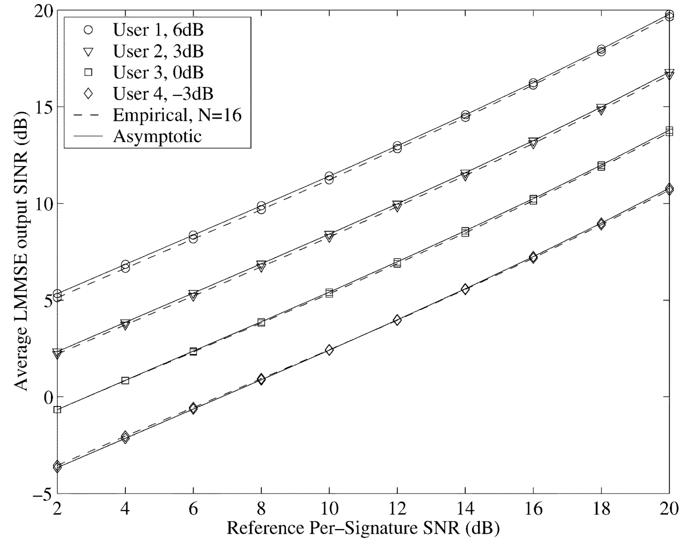

Fig. 4. Asymptotic and empirical SINR for multiuser multi-signature MC-CDMA.

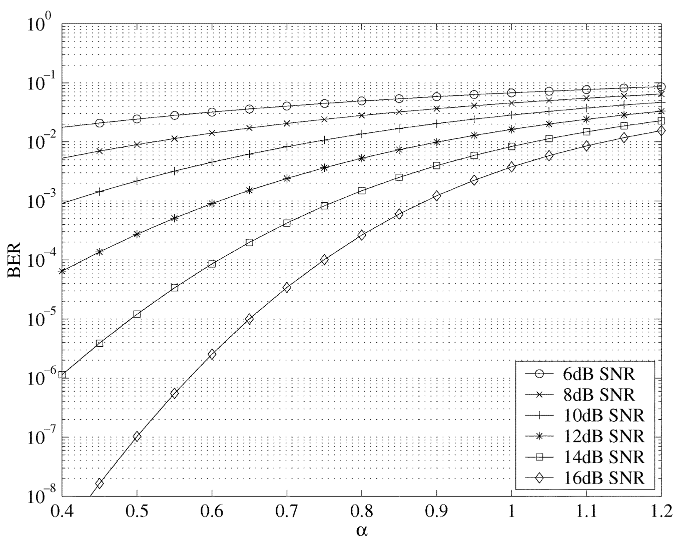

Fig. 3. Asymptotic and empirical BPSK BER for single-user multiple-signature MC-CDMA.

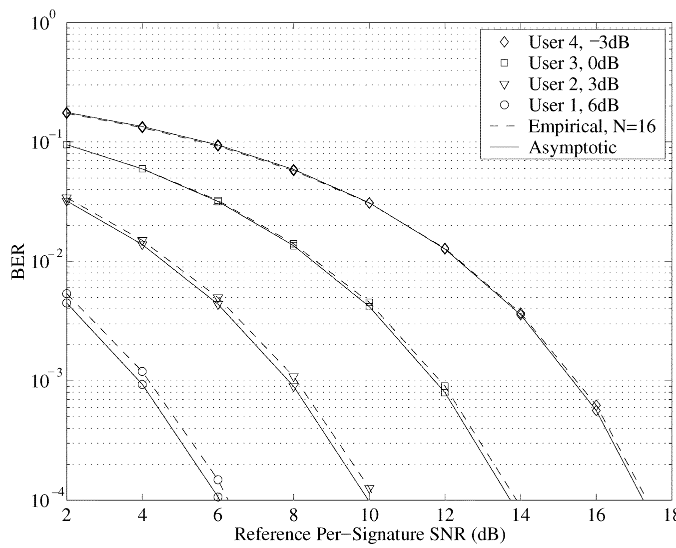

Fig. 5. Asymptotic and empirical BPSK BER for multiuser multi-signature MC-CDMA.

been previously observed for MC-CDMA [9], and confirms the derivations in Section IV.

Fig. 2 shows asymptotic and empirical BER for MC-CDMA systems. The asymptotic BER is computed from (22) and (38), and the empirical BER assumes $N = 16$ with binary symbols. The figure shows that the empirical BER agrees with the analytic results, thus demonstrating the validity of the BER expression (38). Also shown are empirical and analytic BER curves for (uncoded) OFDM, which cross the MC-CDMA curve at low SNR, as expected. When thermal noise dominates, the frequency diversity of MC-CDMA allows it to outperform OFDM, and when intersignature interference dominates, the orthogonality of OFDM gives it the advantage. Since each subcarrier is a Rayleigh flat-fading channel, the analytic BER for uncoded OFDM using BPSK is given by [30, Ch. 14]

$$P_2 = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right)$$

where $\overline{\gamma}$ is the average SNR per subchannel.

Fig. 3 shows the asymptotic BER as a function of system load α. This plot allows achievable data rates to be calculated for the MC-CDMA system. For example, in order to meet a BER $\leq 10^{-4}$ when the received SNR per signature is 14 dB, a system loading of up to 0.6 is allowed, corresponding to 0.6 bits per subcarrier with binary signaling.

B. Multiuser, Multi-Signature Simulations

Fig. 4 shows the analytic and empirical output SINR of a multi-signature multiuser MC-CDMA system with four users. Fig. 5 shows the corresponding empirical and asymptotic BER found using (38). For these two figures, each user has the same number of signatures, and $\alpha = 0.75$. User 3 has per-signature average received power P_{ref} (i.e., the value on the horizontal axis) and users 1, 2, and 4 have per-signature average received power 6, 3, and -3 dB, respectively, relative to P_{ref}.

The figures clearly show that the approximation made in determining β_0 in Section VI-B is valid in this scenario. Also note

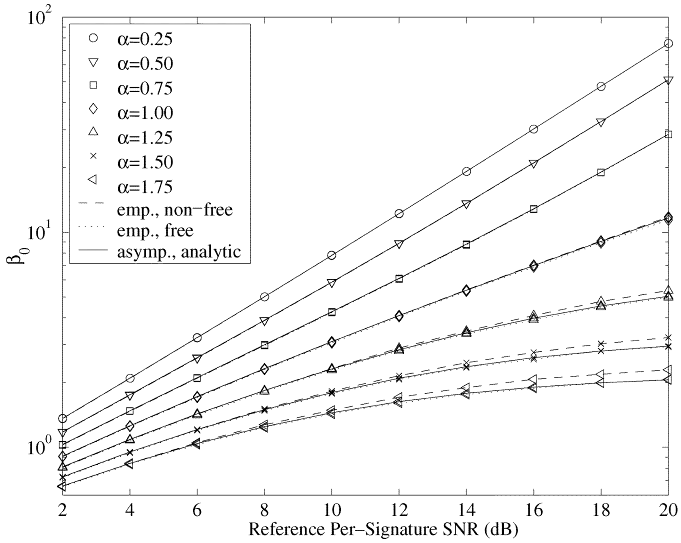

Fig. 6. Empirical and asymptotic values of β_0, and empirical values of $\mathrm{Tr}[\mathbf{R}^{-1}]$ versus per-signature SNR for a range of α for two equal-power, equal-rate users.

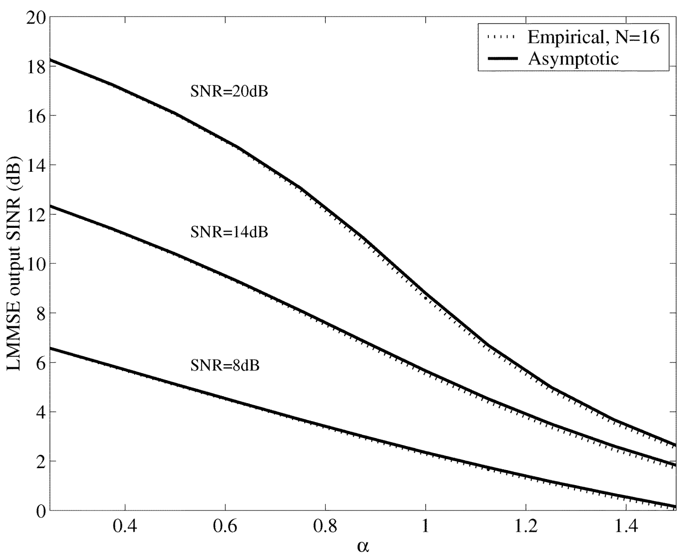

Fig. 7. Asymptotic and empirical SINR versus α for multiuser multi-signature MC-CDMA.

that the approximate asymptotic result more closely matches the empirical result at lower SNRs, as our discussion in Section VI-B indicated.

Let us examine the range of parameters over which the approximation used to determine β_0 is valid. We consider the case of two equal-power, equal α_j users. Fig. 6 shows the empirical finite (true) value of β_0 (with dashed lines) for $N = 16$. It also shows the empirical and asymptotic value of β_0 determined from $\underline{\mathbf{R}}$ (which our analysis uses as an approximation for $\mathbf{R}$). The dotted lines for empirical values cannot be seen, as they coincide with the asymptotic values, shown as solid lines. Clearly, for a wide range of α and SNRs, the approximation is accurate, especially for $\alpha \leq 1$. It remains now to investigate how these small β_0 inaccuracies in the high-SNR/high-α region impact the asymptotic SINR.

Fig. 7 shows the asymptotic and empirical SINR as a function of total system load, α in the same scenario as Fig. 6. Here we observe that the approximation error only increases slightly for higher SNR or higher α; however, it is clear that this approximation error is negligible for all practical values of system load, and over a wide range of SNRs.

IX. Conclusions

We have derived analytic SINR and BER expressions for MC-CDMA systems using an LMMSE receiver in Rayleigh fading channels. We derived results for multiuser and single-user models, and with both multiple and single signatures per user. The asymptotic SINRs are derived as a function of the system load (number of signatures per subcarrier), the noise power, the users' powers, and the fading properties of the channel. Our multiuser, multi-signature analysis relies on approximating certain covariance matrices by unitarily invariant matrices, which are asymptotically free. Comparison with simulation results show that the asymptotic results based on this approximation accurately predict the performance (e.g., output SINR) of finite-size systems over a wide range of system parameters of interest.

References

[1] N. Yee, J. P. Linnartz, and G. Fettweis, "Multi-carrier CDMA in indoor wireless radio networks," in *Proc. Personal, Indoor, Mobile Radio Communications Conf.*, Sept. 1993, pp. 109–113.

[2] P. M. Crespo, M. L. Honig, and J. A. Salehi, "Spread-time code-division multiple access," *IEEE Trans. Commun.*, vol. 43, pp. 2139–2148, June 1995.

[3] S. Zhou and G. B. Giannakis, "Finite-alphabet-based channel estimation for OFDM and related multicarrier systems," *IEEE Trans. Commun.*, vol. 49, pp. 1402–1414, Aug. 2001.

[4] A. C. McCormick, P. M. Grant, J. S. Thompson, and E. A. Al-Susa, "A carrier-frequency offset correction scheme for MC-CDMA," in *Proc. Vehicular Technology Conf.*, May 2001, pp. 1689–1692.

[5] T. M. Lok and T. F. Wong, "Transmitter and receiver optimization in multicarrier CDMA systems," *IEEE Trans. Commun.*, vol. 48, pp. 1197–1207, July 2000.

[6] S. Hara and R. Prasad, "Design and performance of multicarrier CDMA system in frequency-selective Rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1584–1595, Sept. 1999.

[7] B. Smida, C. L. Despins, and G. Y. Delisle, "MC-CDMA performance evaluation over a multipath fading channel using the characteristic function method," *IEEE Trans. Commun.*, vol. 49, pp. 1325–1328, Aug. 2001.

[8] J. Park, J. Kim, S. Choi, N. Cho, and D. Hong, "Performance of MC-CDMA systems in nonindependent Rayleigh fading," in *Proc. Int. Conf. Communications*, June 1999, pp. 506–510.

[9] J. P. Linnartz, "Performance analysis of synchronous MC-CDMA in mobile Rayleigh channel with both delay and Doppler spreads," *IEEE Trans. Veh. Technol.*, vol. 50, pp. 1375–1387, Nov. 2001.

[10] C. Ibars and Y. Bar-Ness, "Comparing the performance of coded multiuser OFDM and coded MC-CDMA over fading channels," in *Proc. GLOBECOM*, Nov. 2001, pp. 881–885.

[11] D. N. C. Tse and S. V. Hanly, "Linear multiuser receivers: Effective interference, effective bandwidth, and user capacity," *IEEE Trans. Inform. Theory*, vol. 45, pp. 641–657, Mar. 1999.

[12] S. Verdú and S. Shamai, "Spectral efficiency of CDMA with random spreading," *IEEE Trans. Inform. Theory*, vol. 45, pp. 622–640, Mar. 1999.

[13] S. Shamai and S. Verdú, "The impact of frequency-flat fading on the spectral efficiency of CDMA," *IEEE Trans. Inform. Theory*, vol. 47, pp. 1302–1327, May 2001.

[14] J. Evans and D. N. C. Tse, "Large-system performance of linear multiuser receivers in multipath fading channels," *IEEE Trans. Inform. Theory*, vol. 46, pp. 2059–2078, Sept. 2000.

[15] R. R. Müller, "Multiuser receivers for randomly spread signals: Fundamental limits with and without decision feedback," *IEEE Trans. Inform. Theory*, vol. 47, pp. 268–283, Jan. 2001.

[16] L. G. F. Trichard, J. S. Evans, and I. B. Collings, "Large-system analysis of linear parallel interference cancellation," in *Proc. Int. Conf. Communications*, June 2001, pp. 26–30.

[17] M. Debbah, W. Hachem, P. Loubaton, and M. de Courville, "MMSE analysis of certain large isometric random precoded systems," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1293–1311, May 2003.

[18] D. Voiculescu, "Addition of certain noncommuting random variables," *J. Funct. Anal.*, vol. 66, pp. 323–346, 1986.

[19] P. Sudarshan and B. L. Hughes, "On the spectral efficiency of CDMA with multiple antennas," in *Proc. IEEE Information Theory Workshop*, Cairns, Australia, Sept. 2001.

[20] R. Wegmann, "The asymptotic eigenvalue distribution for a certain class of random matrices," *J. Math. Anal. Applicat.*, vol. 56, pp. 113–132, 1976.

[21] J. W. Silverstein and Z. D. Bai, "On the empirical distribution of eigenvalues of a class of large dimensional random matrices," *J. Multivariate Anal.*, vol. 54, no. 2, pp. 175–192, 1995.

[22] J. W. Silverstein, "Strong convergence of the empirical distribution of eigenvalues of large dimensional random matrices," *J. Multivariate Anal.*, vol. 55, no. 2, pp. 331–339, 1995.

[23] F. Hiai and D. Petz, *The Semicircle Law, Free Random Variables and Entropy*. Providence, RI: Amer. Math. Soc., 2000, vol. 77, Mathematical Surveys and Monographs.

[24] M. J. M. Peacock, I. B. Collings, and M. L. Honig, "Asymptotic spectral efficiency of multiuser multi-signature CDMA in frequency-selective channels," *IEEE Trans. Inform. Theory*, submitted for publication.

[25] K. L. Chung, *A Course in Probability Theory*, 3rd ed. New York: Academic, 2001.

[26] W. Xiao, "Large-system performance of linear interference suppression for DS-CDMA," Ph.D. dissertation, Northwestern Univ., Evanston, IL, 2001.

[27] M. L. Honig and W. Xiao, "Performance of reduced-rank linear interference suppression," *IEEE Trans. Inform. Theory*, vol. 47, pp. 1928–1946, July 2001.

[28] J. Zhang, E. K. P. Chong, and D. N. C. Tse, "Output MAI distributions of linear MMSE multiuser receivers in DS-CDMA systems," *IEEE Trans. Inform. Theory*, vol. 47, pp. 1128–1144, Mar. 2001.

[29] H. V. Poor and S. Verdú, "Probability of error in MMSE multiuser detection," *IEEE Trans. Inform. Theory*, vol. 43, pp. 858–871, May 1997.

[30] J. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.

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