ORTHOGONALLY ANCHORED BLIND INTERFERENCE SUPPRESSION USING THE SATO COST CRITERION

Michael L. Honig
Department of EECS
Northwestern University
2145 Sheridan Road
Evanston, IL 60208

SUMMARY

1. Introduction

Minimum Mean Squared Error (MMSE) detection has been recently proposed for Direct Sequence-Code Division Multiple Access (DS-CDMA) systems [1]-[3]. MMSE detectors are near-far resistant, in contrast with the conventional matched filter, and can be adapted with standard adaptive algorithms without knowledge of user parameters (i.e., spreading codes). In principle, MMSE detection can therefore alleviate the necessity of tight closed-loop power control in DS-CDMA. However, a remaining problem is that standard adaptive algorithms, such as Least Mean Square (LMS) and Recursive Least Squares (RLS), may not be able to track rapid changes in the interference environment, such as when a new strong user begins transmission. This problem may become critical if power control is relaxed and the algorithm is running in decision-directed mode. In this case, decisions may occasionally become so unreliable that the adaptive algorithm ‘‘derails’’, and fails to converge to the new interference environment. A new training sequence must then be transmitted to guide the adaptive algorithm.

Because of the preceding problem, a blind adaptive interference suppression algorithm, which does not require a training sequence, is desirable for DS-CDMA. Such an algorithm has been recently presented in [4]. This algorithm assumes knowledge of the desired user’s pulse shape and associated timing. The basic approach is to decompose the linear MMSE detector into the sum of two orthogonal components: the matched filter (referred to as the *anchor*) and an adaptive filter. The adaptive filter coefficients are adapted to minimize output variance, subject to the orthogonality constraint. An advantage of this technique is that the cost function has a unique minimum which corresponds to the MMSE solution. However, a disadvantage is that when the desired user’s pulse shape not precisely known, such as when multipath is present, the filter will suppress the desired signal as well as the interference. In addition, the excess MSE produced by a stochastic gradient algorithm based on this approach is significantly greater than that produced by the LMS algorithm with a training sequence [4].

This work is a continuation of the work in [4], and assumes the same orthogonal decomposition of the linear detector. However, instead of using the minimum variance (MV) criterion, we consider an alternative cost function which is more closely related to the actual MSE. This cost criterion was proposed by Sato and Godard [5] for blind equalization of a single-user channel. However, without the orthogonal decomposition presented in [4], this cost function is not suitable for the multi-user application due to the presence of a local minimum associated with each user.

The orthogonally anchored Sato cost function leads to a stochastic gradient algorithm that performs significantly better (i.e., less asymptotic MSE for the same speed of convergence) than the MV algorithm in [4]. Furthermore, the algorithm coherently combines multipath in the presence of mismatch, unlike the MV algorithm. This cost function does have a local minimum associated with each user. However, if the crosscorrelation between any pair of pulse shapes is small, then the orthogonal anchor ensures that the norm of the coefficient vector that achieves any of these local minima must be large. These local minima can then be excluded by an appropriate norm constraint.

2. Orthogonally Anchored Sato Cost Function

For simplicity we consider a synchronous DS-CDMA system. The adaptive algorithms that follow apply to an asynchronous system as well; however, the analysis becomes much more tedious. Assuming a baseband model, the vector of received samples corresponding to the ith transmitted bit at the output of the chip matched filter is given by

$$\mathbf{r}[i] \;=\; \sum_{k=1}^{K} b_k[i] A_k \mathbf{s}_k + \mathbf{n}[i] \tag{1}$$

where K is the number of users, $\mathbf{r}$ has N components, N being the processing gain, $\{b_k[i]\}$ is the sequence of binary symbols corresponding to user k, $\mathbf{s}_k$ is the spreading code for user k where $\|\mathbf{s}_k\| = 1$, A_k is the amplitude for user k, and $\mathbf{n}$ is a noise vector.

The linear MMSE detector for user 1 consists of the coefficient vector $\mathbf{c}_1$ where $\mathbf{c}_1$ minimizes $E[(b_1[i] - \mathbf{c}_1{}'\mathbf{r}[i])^2]$. Here we constrain $\mathbf{c}_1$ to be of the form

$$\mathbf{c}_1 \;=\; \hat{\mathbf{s}}_1 + \mathbf{w}_1 \tag{2}$$

where $\hat{\mathbf{s}}_1$ is an estimate of $\mathbf{s}_1$, and $\mathbf{w}_1$ is *orthogonal* to $\hat{\mathbf{s}}_1$. Alternatively, we express this constraint as

$$\mathbf{c}_1{}'\hat{\mathbf{s}}_1 \;=\; 1. \tag{3}$$

Rather than select $\mathbf{c}_1$ to minimize output variance, as in [4], we choose $\mathbf{c}_1$ to minimize the Sato cost function

$$F(\mathbf{c}_1) \;=\; E\left\{\left[\mathbf{c}_1{}'\mathbf{r}[i] - \operatorname{sgn}\,(\mathbf{c}_1{}'\mathbf{r}[i])\right]^2\right\} \tag{4}$$

where $\operatorname{sgn}\,(x) = x/|x|$.

If reliable detection of user 1 is possible, then selecting $\mathbf{c}_1$ to minimize the MSE for user 1 means that with high probability $\operatorname{sgn}\,(\mathbf{c}_1{}'\mathbf{r}[i]) = b_1[i]$, so that $F(\mathbf{c}_1)$ is nearly equal to the MSE for $\mathbf{c}_1$ in the neighborhood of the MMSE solution. Notice, however, that a local minimum exists for each user. To achieve the local minimum associated with user k, $\mathbf{w}_1$ must be selected so that $\hat{\mathbf{s}}_1 + \mathbf{w}_1$ is nearly coincident with $\mathbf{s}_k$. Consequently, if $\hat{\mathbf{s}}_1{}'\mathbf{s}_1$ is close to one and if $\mathbf{s}_k{}'\mathbf{s}_m$ is small for each pair of k and m, then $\|\mathbf{w}_1\|$ in

(2) must be very large to achieve this local minimum. We can therefore exclude local minima associated with interferers by enforcing an appropriate norm constraint on the coefficient vector $\mathbf{c}_1$.

3. Cost Function for Two Users

For the case $K = 2$ the cost function (4) can be evaluated explicitly in terms of the signal vectors and associated amplitudes, assuming Gaussian noise. Because this cost function is complicated, it appears to be difficult to show that only two local minima exist for any combination of signal vectors and background noise level. However, if the background noise level is small, then we show that all local minima for the vector $\mathbf{c}_1$ must be confined to two small neighborhoods, each of which corresponds to the MMSE solution for a particular user. This is shown in the presence of a mismatched anchor. As the noise level goes to zero, these neighborhoods shrink to the MMSE solutions, where the orthogonal decomposition (2) is assumed.

4. Adaptive Algorithms

A stochastic gradient algorithm that minimizes (4), subject to the orthogonal decomposition (2), is given by

$$\mathbf{w}[i] = \mathbf{w}[i-1] - \mu e[i]\left(\mathbf{r}[i] - (\mathbf{r}'[i]\hat{\mathbf{s}}_1)\hat{\mathbf{s}}_1\right) \tag{5}$$

where

$$e[i] = \mathbf{c}_1{}'\mathbf{r}[i] - \operatorname{sgn}(\mathbf{c}_1{}'\mathbf{r}[i]), \tag{6}$$

and μ is the step-size. (The MV stochastic gradient algorithm presented in [4] simply replaces $e[i]$ by the output sample $\mathbf{c}_1{}'\mathbf{r}[i]$.) We show that the excess MSE associated with this algorithm is considerably less than that associated with the MV stochastic gradient algorithm. This is due to the smaller stochastic driving term in (5).

A least squares adaptive algorithm based on the preceding approach chooses $\mathbf{c}[i]$ to minimize $\sum_{m=0}^{i} e^2[m]$ subject to $\mathbf{c}_1{}'[i]\hat{\mathbf{s}}_1 = 1$ and $\|\mathbf{c}[i]\| \le C$. In this case $\mathbf{c}_1[i]$ must satisfy

$$\mathbf{c}_1[i] = \left[\sum_{m=0}^{i} \mathbf{r}[m]\mathbf{r}'[m] + \lambda_1 \mathbf{I}\right]^{-1} \left(\sum_{m=0}^{i} \operatorname{sgn}(\mathbf{c}_1{}'[i]\mathbf{r}[m]) \cdot \mathbf{r}[m]\right) + \lambda_2 \hat{\mathbf{s}}_1 \tag{7}$$

where λ_1 and λ_2 are Lagrange multipliers selected to satisfy the constraints. In general, because the right hand side depends in a complicated way on $\mathbf{c}_1[i]$, solving this set of equations is computationally expensive. However, an "approximate" solution can be obtained by replacing $\operatorname{sgn}(\mathbf{c}_1{}'[i]\mathbf{r}[m])$ by past decisions.

The performance (convergence speed and excess MSE) of the preceding algorithms will be illustrated numerically for some specific system models. The transient behavior of averaged signal-to-interference ratio in response to the onset of a new strong user will be presented, and will be compared with the performance of the MV algorithm in [4] and with the conventional LMS algorithm.

References

1. M. Abdulrahman, A. U. H. Sheikh, and D. D. Falconer, "Decision Feedback Equalization for CDMA in Indoor Wireless Communications", *IEEE JSAC",* vol. 12, no. 4, pp. 698-704, May 1994.

2. P. B. Rapajic and B. S. Vucetic, "Adaptive Receiver Structures for Asynchronous CDMA Systems", *IEEE JSAC,* vol. 12, no. 4, pp. 685-697, May 1994

3. U. Madhow and M. L. Honig, "MMSE interference suppression for DS/SS CDMA", *IEEE Trans. on Communications,* Dec. 1994.

4. M. L. Honig, U. Madhow, and S. Verdú, "Blind Adaptive Interference Suppression for Near-Far Resistant CDMA", *Proceedings Globecom '94,* San Francisco, CA, Dec. 1994.
   (An expanded version has been submitted to the *IEEE Trans. on IT*.)

5. D. N. Godard, "Self-Recovering Equalization and Carrier Tracking in Two-Dimensional Data Communication Systems", *IEEE Trans. on Comm.,* Vol. COM-28, No. 11, pp. 1867-1875, Nov. 1980.