

Spread-Time Code-Division Multiple Access

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Abstract— An alternative code-division multiple-access (CDMA) scheme to spread spectrum (SS), called spread time (ST) is proposed for bandlimited multiple-access channels. ST-CDMA can be considered the time-frequency dual of SS-CDMA. In ST-CDMA pseudorandom (PN) sequences are assigned to each user, and the Fourier transform of the transmitted pulse for a given user is determined by modulating the phase of the desired transmitted spectrum by the user's PN-sequence. The transmitted data for a particular user can be recovered by sampling the output of a filter matched to the user's pulse. Implementations are described in which surface acoustic wave devices are used to perform the matched filtering or Fourier transformation. Averaged signal-to-interference plus noise ratio (SIR) and spectral efficiency are computed for both asynchronous ST and direct-sequence SS-CDMA systems, assuming an arbitrary channel transfer function $H(f)$, which is the same between all pairs of users. The results are the same for SS and ST provided that the magnitude of the Fourier transform of the chip shape in the SS system is the same as the magnitude of the Fourier transform of the ST pulse shape. The main advantage of the ST technique is the flexibility with which the transmitted spectrum can be selected. We derive the transmitted spectrum that maximizes the SIR subject to an average power constraint.

I. INTRODUCTION

THE POTENTIAL demand for ubiquitous wireless communications combined with restricted availability of the radio frequency spectrum has motivated intense research into bandwidth-efficient multiple-access schemes. Code-division multiple access (CDMA), or spread-spectrum (SS) schemes have recently received attention as potentially attractive candidates for these types of multiple access channels [1], [2].

We present an alternative CDMA scheme to direct sequence spread-spectrum (DS-SS), called *spread time* (ST), since it can be viewed as the time-frequency dual of spread spectrum. This CDMA technique was motivated by work on optical CDMA by Salehi, Weiner, and Heritage [3]. Here, we examine ST-CDMA in the context of bandwidth-limited wireless communications, and compare its performance to that of conventional DS-SS.

As in SS, in ST, each user is assigned a pseudorandom (PN) sequence, and the data is transmitted by pulse-amplitude modulation. However, in ST, the transmitted pulse for a given

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user is determined by modulating the phase of the desired transmitted power spectral density.¹ That is, the available bandwidth is partitioned into M subbands, or frequency bins, where M is the processing gain. Each bin is assigned a phase (i.e., $0, \pi, \pm\pi/2$), which depends on the user's PN-sequence. The pulse is then obtained by taking the inverse Fourier transform of the resulting frequency response. At the receiver, the desired data can be recovered by sampling the output of a filter matched to the specific user pulse. Alternatively, the receiver can compute the Fourier transform of the time-windowed received signal at each symbol interval, which, in the absence of interference and channel impairments, produces the PN-sequence assigned to the transmitter. Subsequent processing then proceeds as in the case of a conventional DS-SS receiver. In the next section, implementations of an ST-CDMA system are described in which surface acoustic wave (SAW) devices [4] perform the matched filtering or Fourier transformation.

A performance analysis of asynchronous ST- and SS-CDMA is given in Section III. The performance measures considered are the ratio of rms signal level to rms interference plus noise in the output samples of the matched filter, or correlator (i.e., signal to interference plus noise ratio, or SIR), and the ratio of rms received signal to rms transmitted signal for a particular user (spectral efficiency). This analysis differs from previous analyses, i.e., [5]–[7], in that our system model includes a channel transfer function, which is assumed to be the same between all pairs of users.

We show that the preceding performance measures are the same for SS and ST provided that the magnitude of the Fourier transform of the SS chip shape is the same as the magnitude of the Fourier transform of the ST pulse shape. If the channel between each pair of users is ideal bandlimited, then the SIR for ST (or SS) with an optimized transmitted pulse (chip) shape is 2.1 dB greater than the SIR for SS with rectangular chips. This result assumes that the product of bandwidth with symbol time is the same for both systems (for SS with rectangular chips 90% of the pulse energy is confined to the channel bandwidth), that for the optimized system, the transmitted pulse and the matched filter impulse response are not truncated in time, and that the background noise level is zero.

For the ideal bandlimited channel the spectral efficiency, i.e., fraction of transmitted power which appears at the output of the channel, is slightly better for ST than for SS with rectangular chips. This is because the transmitted spectrum in ST can be closely matched to the channel transfer function. However, this difference in performance becomes very small when chip shaping is used in the SS system to increase spectral

¹Throughout this paper "transmitted spectrum" refers to the squared magnitude of the Fourier transform of the transmitted pulse.

efficiency. Nevertheless, the main advantage of ST-CDMA is the flexibility with which power-limited pulses can be designed with particular spectral characteristics. For example, the transmitted spectrum can have support on disconnected frequency bands, which is relatively difficult to achieve by chip shaping in SS systems.

The effect of increasing the number of chips per symbol and simultaneously allowing ST or SS pulses longer than the symbol interval is also considered. Although our analysis shows that the SIR remains unaffected, the correlation properties of the PN sequences assigned to each user may be improved in this way.

II. DESCRIPTION OF ST-CDMA

Multiplexing in conventional CDMA is achieved by assigning a different code, or signature sequence, to each transmitter, which uses this code to generate a data signal that can be decoded at each receiver. Specifically, it is assumed that the transmitted signal for user i is of the form

$$s_i(t) = \sum_k b_k^{(i)} p_i(t - kT) \quad (1)$$

where $\{b_k^{(i)}\}$ is the sequence of information bits for user i , $p_i(t)$ is the baseband pulse assigned to user i , and $1/T$ is the symbol rate. Throughout this paper, we assume binary signaling, i.e., $b_k^{(i)} \in \{\pm 1\}$. It is desired that the pulses $p_i(t)$, $i = 1, \dots, K$, where K is the number of users, be nearly orthogonal for all time shifts, that is

$$\left| \int_0^T p_i(t) s_l(t - \tau) dt \right| < \epsilon$$

for all $l \neq i$ and τ , where ϵ is some suitably small constant. In this case, the intended receiver can recover the data from transmitter i in the presence of interferers by sampling the output of a filter matched to $p_i(t)$. Typically, the pulse p_i is obtained by modulating a single-square pulse of width T by a PN sequence. We now describe an alternative method for generating pulses in which the square root of the desired transmitted spectrum $S(f)$ is modulated.

An ST-CDMA encoder and decoder is shown in Fig. 1. The pulse generator for user i , shown in Fig. 1(a) consists of a modulator followed by an inverse Fourier transform. The resulting pulse is then truncated by the time window $w(t)$. The input to the encoder is the square root transmitted spectrum $S(f)$. If the modulator multiplies $S(f)$ by a complex function which has modulus one, and $w(t) = 1$, then the transmitted spectrum is $|S(f)|^2$. We refer to this encoder as a "spectral encoder." The ST-CDMA, or "spectral," decoder shown in Fig. 1(b) may consist of a Fourier transformer, which computes the Fourier transform of the windowed data signal, conjugate modulator, and integrator, which is equivalent to a filter matched to the transmitted pulse.

The code assigned to a particular transmitter, which modulates $S(f)$, can be a complex-valued PN-sequence. For example, each sequence element can be chosen from a set of uniformly spaced points on the unit circle. Assuming the intended receiver is properly synchronized, then demodulation

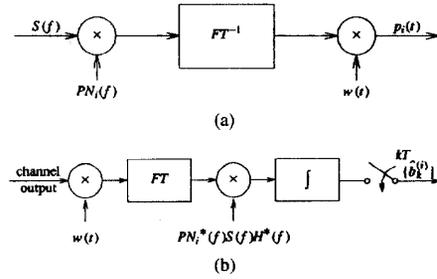


Fig. 1. (a) ST encoder. $FT^{-1} \equiv$ inverse Fourier transform. (b) ST decoder. $FT \equiv$ Fourier transform.

by the "conjugate" code, in which each PN-sequence element is replaced by its conjugate, enables detection of the transmitted data sequence. If, however, the decoder is matched to a different PN-sequence, then the output signal is additive low-intensity interference.

The spectral encoder can be implemented in one of two ways. In the first, the inverse Fourier transform shown in Fig. 1(a) can be implemented by SAW chirp filters [4]. The second method is to precompute $p_i(t)$, and then synthesize a filter with this impulse response. The transmitted signal is then the output of this filter in response to a series of short pulses modulated by the data at the rate $1/T$. These short pulses are "spread" in time by the spectral encoder (hence the name "spread-time CDMA"). In either case, the resulting transmitted waveform is a noiselike signal which requires more accurate processing than standard DS-SS signals.

An example of an ST pulse obtained from a transmitted spectrum which is constant over the frequency interval $[-1/2, 1/2]$ is shown in Fig. 2. The random sequence used to modulate the spectrum has length 256. Only 128 sequence elements are chosen randomly, however, since this random sequence and its conjugate modulate the positive and negative halves of the spectrum, respectively. This guarantees that the pulse is real-valued. The sequence elements a_k , $k = 0, \dots, 127$, were randomly chosen from the set $\{1, e^{j\pi/2}, -1, e^{-j\pi/2}\}$. It is easily verified that this pulse is given by

$$p(t) = \frac{\sin \pi f_c t}{\pi t} \sum_{k=-M/2}^{M/2-1} a_k e^{j2\pi f_c (k + \frac{1}{2})t}$$

where $M = 256$, and the chip width $f_c = 1/M$. Note that $p(t)$ is of infinite duration, and therefore must be truncated in time.

The spectral decoder can also be implemented in two ways. The first consists of Fourier transformation of the time-windowed received signal followed by correlation with the PN-sequence matched to the transmitter code. In this case, processing after the Fourier transformation proceeds as in the DS-SS case. Alternatively, Fig. 3(a) shows a spectral decoder consisting of a matched filter (correlator), which can, again, be implemented as a SAW device. Synchronization may be more difficult in the former case; however, correlation with the PN-sequence shown in Fig. 1(b) is simpler than correlation with the transmitted pulse as shown in Fig. 3(a).

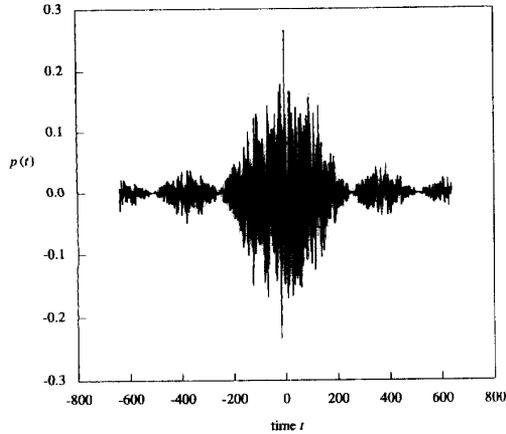


Fig. 2. Example of a spread-time pulse for an ideal bandlimited spectrum ($M = 256$).

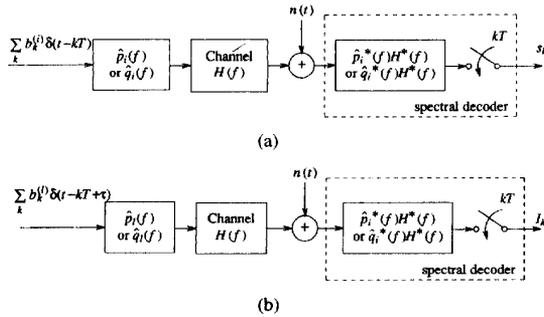


Fig. 3. (a) Generation of desired signal. (b) Generation of interference.

III. SYSTEM MODELS

It is of interest to evaluate the performance of ST- and SS-CDMA assuming a channel transfer function $H(f)$, which is the same between all pairs of K users, and a rate R bits/s for each user. In both cases, the transmitted signal for user i is assumed to be of the form (1).

Here, we focus on two performance parameters: signal-to-interference plus noise ratio (SIR), and the fraction of transmitted power which is passed by the channel. The following analysis assumes that for SS

$$p_i(t) = \sum_{k=0}^{nM-1} a_k^{(i)} g(t - kt_c) \quad (2)$$

where $\{a_k^{(i)}\}$ is a sequence of independent random variables for user i that take on values ± 1 with equal probability, nM is the code length, t_c is the chip duration, $g(t)$ is the chip shape, and n is a positive integer which will be explained shortly. For the time being, we assume that $n = 1$ so that the code length is M . Similarly, for ST, we assume that

$$\hat{q}_i(f) = S(f) \sum_{k=-nM/2}^{nM/2-1} \tilde{a}_k^{(i)} r_{f_c}(f - kf_c) \quad (3)$$

assuming M is even, where $\hat{q}_i(f)$ is the Fourier transform of the ST pulse $q_i(t)$, $|S(f)|^2$ is the desired transmitted spectrum, f_c is the chip width in the frequency domain, $\{\tilde{a}_k^{(i)}\}$ is a pseudorandom complex sequence, $r_\tau(t)$ is the rectangle function

$$r_\tau(t) = \begin{cases} 1 & 0 \leq t < \tau \\ 0 & \text{elsewhere} \end{cases} \quad (4)$$

and the remaining quantities are defined as before. Throughout this paper a “ $\hat{\cdot}$ ” denotes the Fourier transform of the associated time function. Note that for conventional DS-SS, $g(t) = r_{t_c}(t)$.

Choosing the $\tilde{a}_k^{(i)}$ s in (3) to be $e^{j\theta_k}$ for particular values of θ_k , i.e., $\theta_k \in \{\pm\pi/2, 0, \pi\}$, implies that the transmitted spectrum is given by $|S(f)|^2$ for $|f| < nMf_c/2$. Whereas, in SS, the magnitude of the transmitted pulse is selected *a priori*, in ST the transmitted spectrum is selected *a priori*. Both transmitted pulses specified by (2) and (3) can be scaled in order to achieve a desired transmitted power; however, this scaling does not affect the results that follow.

To ensure that the transmitted time waveform associated with $\hat{q}_i(f)$ in (3) is real, the corresponding PN-sequence must satisfy $\tilde{a}_k^{(i)} = \tilde{a}_{-k-1}^{(i)*}$, where “ $*$ ” denotes complex conjugate. Although the length of the PN-sequence assigned to each user is therefore one half the length of a PN-sequence in the analogous SS system, this is offset by the additional flexibility of being able to choose complex $\tilde{a}_k^{(i)}$ s. In what follows, we assume that $\tilde{a}_k^{(i)}$ is chosen from one of four uniformly spaced points on the unit circle, so that there is an isomorphic mapping between the SS and ST sequences $\{a_k^{(i)}\}$ and $\{\tilde{a}_k^{(i)}\}$. That is, $\tilde{a}_k^{(i)} = e^{j(\theta + n_k\pi/2)}$, where θ is a constant angle in $[0, \pi/2)$, and n_k takes on values 0, 1, 2, or 3 with equal probability. Consequently,

$$E(\tilde{a}_k^{(i)*} \tilde{a}_l^{(i)}) = \delta_{kl} \quad (5a)$$

and

$$E(a_k^{(i)} a_l^{(i)}) = 0 \quad (5b)$$

assuming the $\tilde{a}_k^{(i)}$ s are independent. The following results are therefore independent of θ .

Suppose now that $H(f) = 1$ for $|f| < W$, and is zero elsewhere. Typically, t_c in the SS system is selected so that $W = 1/t_c = M/T$, where $1/T$ is the symbol rate. The output of the channel in response to the input $p_i(t)$, given by (2), is therefore

$$y(t) = \sum_{k=0}^{M-1} a_k^{(i)} (h * r_{t_c})(t - kt_c) \quad (6)$$

where $*$ denotes convolution, and $h(t) = 2W \text{sinc}(2Wt)$ is the channel impulse response where $\text{sinc } t = \sin(\pi t)/(\pi t)$.

Similarly, assuming that for ST the transmitted spectrum is matched to the ideal bandlimited channel, i.e., $S(f) = |H(f)|$, choosing $f_c = 2/T$ implies that $W = Mf_c/2 = M/T$. Since the pulse $q_i(t)$ that results from taking the inverse Fourier transform of $\hat{q}_i(f)$ in (3) is of infinite duration, it must be windowed by some function $w(t)$. For example, in analogy

with the ideal bandlimited channel $H(f)$, we can choose the ideal time-limited function $w(t) = 1$ for $|t| < T/2$, and $w(t) = 0$ elsewhere. The intensity of the ST pulse $q_i(t)$ varies as $f_c \text{sinc}(f_c t)$ [3]. Consequently, most (approximately 90%) of the energy in the pulse $q_i(t)$ is contained in the interval $[-T/2, T/2]$. Of course, chip shapes in the frequency domain other than $r_{f_c}(f)$ can be selected that confine more of the pulse energy to within this interval. If $w(t)$ is nonzero over an interval of length greater than T , then the transmitted pulses will overlap, although it will be shown that the received samples at the output of the matched filter do not necessarily contain intersymbol interference (ISI). The Fourier transform of the output of the channel $H(f)$ in response to the ST input $w(t)q_i(t)$ is

$$\hat{y}(f) = H(f)S(f) \left(\sum_{k=-nM/2}^{nM/2-1} \hat{a}_k^{(i)}(\hat{w} * r_{f_c})(f - kf_c) \right) \quad (7)$$

where $\hat{w}(f) = T \text{sinc} fT$ is the Fourier transform of the ideal time-limited window $w(t)$.

Whereas in SS, the transmitted pulse is typically confined to a symbol interval, which causes the transmitted spectrum to extend beyond the channel bandwidth, in ST, the transmitted spectrum can be confined to the channel bandwidth, but the resulting time waveform must extend beyond the symbol interval. In SS, the transmitted spectrum that falls outside the passband of the channel is attenuated, whereas, in ST, the portion of the time waveform that extends beyond the symbol interval causes ISI if the duration of the matched filter impulse response is truncated to the symbol interval.

It is also of interest to consider the effect of increasing the code length nM for both ST and SS while keeping the transmitted spectrum the same. In what follows, we assume that M is fixed, i.e., $M = WT$ for the ideal bandlimited channel, and n is a positive integer. In ST-CDMA, this implies that f_c must decrease in order to satisfy $nMf_c = 2W$ for the ideal bandlimited channel. Consequently, the pulse $q_i(t)$ must expand accordingly, although its energy remains the same. That is, if the symbol interval T remains constant, then $f_c < 2/T$ implies that the pulses $q_i(t - kT)$, $k = 0, 1, 2, \dots$ overlap significantly with each other. For example, if $n = 2$, then $f_c = 2W/(2M) = 1/T$, and most (90%) of the energy in each pulse will be confined to the time interval $[-T, T]$, which implies that each pulse will significantly overlap with the two adjacent pulses on either side.

In conventional DS-SS, the transmitted spectrum depends primarily on t_c . Fixing M and t_c and increasing n implies that the length of the transmitted pulse must increase. If $nMt_c > T$, then the pulses $p_i(t - kT)$, $k = 0, 1, 2, \dots$ overlap. We will show that the SIR for SS and ST-CDMA is approximately unaffected by the type of pulse overlap just described. However, by increasing nM , it may be possible to improve the correlation properties of the codes assigned to each user.

The following analysis therefore assumes that the number of chips (in time for SS, and in frequency for ST) per symbol is nM , where $M = WT$ for the ideal bandlimited channel, and n represents the number of symbol intervals a transmitted pulse occupies. The pulses for SS and ST are then given by

(2) and (3), respectively, and the transmitted signal is given by (1), where p_i is replaced by q_i for ST. The chip width for ST is, therefore, $f_c = 2W/(nM) = 2/(nT)$; however, the chip width for SS, $t_c = 1/W$, is independent of n . For both SS and ST, if $n = 1$, then there is little overlap between transmitted pulses, and if $n = 2$, then each pulse overlaps primarily with the two adjacent pulses on either side.

IV. COMPUTATION OF SIR

We first compute the SIR for SS. Referring to Fig. 3(a), the average received signal energy is $E(s_k^2)$ where s_k is the output of the matched filter $\hat{p}_i^* H^*$ at time kT , so that

$$s_0 = \int_{-\infty}^{\infty} |H(f)|^2 |\hat{p}_i(f)|^2 \left(\sum_k b_k^{(i)} e^{-j2\pi f kT} \right) df \quad (8)$$

where the filter with transfer function $\hat{p}_i(f)$ has impulse response $p_i(t)$ given by (2). If the chip shape $g(t)$ has duration t_c , then $p_i(t)$ has duration nT . Assuming that the transmitted symbols $b_k^{(i)}$ in (1) are iid and are ± 1 , then it is shown in the Appendix that

$$E(s_k^2) = n^2 M \left(M + \frac{n-1}{n} \right) v^2(0) + 2n^2 M \sum_{l=1}^{M-1} v^2(lt_c) + 2 \sum_{l=1}^{M-1} (nM - l) v^2(lt_c) \quad (9)$$

where

$$v(t) = \int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^2 e^{j2\pi ft} df \quad (10)$$

and $\hat{g}(f)$ is the Fourier Transform of the SS chip shape. The expression (9) also assumes that $v(t) = 0$ for $t > T$, which is reasonable for $M \gg 1$. That is, $v(t)$ is the impulse response corresponding to the transfer function $|H(f)\hat{g}(f)|^2$, which is assumed to have bandwidth on the order of $1/t_c$. The duration of $v(t)$ should, therefore, be on the order of $t_c = T/M$. We can rewrite (9) as

$$E(s_k^2) = P_0 + P_I \quad (11)$$

where $P_0 = n^2 M [M + (n-1)/n] v^2(0)$ is the desired signal, and P_I , which is equal to the sum of the remaining terms on the right-hand side of (9), is ISI. In the usual case where $g(t) = r_{t_c}(t)$, and the channel is ideal bandlimited with bandwidth $2W$, $v(t)$ is an even function, and has duration approximately $2t_c$, so that $P_I \approx 0$. This remains true when $n > 1$ provided that the chip length is t_c . That is, even though the duration of the transmitted pulses may be greater than T , they are compressed to length T by the matched filter.

Referring to Fig. 3(b), the interference from a single interferer, i.e., user l , at time kT is I_k , and is the sampled output of the matched filter $\hat{p}_i^* H^*$ in response to the data signal from user l filtered by the channel. That is,

$$I_0(\tau) = \int_{-\infty}^{\infty} |H(f)|^2 \hat{p}_i^*(f) \hat{p}_l(f) \left(\sum_m b_m e^{-j2\pi f(mT+\tau)} \right) df \quad (12)$$

where $\hat{p}_i(f)$ is the Fourier transform of $p_i(t)$, and τ is a random phase offset in $[0, T)$. It is shown in the appendix that

$$E(I_0^2) = \frac{n^2 M^2}{T} \int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^4 df \quad (13)$$

where the expectation is with respect to the transmitted data sequence, PN-sequences, and phase offset τ , and τ is assumed to be uniformly distributed in $[0, T)$. For simplicity, in (12), we have omitted the phase of user l 's carrier frequency relative to that of the desired signal. However, assuming that this phase is a uniformly distributed random variable, then subsequent averaging will decrease the mean-squared interference in (13) by a factor of one half.

Finally, we assume that white noise $n(t)$ with spectral density $nN_0/2$ is added to the output of the channel in the SS system. The additional factor of n ensures that the ratio of transmitted signal power to additive noise power remains the same when the transmitted pulse width is increased beyond one symbol interval. Note this additional factor of n should not be included in the noise variance for ST, since the transmitted pulse energy in ST is independent of n . The noise variance at the sampled output of the matched filter in this case is therefore

$$\sigma^2 = \int_{-\infty}^{\infty} |\hat{p}_i(f)H(f)|^2 \frac{nN_0}{2} df = \frac{n^2 MN_0 v(0)}{2}. \quad (14)$$

The SIR is defined as

$$\text{SIR} = \left(\frac{P_0}{(K-1)E(I_0^2) + P_I + \sigma^2} \right)^{1/2} \quad (15)$$

where P_0 is the desired signal energy, P_I represents ISI, $K-1$ is the number of interferers, so that $(K-1)E(I_0^2)$ is the interference energy, and σ^2 is the term due to additive noise. Combining (9)–(15) and the fact that $T = Mt_c$ gives (16) shown at the bottom of the page. This expression can be simplified by noting that

$$2t_c \sum_{l=1}^{M-1} \left(n + 1 - \frac{l}{nM} \right) v^2(lt_c) < 2(n+1) \sum_{l=1}^{M-1} v^2(lt_c)t_c \approx 2(n+1) \int_{t_c}^{\infty} v^2(t) dt \quad (17)$$

and from Parseval's theorem

$$\int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^4 df = 2 \int_0^{\infty} v^2(t) dt. \quad (18)$$

Since the most of the energy in $v(t)$ is concentrated in the interval $(-t_c, t_c)$, assuming the channel bandwidth is approximately $2W$, it follows that for large K

$$\begin{aligned} nM(K-1) \int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^4 df \\ \gg 2(n+1)M \int_{t_c}^{\infty} v^2(t) dt \end{aligned}$$

$$> 2Mt_c \sum_{l=1}^{M-1} \left(n + 1 - \frac{l}{nM} \right) v^2(lt_c) \quad (19)$$

and

$$\text{SIR}^2 \approx \frac{T(M+1-\frac{1}{n})v^2(0)}{M(K-1) \int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^4 df + \frac{N_0 T v(0)}{2}}. \quad (20)$$

To compute the signal energy for ST, we again refer to Fig. 3(a), and note that the filter \hat{q}_i should be replaced by $\hat{w} * \hat{q}_i$, where $w(t)$ is the time window used to truncate the received and transmitted pulses. However, to simplify the following analysis, we will assume that $w(t) = 1$ for all t . That is, the transmitted pulse and the matched filter impulse response can be of infinite duration. For the case where $S(f) = H(f) = 1$ for $|f| < W$, nearly all of the pulse energy is contained within the time interval $[-mT/2, mT/2]$ where m is relatively small, (i.e., 2 or 3). The effect of truncating the transmitted pulse and matched filter response to this interval is therefore minor. In what follows, we also assume that the transmitted spectrum $|S(f)|^2$ is zero outside of the frequency band $[-W, W]$. When $H(f)$ is not strictly bandlimited, W can be chosen arbitrarily large.

The signal at the output of the matched filter at time $t = 0$, assuming no additive noise and interference from other users, is

$$s_0 = \int_{-W}^W |H(f)\hat{q}_i(f)|^2 \left(\sum_m b_m^{(i)} e^{-j2\pi f m T} \right) df \quad (21)$$

where $\hat{q}_i(f)$ is given by (3). It is shown in the Appendix that

$$E(s_0^2) = u^2(0) + \sum_{m \neq 0} u^2(mT) \quad (22)$$

where

$$u(t) = \int_{-W}^W |H(f)S(f)|^2 e^{j2\pi f t} df. \quad (23)$$

Assuming that the desired transmitted spectrum has bandwidth $2W$, then $u(t)$ is a short pulse with nearly all of its energy confined to within a small multiple of $1/W = T/M$. Consequently, in this case, $u(mT) = 0$, for $m \neq 0$, and the second term on the right-hand side of (22), which represents ISI, is zero.

To compute the interference for ST, we consider Fig. 3(b) where the input to the filter $\hat{q}_i(f)e^{j2\pi f \tau}$ is the impulse train $\sum_k b_k^{(l)} \delta(t-kT)$, where $\{b_k^{(l)}\}$ is the sequence of data symbols from user l . Assuming that the output of the matched filter on the right is sampled at time $t = 0$, we have that

$$I_0(\tau) = \int_{-W}^W |H(f)|^2 \hat{q}_i^*(f) \hat{q}_i(f) \left(\sum_k b_k^{(l)} e^{-j2\pi f(kT+\tau)} \right) df \quad (24)$$

$$\text{SIR}^2 = \frac{nT(M+1-\frac{1}{n})v^2(0)}{nM(K-1) \int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^4 df + \frac{nN_0 T v(0)}{2} + 2T \sum_{l=1}^{M-1} \left(n + 1 - \frac{l}{nM} \right) v^2(lt_c)} \quad (16)$$

where τ is the phase offset of the interferer, and is assumed to be in $[0, T)$. Substituting for \hat{q}_i and \hat{q}_j from (3), squaring, and averaging over the data and PN-sequences gives

$$E(I_0^2 | \tau) = 2 \sum_m \sum_{k=0}^{nM/2-1} \left| \int_{-W}^W |H(f)S(f)|^2 r_{f_c}(f - kf_c) e^{-j2\pi f(mT+\tau)} df \right|^2. \quad (25)$$

If it is assumed that $H(f) = 1$ for $|f| < W$ and $H(f) = 0$ elsewhere, and that $S(f) = H(f)$, then (25) becomes

$$E(I_0^2 | \tau) = nMf_c^2 \sum_k \text{sinc}^2 [f_c(kT + \tau)]. \quad (26)$$

Each term in the sum is the mean-squared interference due to an interfering pulse shifted by $kT + \tau$. Note that $E(I_0^2 | \tau = 0) = Mf_c^2$ when $n = 1$, and combining this with (15) and (22) gives $\text{SIR} = 4W^2 / [(K-1)Mf_c^2] = M / (K-1)$ in the absence of additive noise, which is the same as the SIR for baseband chip-synchronous DS-SS with rectangular chips. Because the ST pulse envelope is time varying, the SIR is a function of τ . For the ideal bandlimited transmitted spectrum, this intensity is given by $f_c^2 \text{sinc}^2(f_c t)$. Assuming that $f_c = 2/T$, then $E(I_0^2 | \tau)$ is minimized by taking $\tau = -T/2$, in which case $E(I_0^2 | \tau = -T/2) = 0$. If τ is uniformly distributed in $[0, T)$, then it is shown in the Appendix that averaging over τ in (25) gives

$$E(I_0^2) = \frac{1}{T} \int_{-W}^W |H(f)S(f)|^4 df. \quad (27)$$

As in the SS case, averaging over a uniformly distributed carrier phase decreases $E(I_0^2)$ by a factor of one half.

The noise variance at the output of the matched filter is

$$\sigma^2 = \frac{N_0}{2} \int_{-W}^W |H(f)S(f)|^2 df \quad (28)$$

and combining (15), (22), (27), and (28) gives

$$\text{SIR}^2 = \frac{MTu^2(0)}{M(K-1) \int_{-W}^W |H(f)S(f)|^4 df + \frac{N_0 T u(0)}{2}}. \quad (29)$$

Note that this expression for SIR is the same as (20) when $n = 1$, if the Fourier transform of the chip shape in (20), $\hat{g}(f)$, is equal to $S(f)$ in (29). The ST-CDMA scheme described here therefore gives the same SIR performance as an SS-CDMA scheme in which the chips in the SS waveform are shaped to have the specified ST spectrum. Note that the SIR for SS, given by (20), depends weakly on n , and the SIR for ST is independent of n . Although the SIR does not change significantly when a transmitted pulse of duration greater than a symbol interval is used, the longer PN-sequences associated with these longer pulses have improved correlation properties.

As an example, suppose that $H(f) = 1$ for $|f| < W$. For DS-SS with rectangular chips, $g(t) = r_{t_c}(t)$ and $t_c = 1/W$, so that

$$v(0) = \int_{-W}^W |\hat{r}_{t_c}(f)|^2 df = t_c^2 \int_{-1/t_c}^{1/t_c} \text{sinc}^2(t_c f) df \quad (30)$$

where $\hat{r}_{t_c}(f)$ is the Fourier Transform of $r_{t_c}(t)$. We also have that

$$\int_{-W}^W |H(f)\hat{g}(f)|^4 df = t_c^4 \int_{-1/t_c}^{1/t_c} \text{sinc}^4(t_c f) df. \quad (31)$$

Substituting into the expression for SIR (20), and assuming $n = 1$ gives

$$\text{SIR}^2 = \frac{\kappa_1 \gamma}{1 + \kappa_2 \gamma (K-1)/M} \quad (32)$$

where $\kappa_1 = \int_{-1}^1 \text{sinc}^2(t) dt \approx 0.902$, $\kappa_2 = \int_{-1}^1 \text{sinc}^4(t) dt / \kappa_1 \approx 0.736$, and $\gamma = 2T/N_0$ is the ratio of transmitted pulse energy to background noise level. As the noise level $N_0 \rightarrow 0$, (32) becomes

$$\text{SIR}^2 = \frac{\kappa_1 M}{\kappa_2 (K-1)} = \frac{\kappa_1 WT}{\kappa_2 (K-1)}. \quad (33)$$

As the bandwidth $W \rightarrow \infty$, with t_c and M held constant, then the preceding discussion shows that the corresponding SIR without background noise is given by

$$\begin{aligned} \text{SIR}^2 &= \frac{M \left(\int_{-\infty}^{\infty} \text{sinc}^2(t) dt \right)^2}{(K-1) \int_{-\infty}^{\infty} \text{sinc}^4(t) dt} = \frac{3M}{2(K-1)} \\ &= \frac{3WT}{2(K-1)} \end{aligned} \quad (34)$$

which is the standard result for baseband asynchronous DS-SS [5].²

Consider now ST where $S(f) = 1$ for $|f| < W$ and $N_0/2 = 0$. Then from (23), $u(0) = 2W$, and from (29)

$$\text{SIR}^2 = \frac{2WMT}{M(K-1) + N_0 T/2} = \frac{\gamma WT}{1 + \gamma(K-1)/2} \quad (35)$$

where $\gamma = (2W)/(N_0/2)$ is the ratio of transmitted pulse energy to background noise level. From (20), this is also the SIR for an SS system in which the chip shape $g(t) = (2/t_c) \text{sinc}(2t/t_c)$. Letting $N_0 \rightarrow 0$ gives $\text{SIR}^2 = (2WT)/(K-1)$. Comparing this with the expression (33), we find that the SIR for ST in the absence of background noise is a factor of $\sqrt{2\kappa_2/\kappa_1}$, or 2.1 dB, greater than the SIR for SS with rectangular chips. As $W \rightarrow \infty$ with t_c and M fixed, this improvement drops to $2/\sqrt{3}$, or 1.25 dB.

V. OPTIMIZATION OF TRANSMITTED SPECTRUM

Since the SIR is a functional of the transmitted spectrum, we can find the transmitted spectrum that maximizes the SIR subject to an average transmitted power constraint. That is, for ST, we wish to

$$\max_{S(f)} \text{SIR}$$

subject to

$$\int_{-W}^W |S(f)|^2 df = 1. \quad (36)$$

²Averaging over a uniformly distributed random carrier phase for the interferer increases this expression by a factor of two.

A standard variational argument can be used to show that the optimal spectrum is given by

$$|S(f)|^2 = \begin{cases} \frac{c_1}{|H(f)|^2} \left(1 - \frac{c_2}{|H(f)|^2}\right) & \text{if } |H(f)|^2 > c_2 \\ 0 & \text{if } |H(f)|^2 \leq c_2 \end{cases} \quad (37)$$

where

$$c_1 = \frac{1 + \alpha I}{I + \alpha(I^2 - \mu J)}, \quad c_2 = \frac{\alpha \mu}{1 + \alpha I}, \quad (38)$$

$$I = \int_{B(f)} \frac{1}{|H(f)|^2} df, \quad J = \int_{B(f)} \frac{1}{|H(f)|^4} df, \quad (39)$$

$$B(f) = \{f: |H(f)|^2 > c_2\} \quad (40)$$

$\alpha = (N_0 T) / [4M(K - 1)]$, and $\mu = \text{meas } B(f)$. Note that c_1 is simply a normalization constant that enforces the constraint (36).

When the additive channel noise is small, i.e., if $N_0/2 \rightarrow 0$, then $c_2 \rightarrow 0$ and

$$|S(f)|^2 \approx \frac{c_1}{|H(f)|^2} \quad (41)$$

for all f such that $|H(f)|^2 \gg c_2$. If $|H(f)| > 0$ for all f , $c_2 \rightarrow 0$ implies that the support of $S(f)$ increases without bound. As the channel noise level increases, c_2 also increases. If it is true that $I \gg 1/\alpha$ as $N_0/2 \rightarrow \infty$, then

$$c_2 \rightarrow \sup_f |H(f)|^2.$$

This will be illustrated by an example; however, in general, it is easily verified that this is true if $|H(f)|$ is continuous and differentiable at its maximum value(s). If this maximum occurs at a finite set of frequencies, then the optimal transmitted spectrum becomes narrowly centered around the f s for which $|H(f)|$ is maximized.

As an example, suppose that $|H(f)| = 1$ for $|f| < W$. In this case

$$c_2 = \frac{2\alpha W}{1 + 2W\alpha} < 1 \quad (42)$$

so that from (40), $B(f) = \{f: |f| < W\}$. In this case, $c_1 = (1 + 2W\alpha)/2W$, and it is easily verified that

$$|S(f)|^2 = \begin{cases} 1/(2W) & \text{for } |f| < W \\ 0 & \text{for } |f| > W \end{cases} \quad (43)$$

Substituting into the expression for SIR gives $\text{SIR}^2 = 2WT / [(K - 1) + N_0 T / (2M)]$.

Assume now that the channel impulse response is $h(t) = e^{-t}$, or $|H(f)|^2 = 1/(1 + 4\pi^2 f^2)$. We have that $|H(f)|^2 > c_2$ when

$$|f| < f_0 = \frac{1}{2\pi} \left(\frac{1}{c_2} - 1 \right)^{1/2} \quad (44)$$

so that from (40), $\mu = \text{meas } B(f) = 2f_0$. From (38) and (39) it follows that

$$c_2 = \frac{[2\alpha/(3\pi)]^{2/3}}{1 + [2\alpha/(3\pi)]^{2/3}}. \quad (45)$$

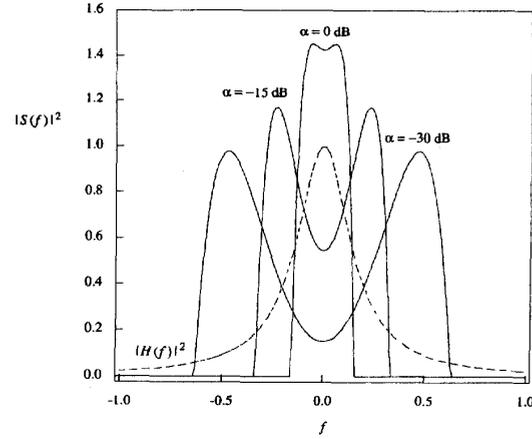


Fig. 4. Optimized transmitted spectra for different values of α , assuming the channel impulse response is e^{-t} .

As $N_0/2$ increases from zero to infinity, c_2 therefore increases from zero to $\sup_f |H(f)|^2 = 1$. A plot of optimized transmitted spectra for different values of α is shown in Fig. 4.

VI. SPECTRAL EFFICIENCY

We now examine the spectral efficiency of SS and ST. Here, we define the spectral efficiency to be the fraction of transmitted power which is present at the receiver, i.e.,

$$\eta = \left(\frac{[E(s_k^2)]}{P} \right)^{1/2} \quad (46)$$

where P is the energy in the transmitted pulse. Note that $0 \leq \eta \leq 1$. In the case of SS, if the input pulse (2) is time-limited to $[0, T]$, then $P = \int_0^T |p_i(t)|^2 dt = 1$, so that the spectral efficiency is simply $\eta = [E(s_k^2)]^{1/2}$, and $E(s_k^2)$ is given by (9). For this case of the ideal bandlimited channel with $W = 1/t_c$ and rectangular chips, the spectral efficiency is approximately $\eta \approx \int_{-1}^1 \text{sinc}^2 t dt \approx 0.90$. That is, on average, approximately 10% of the input power is lost in the channel in this case. Of course, spectral efficiency can be improved by chip shaping.

For ST the spectral efficiency is given by

$$\eta^2 = \frac{\int_{-W}^W |H(f)|^2 |\hat{w} * \hat{q}_i(f)|^2 df}{\int_{-\infty}^{\infty} |\hat{w} * \hat{q}_i(f)|^2 df} \quad (47)$$

where $\hat{w}(f)$ is the Fourier transform of the time window $w(t)$. The ideal bandlimited channel therefore truncates the "spillover" outside the band $[-W, W]$ caused by the time window $w(t)$. If $w(t)$ has width equal to the symbol interval $T = 2/f_c$, then $\hat{w}(f) = (2/f_c) \text{sinc}(2f/f_c)$. Since the width of $\hat{w}(f)$ is on the order of $f_c = 2W/(nM)$, for large M the energy loss due to the channel is negligible. Consequently, as a first-order approximation, $\eta \approx 1$.

Assuming an ideal bandlimited channel, we therefore conclude that the spectral efficiency for ST is only slightly (10%) better than that for conventional DS-SS with rectangular chips. For other types of channels, however, ST may offer

a significant advantage when compared to SS. For example, suppose that $H(f)$ consists of two ideal bandpass channels each having bandwidth W with respective support on disconnected frequency bands. Since $H(f)$ is even, the total available channel bandwidth is $4W$. The transmitted spectrum for ST can be matched to the channel, which results in essentially the same performance as for the case of an ideal bandlimited channel with bandwidth $4W$. That is, $\text{SIR} = [4WT/(K-1)]^{1/2}$ and $\eta \approx 1$. In contrast, a conventional SS system would most likely treat this channel as two independent channels each having bandwidth $2W$. An additional channel assignment scheme is then needed to assign users to one of the two bandpass channels.

VII. CONCLUSIONS

An alternative CDMA technique to spread spectrum, called spread-time, has been presented for multiple access communications over bandlimited channels. Assuming an ideal bandlimited channel between each pair of users, our results show that the signal to interference ratio for ST (and for SS with optimal chip shapes) is 2.1 dB greater than that of conventional asynchronous DS-SS with rectangular chips, while the spectral efficiency for ST is slightly better than that of conventional SS. These results assume that for ST, both the transmitted pulse shape and matched filter impulse response are not truncated in time. Because ST, in principle, offers complete flexibility in choosing the transmitted spectrum, this technique may be attractive in applications where the channel transfer function deviates considerably from a "boxcar" shape. An example which may be of practical interest is when the channel has support on disconnected frequency bands.

In this paper, we have not considered important issues related to the implementation of an ST system. For example, questions such as how sensitive the system is to inaccuracies in the matched filter, the accuracy with which the transmitted signal must be generated, synchronization, and the effects of system nonlinearities have not been addressed. Other interesting problems related to channel impairments, such as fading, are also left for future investigation.

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APPENDIX

Here we compute the average signal energy $E(s_0^2)$ and the interference energy $E(I_0^2)$ for both SS and ST. We start with the computations for SS. Squaring the expression for s_0 given by (8), and averaging over the PN-sequence using the fact that

$$E[a_k^{(i)} a_l^{(i)} a_{k'}^{(i)} a_{l'}^{(i)}] = \begin{cases} 1 & \text{for } k = l, k' = l' \\ 1 & \text{for } k = k', l = l' \\ 1 & \text{for } k = l', l = k' \\ 0 & \text{otherwise} \end{cases} \quad (\text{A1})$$

gives

$$E(s_0^2 | \{b_k^{(i)}\})$$

$$\begin{aligned} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |H(f)|^2 |H(f')|^2 |\hat{g}(f)|^2 |\hat{g}(f')|^2 \\ &\times \left(\sum_{k=0}^{nM-1} \sum_{l=0}^{nM-1} 1 + \sum_{k=0}^{nM-1} \sum_{\substack{l=0 \\ l \neq k}}^{nM-1} (e^{j2\pi(f-f')t_c(k-l)} \right. \\ &\left. + e^{j2\pi(f-f')t_c(k-l)}) \right) \left(\sum_m \sum_{m'} b_m^{(i)} b_{m'}^{(i)} e^{-j2\pi f m T} \right. \\ &\left. \cdot e^{-j2\pi f' m' T} \right) df df'. \end{aligned} \quad (\text{A2})$$

Averaging over the data and rearranging terms gives

$$\begin{aligned} E(s_0^2) &= \sum_m \left\{ \left(nM \int_{-\infty}^{\infty} |H(f)|^2 |\hat{g}(f)|^2 e^{-j2\pi f m T} df \right)^2 \right. \\ &+ \sum_{k=0}^{nM-1} \sum_{\substack{l=0 \\ l \neq k}}^{nM-1} \left[\left(\int_{-\infty}^{\infty} |H(f)|^2 |\hat{g}(f)|^2 e^{j2\pi f(k-l)t_c} \right. \right. \\ &\left. \left. \cdot e^{-j2\pi f m T} df \right)^2 \right. \\ &+ \left(\int_{-\infty}^{\infty} |H(f)|^2 |\hat{g}(f)|^2 e^{j2\pi f(k-l)t_c} e^{-j2\pi f m T} df \right) \\ &\left. \cdot \left(\int_{-\infty}^{\infty} |H(f')|^2 |\hat{g}(f')|^2 e^{-j2\pi f'(k-l)t_c} \right. \right. \\ &\left. \left. \cdot e^{-j2\pi f' m' T} df' \right) \right] \}. \end{aligned} \quad (\text{A3})$$

Letting $v(t)$ denote the inverse Fourier transform of $|H(f)\hat{g}(f)|^2$, i.e., (10), then (A3) can be rewritten as

$$\begin{aligned} E(s_0^2) &= [nMv(0)]^2 + n^2 M^2 \sum_{m \neq 0} v^2(mT) \\ &+ \sum_m \sum_{k=0}^{nM-1} \sum_{\substack{l=0 \\ l \neq k}}^{nM-1} v^2[(k-l)t_c - mT] \\ &+ \sum_m \sum_{k=0}^{nM-1} \sum_{\substack{l=0 \\ l \neq k}}^{nM-1} v[(k-l)t_c - mT] \\ &\cdot v[(k-l)t_c + mT]. \end{aligned} \quad (\text{A4})$$

If the bandwidth of $|H(f)\hat{g}(f)|^2$ is approximately $1/t_c$, then we can assume that $v(t) = 0$ for $t \geq Mt_c$ since $M \gg 1$. The right-most triple sum in (A4) then contains only terms corresponding to $m = 0$, and (A4) can be rewritten as

$$\begin{aligned} E(s_0^2) &= [nMv(0)]^2 + \sum_m \sum_{k=0}^{nM-1} \sum_{\substack{l=0 \\ l \neq k}}^{nM-1} v^2[(k-l)t_c - mT] \\ &+ \sum_{k=0}^{nM-1} \sum_{\substack{l=0 \\ l \neq k}}^{nM-1} v^2[(k-l)t_c] \end{aligned} \quad (\text{A5})$$

where the fact that $v(t)$ is an even function has been used. Counting terms in the summations gives (9).

To compute $E(I_0^2)$ for SS, we first square the expression for I_0 , (12), and average over the data, which gives

$$E(I_0^2|\tau) = \sum_m \sum_{k=0}^{nM-1} \sum_{l=0}^{nM-1} \left(\int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^2 \cdot e^{j2\pi f[t_c(k-l)-mT-\tau]} df \right)^2 \quad (A6)$$

Letting $\hat{\tau} = t_c(k-l) - mT - \tau$, we then have that

$$\begin{aligned} E(I_0^2) &= \frac{1}{T} \int_0^T E(I_0^2|\tau) d\tau \\ &= \sum_m \sum_{k=0}^{nM-1} \sum_{l=0}^{nM-1} \frac{1}{T} \int_{t_c(k-l)-(m+1)T}^{t_c(k-l)-mT} \left(\int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^2 e^{j2\pi f\hat{\tau}} df \right)^2 d\hat{\tau} \\ &= \sum_{k=0}^{nM-1} \sum_{l=0}^{nM-1} \frac{1}{T} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^2 e^{j2\pi f\hat{\tau}} df \right)^2 d\hat{\tau} \\ &= \sum_{k=0}^{nM-1} \sum_{l=0}^{nM-1} \frac{1}{T} \int_{-\infty}^{\infty} v^2(t) dt \\ &= \frac{n^2 M^2}{T} \int_{-\infty}^{\infty} |H(f)\hat{g}(f)|^4 df. \end{aligned} \quad (A7)$$

We now compute $E(s_0^2)$ and $E(I_0^2)$ for ST. Substituting (3) into (8), and noting that $\hat{a}_k^{(i)} = \hat{a}_{-k-1}^{(i)*}$, and that $a_k^{(i)} a_k^{(i)*} = 1$, gives

$$\begin{aligned} s_0 &= \int_{-W}^W |H(f)|^2 |S(f)|^2 \left[\sum_{k=0}^{nM/2-1} (r_{f_c}(f - kf_c) \right. \\ &\quad \left. + r_{f_c}[f + (k+1)f_c]) \left(\sum_m b_m^{(i)} e^{-j2\pi f mT} \right) \right] df. \end{aligned} \quad (A8)$$

Squaring, averaging over the data, and rearranging terms gives

$$\begin{aligned} E(s_0^2) &= \sum_m \left(\sum_{k=0}^{nM/2-1} \int_{-W}^W |H(f)|^2 |S(f)|^2 r_{f_c}(f - kf_c) \right. \\ &\quad \cdot e^{-j2\pi f mT} df + \sum_{k=0}^{nM/2-1} \int_{-W}^W |H(f)|^2 |S(f)|^2 \\ &\quad \left. \cdot r_{f_c}(f - kf_c) e^{j2\pi f mT} df \right)^2 \end{aligned} \quad (A9)$$

where the fact that $r_{f_c}(-f - kf_c) = r_{f_c}[f + (k+1)f_c]$ has been used. Applying the definition of $r_{f_c}(f)$ then gives (22). Notice that in this case it is not necessary to average over the elements of the PN-sequence.

Substituting (3) into the expression (24) for I_0 gives

$$\begin{aligned} I_0(\tau) &= \int_{-W}^W |H(f)|^2 |S(f)|^2 \\ &\quad \times \left[\sum_{k=0}^{nM/2-1} (a_k^{(l)} a_k^{(i)*} r_{f_c}(f - kf_c) + a_k^{(i)} a_k^{(l)*} \right. \end{aligned}$$

$$\left. \cdot r_{f_c}[f + (k+1)f_c] \left(\sum_m b_m^{(l)} e^{-j2\pi f(mT+\tau)} \right) \right] df. \quad (A10)$$

Squaring, averaging over the data and PN-sequences using (5), and rearranging the terms gives (25). Averaging over the phase offset τ , letting $\hat{\tau} = -(mT + \tau)$, and using the definition (23) gives

$$\begin{aligned} E(I_0^2) &= \frac{2}{T} \sum_{k=0}^{nM/2-1} \sum_m \int_{mT}^{(m+1)T} \left(\int_{-\infty}^{\infty} |H(f)S(f)|^2 \right. \\ &\quad \left. \cdot r_{f_c}(f - kf_c) e^{j2\pi f\hat{\tau}} df \right)^2 d\hat{\tau} \\ &= \frac{2}{T} \sum_{k=0}^{nM/2-1} \int_{-W}^W |u(t)|^2 dt \\ &= \frac{2}{T} \sum_{k=0}^{nM/2-1} \int_{-W}^W |H(f)S(f)|^4 r_{f_c}(f - kf_c) df \\ &= \frac{1}{T} \int_{-W}^W |H(f)S(f)|^4 df. \end{aligned} \quad (A11)$$

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