



N O R T H W E S T E R N
U N I V E R S I T Y

MSIT | Master of Science in Information Technology

MSIT 413: Wireless Technologies

Week 5

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Outline

- Finish discussion of modulation
 - Signal constellations
 - Orthogonal Frequency Division Multiplexing (OFDM)
 - Effect of fading
- Diversity
- Error control coding



Why Digital Communications?

1G (analog) → 2G (digital) → 3G (digital)

Digitized voice requires about 64 kbps, therefore the required bandwidth is >> the bandwidth of the voice signal (3—4 kHz)!

- Can combine with sophisticated signal processing (voice compression) and error protection.
- Greater immunity to noise/channel impairments.
- Can multiplex different traffic (voice, data, video).
- Security through digital encryption.
- Flexible design possible (software radio).

VLSI + special purpose digital signal processing
→ digital is more cost-effective than analog!



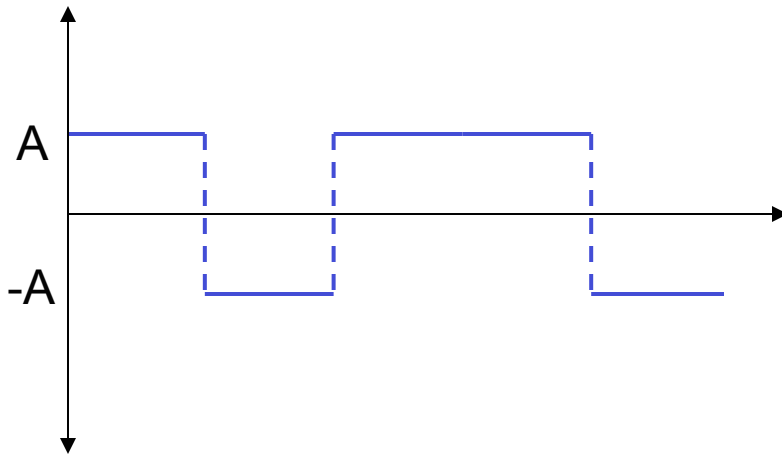
Selection Criteria

How do we decide on which modulation technique to use?

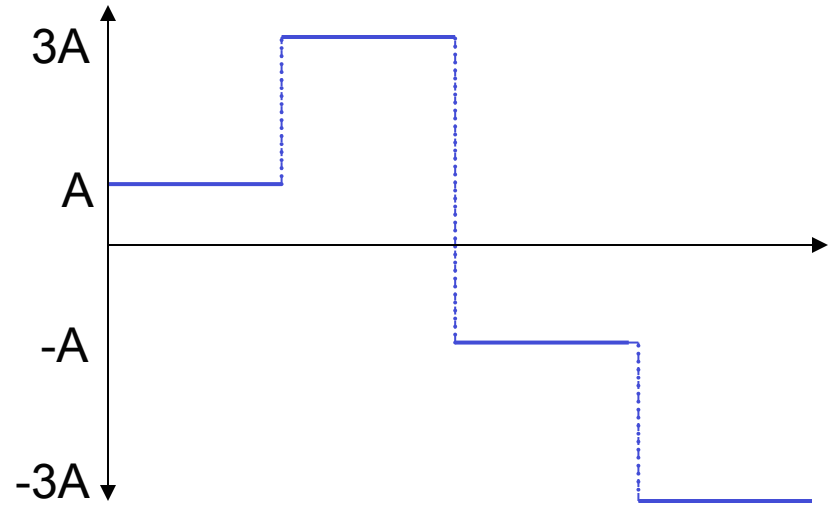
- Performance: probability of error P_e .
 - Probability that a 0 (1) is transmitted and the receiver decodes as a 1 (0).
- Complexity: how difficult is it for the receiver to recover the bits (demodulate)?
 - FSK was used in early voiceband modems because it is simple to implement.
- Bandwidth or spectral efficiency:
bandwidth (B) needed to accommodate data rate R bps, i.e.,
R/B measured in bits per second per Hz.
- Power efficiency: energy needed per bit to achieve a satisfactory P_e .
- Performance in the presence of fading, multipath, and interference.



Example: Binary vs. 4-Level ASK



Rate = $1/T$ symbols/sec
Bandwidth is roughly $1/T$ Hz
Bandwidth efficiency = 1 bps/Hz

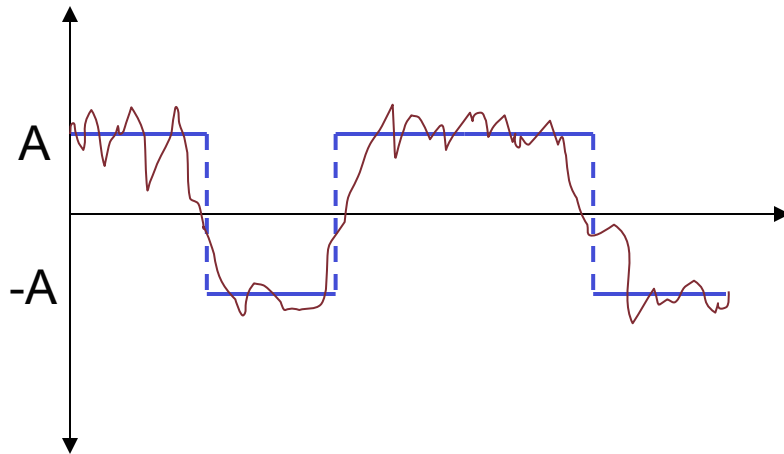


Rate = $2/T$ symbols/sec
Bandwidth is roughly $1/T$ Hz
Bandwidth efficiency = 2 bps/Hz

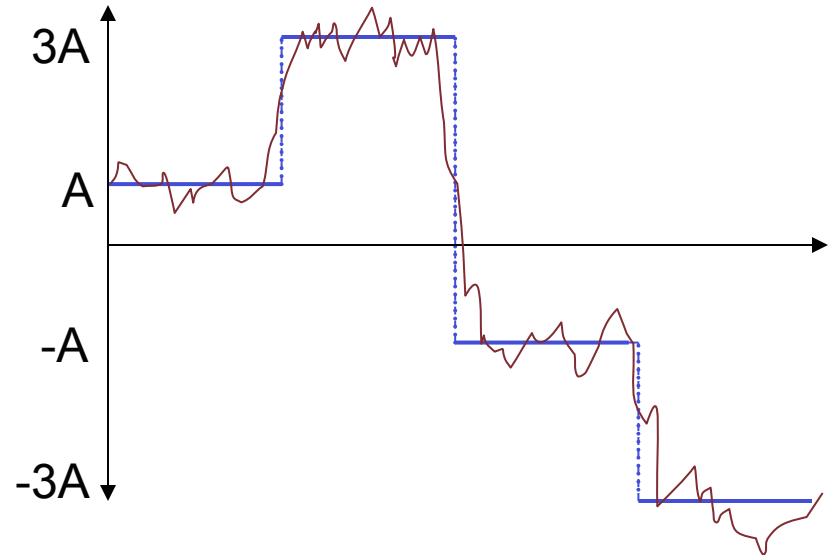
What about power efficiency?



Noisy Baseband Signals



Rate = $1/T$ symbols/sec
Bandwidth is roughly $1/T$ Hz
Bandwidth efficiency = 1 bps/Hz
Power = A^2 (amplitude squared).

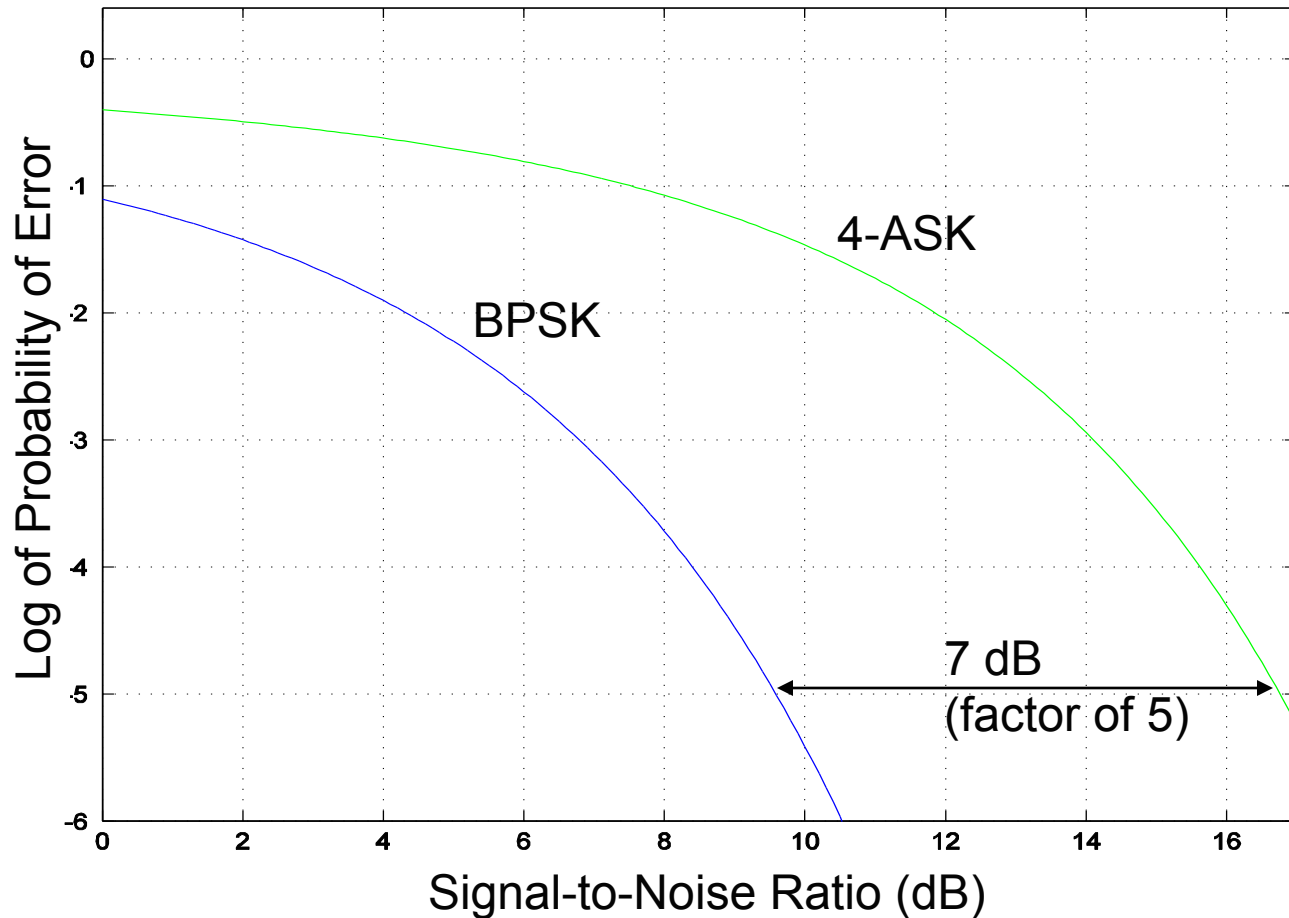


Rate = $2/T$ symbols/sec
Bandwidth is roughly $1/T$ Hz
Bandwidth efficiency = 2 bps/Hz
Power = $(A^2 + 9A^2)/2 = 5A^2$

What about probability of error vs transmitted power?



Probability of Error





How to Increase Bandwidth Efficiency?



How to Increase Bandwidth Efficiency?

- Increase number of signal levels.
- Use more bandwidth efficient modulation scheme (e.g., PSK).
- Apply coding techniques:
protect against errors by adding redundant bits.
- Note that reducing T increases the symbol rate, but also increases the signal bandwidth.

There is a fundamental tradeoff between power efficiency and bandwidth efficiency.



The Fundamental Question

Given:

- B Hz of bandwidth
- S Watts of transmitted signal power
- N Watts per Hz of background noise
(or interference) power

What is the maximum achievable data rate?
(Note: depends on P_e .)



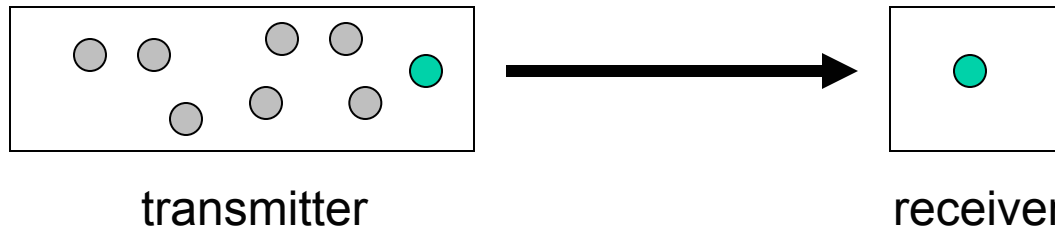
Claude Shannon (1916-2001) Father of “Information Theory”

Shannon’s 1948 paper

“*A Mathematical Theory of Communications*”

laid the foundations for modern communications and networking:

“The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point...”

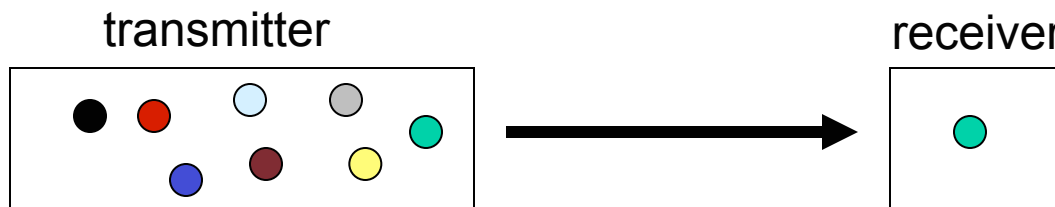




Claude Shannon (1916-2001) Father of “Information Theory”

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“The significant aspect is that the actual message is one
selected from a set of possible messages. The system must
be designed to operate for each possible selection, not just
the one which will actually be chosen since this is unknown
at the time of design.”





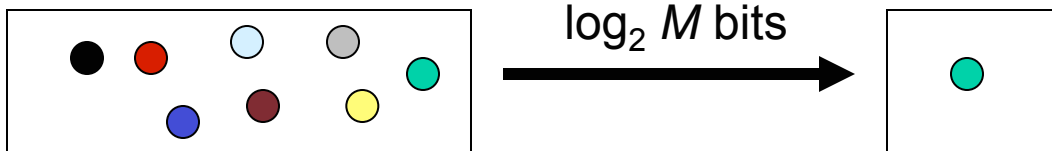
Claude Shannon (1916-2001) Father of “Information Theory”

Shannon’s 1948 paper

“*A Mathematical Theory of Communications*”

laid the foundations for modern communications and networking:

“The choice of a logarithm base corresponds to the choice of a unit for measuring information. If the base 2 is used the resulting units may be called binary digits, or more briefly *bits*, a word suggested by J. W. Tukey.



Transmitter (M possible messages)

receiver





Claude Shannon (1916-2001)

Father of “Information Theory”

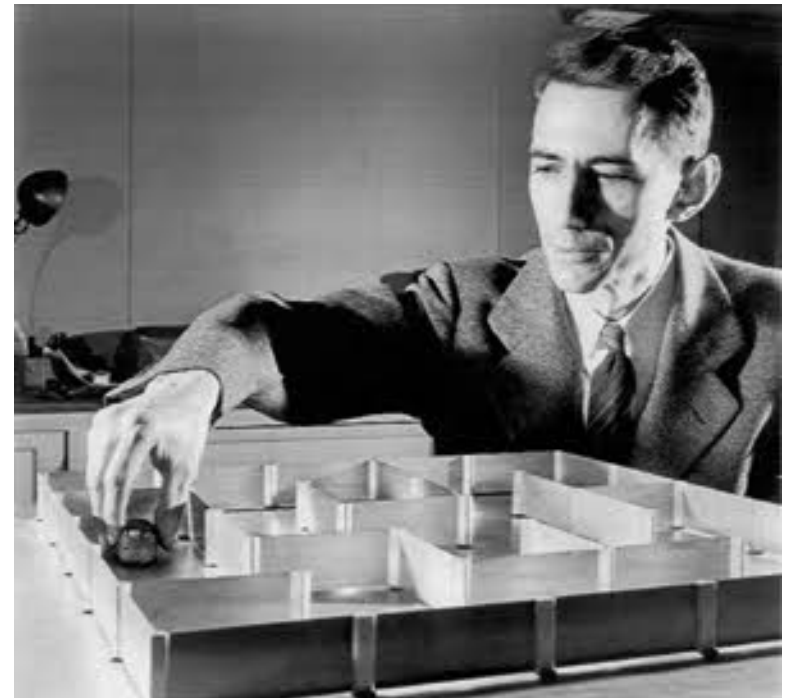
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laid the foundations for modern
communications and networking.

Other contributions and interests:
digital circuits, genetics, cryptography,
investing, chess-playing computer,
roulette prediction, maze-solving,
unicycle designs, juggling

Videos:

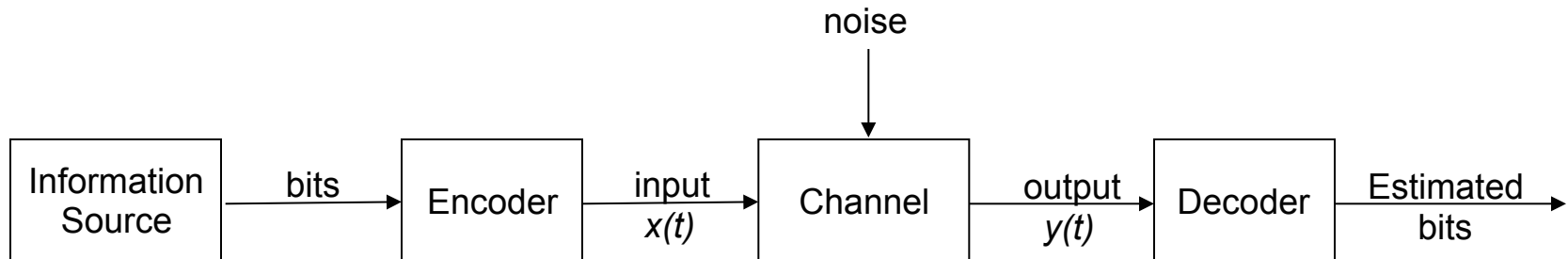
[Father of the Information Age](#)

[Juggling video](#)





Shannon's Channel Coding Theorem (1948)



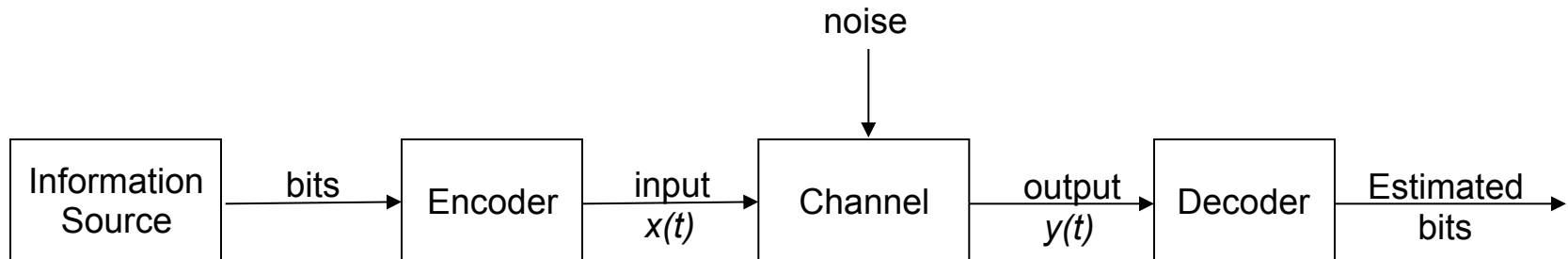
Information rate:	R bits/second
Channel capacity:	C bits/second

$R < C \rightarrow$ There exists an encoder/decoder combination that achieves arbitrarily low error probability.

$R > C \rightarrow$ The error probability cannot be made small.



Shannon Capacity



Channel capacity: $C = B \log(1+S/N)$ bits/second

B= Bandwidth, S= Signal Power, N= Noise Power
No fading



Caveats

- “There exists” does not address complexity issues.
 - As the rate approaches Shannon capacity, to achieve small error rates, the transmitter and (especially) the receiver are required to do more and more computations.
- The theorem does not say anything about delay.
 - To achieve Shannon capacity the length of the transmitted code words must tend to infinity!
- The previous formula does not apply with fading, multipath, frequency-selective attenuation.
- It has taken communications engineers more than 50 years to find practical coding and decoding techniques, which can achieve information rates close to the Shannon capacity.



Example: GSM/EDGE

Bandwidth = 200 kHz, S/I = 9 dB = 7.943

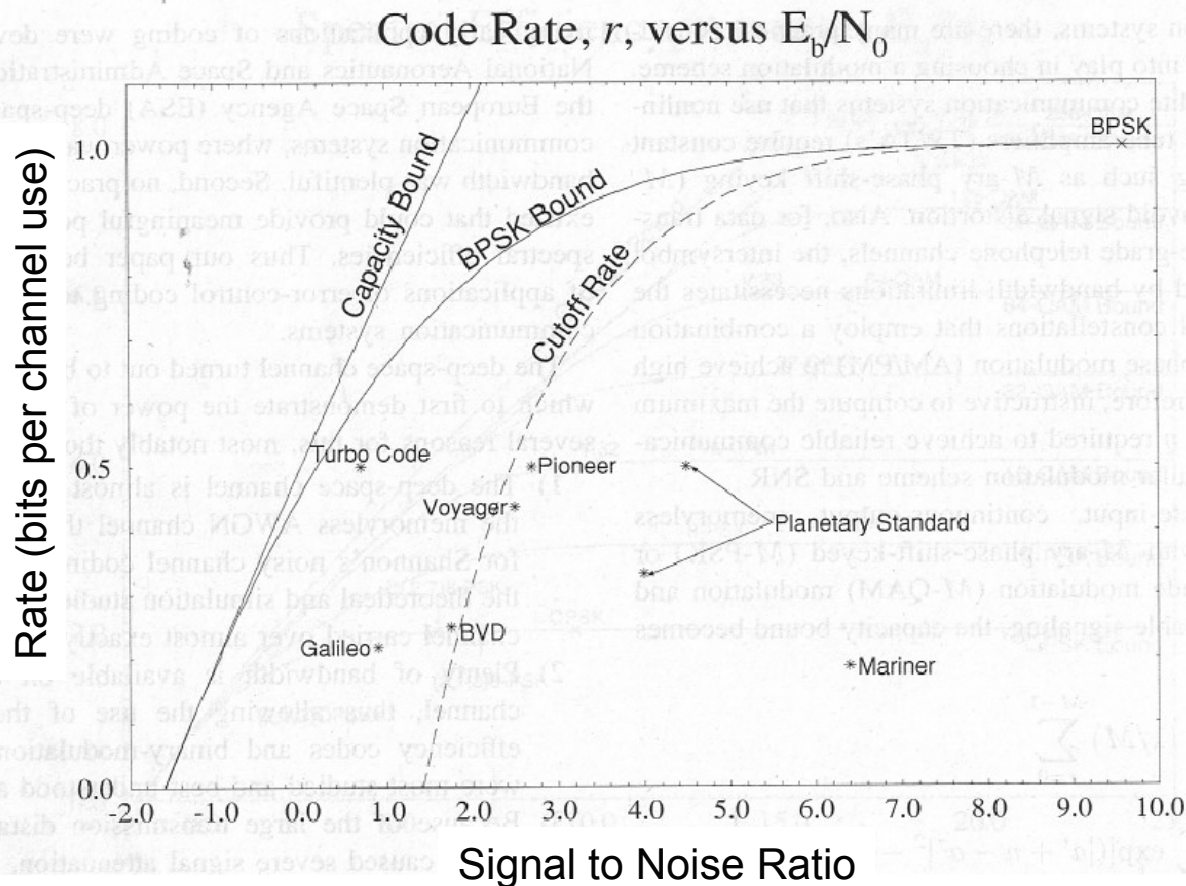
→ $C = 200,000 \times \log(8.943) \approx 632$ kbps

This is what would be achievable in the absence of fading, multipath, etc.

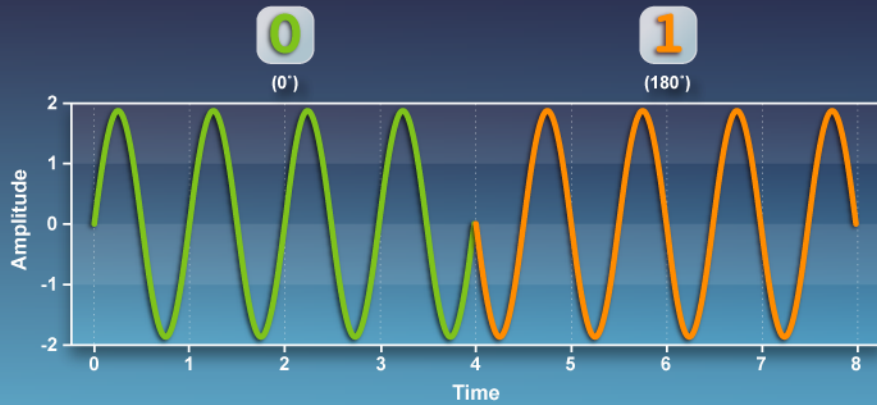
- Currently, EDGE provides throughputs of about 230 kbps.
- Up to 470 kbps possible using additional “tricks”, such as adapting the modulation and coding format to match the channel
 - Preceding Shannon formula is not directly applicable.



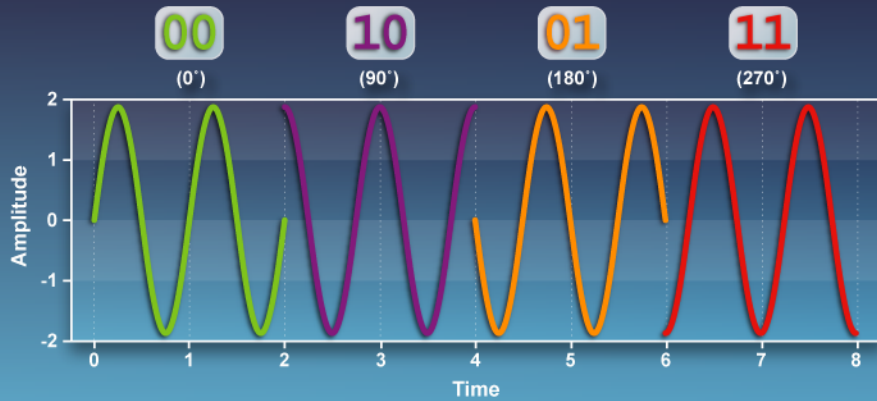
Data Rates for Deep Space Applications



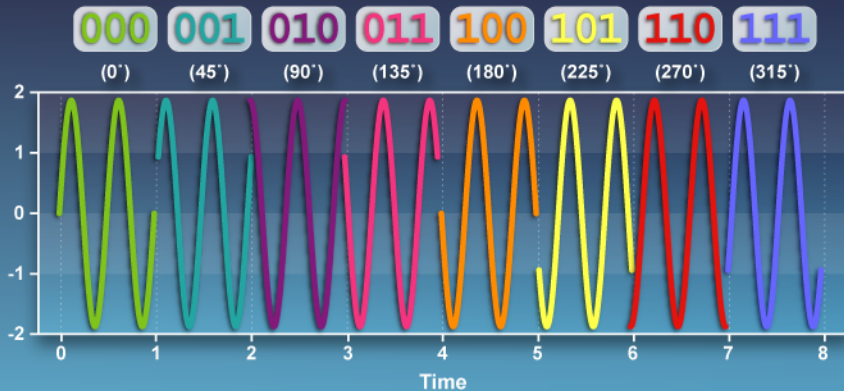
- Mariner: 1969 (Mars)
- Pioneer 10/11: 1972/3 (Jupiter/Saturn fly-by)
- Voyager: 1977 (Jupiter and Saturn)
- Planetary Standard: 1980's (military satellite)
- BVD: "Big Viterbi Decoder"
- Galileo: 1992 (Jupiter) (uses BVD)
- Turbo Code: 1993



Binary Phase Shift Keying

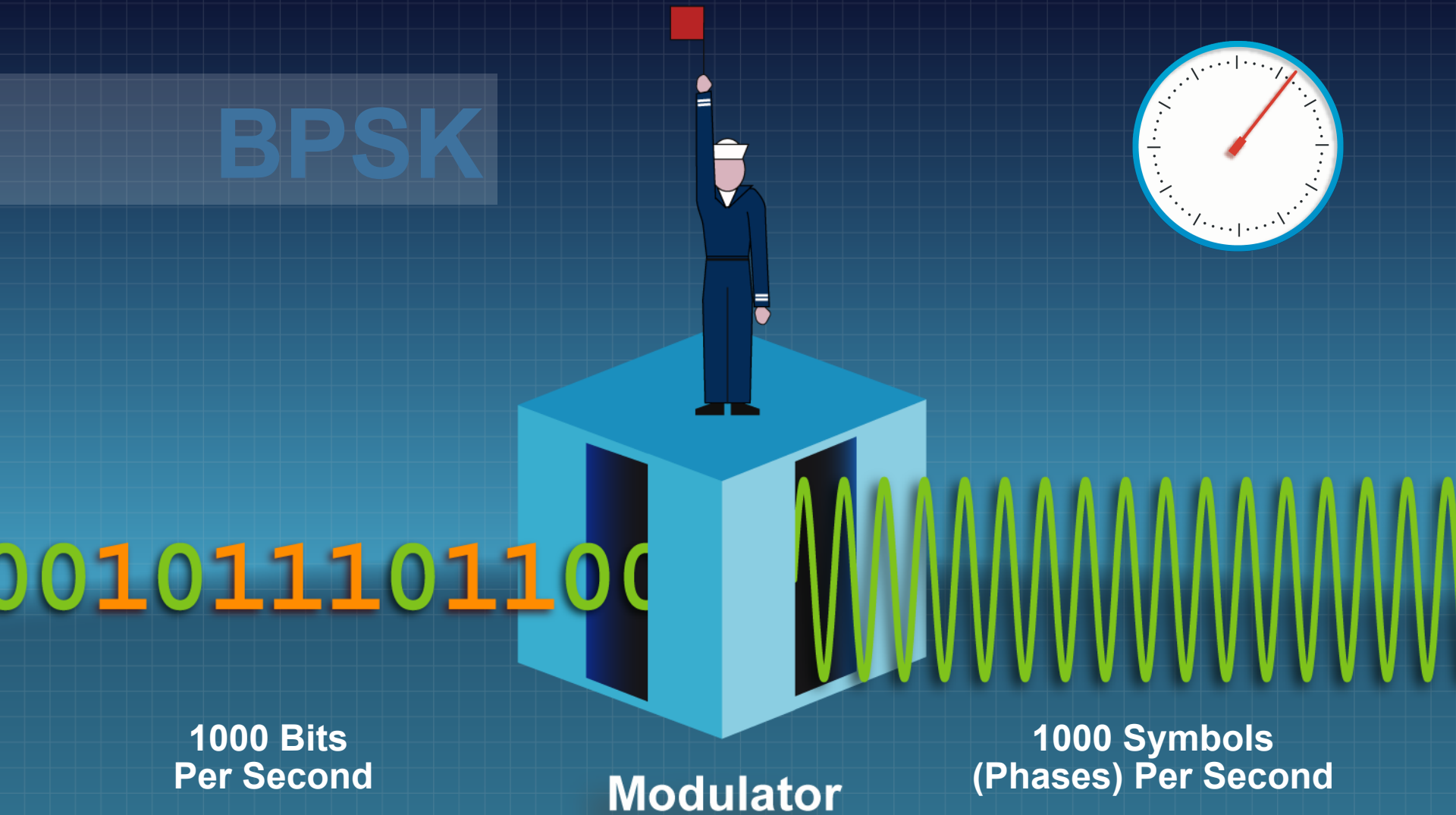


Quadrature Phase Shift Keying



8 Phase Shift Keying

BPSK

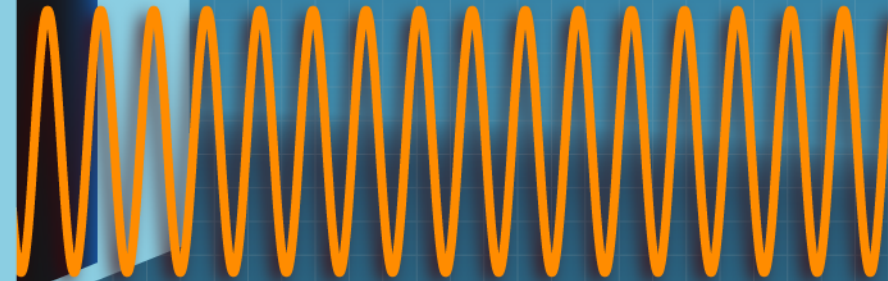
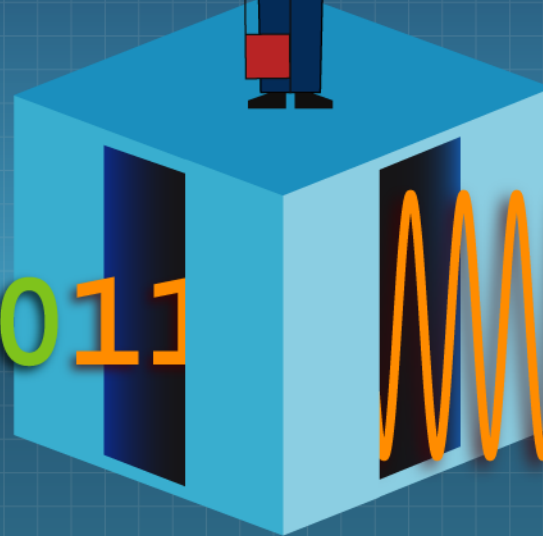


1000 Symbols Per Second x 1 Bit Per Symbol = 1000 Bits Per Second

BPSK



110010111011



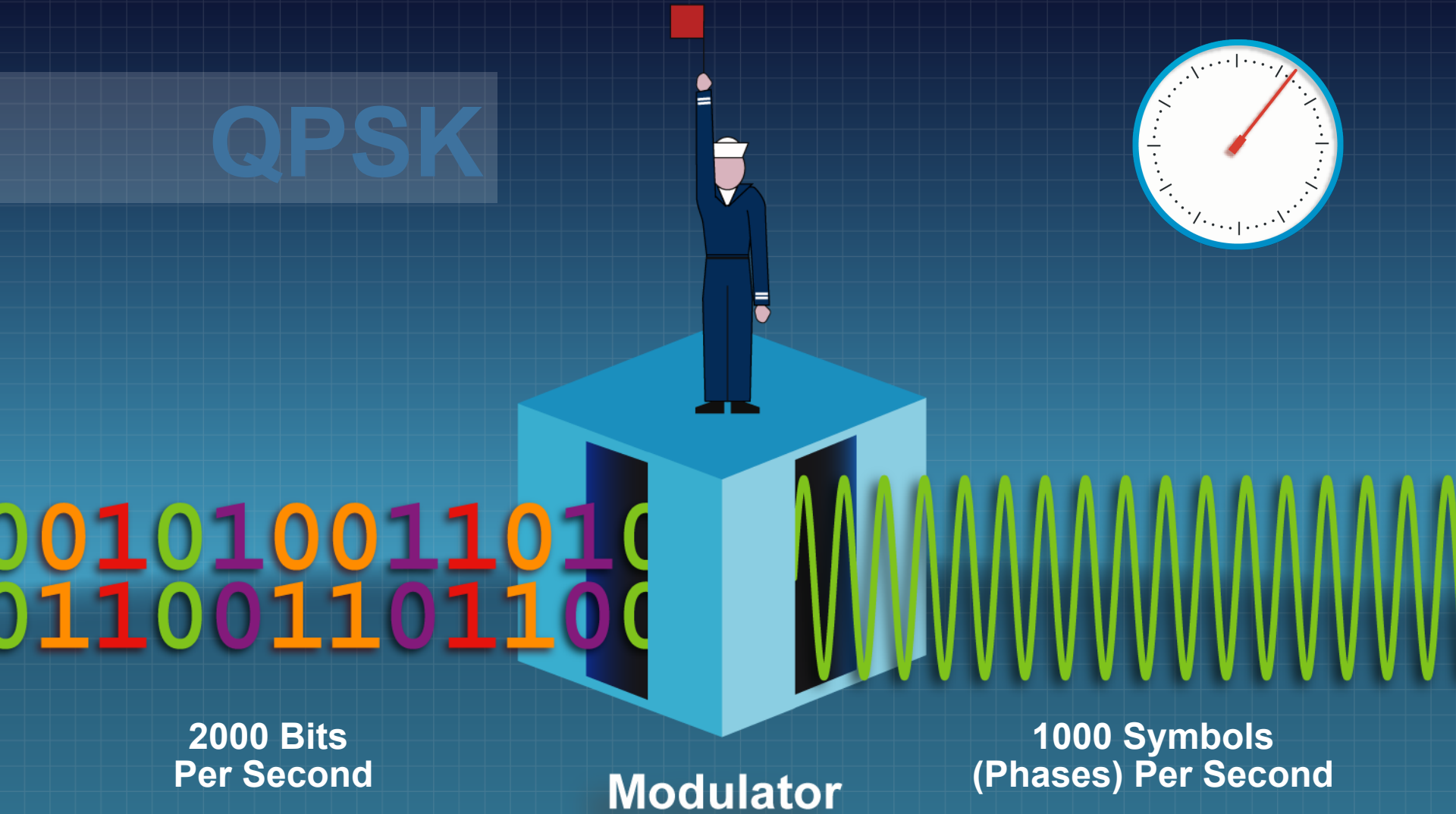
1000 Bits
Per Second

Modulator

1000 Symbols
(Phases) Per Second

1000 Symbols Per Second x 1 Bit Per Symbol = 1000 Bits Per Second

QPSK



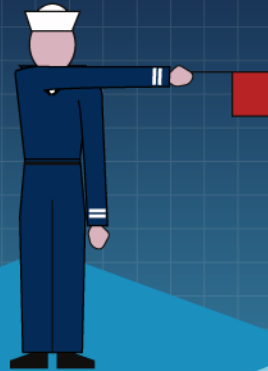
2000 Bits
Per Second

Modulator

1000 Symbols
(Phases) Per Second

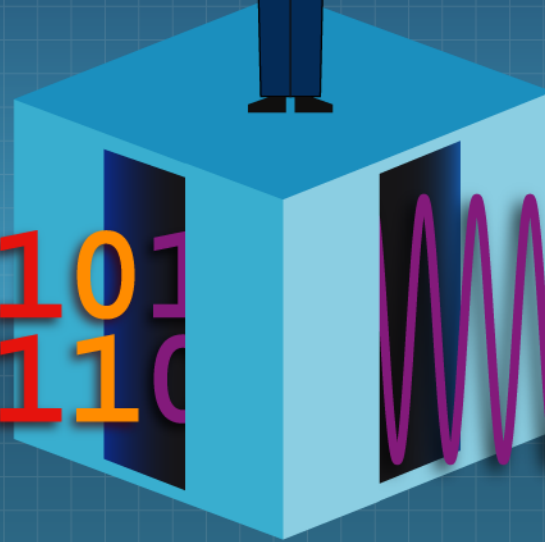
1000 Symbols Per Second x 2 Bits Per Symbol = 2000 Bits Per Second

QPSK

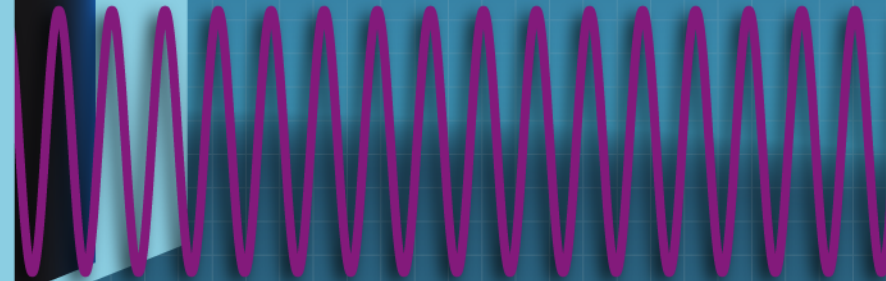


100101001101
001100110110

2000 Bits
Per Second



Modulator



1000 Symbols
(Phases) Per Second

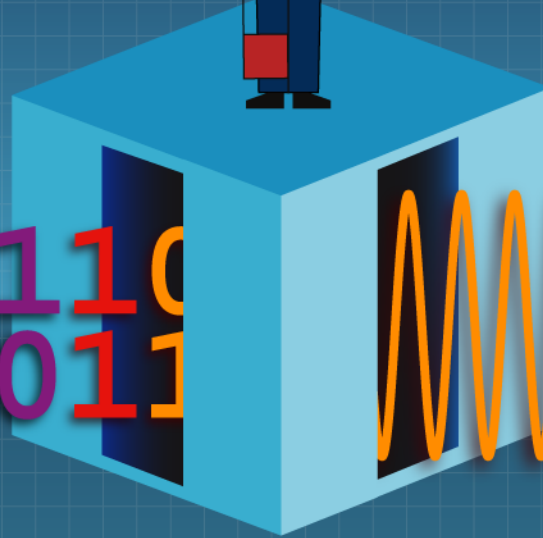
1000 Symbols Per Second x 2 Bits Per Symbol = 2000 Bits Per Second

QPSK

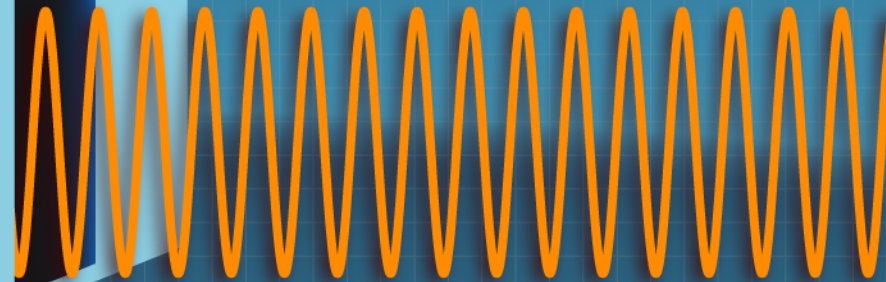


010010100110
000110011011

2000 Bits
Per Second



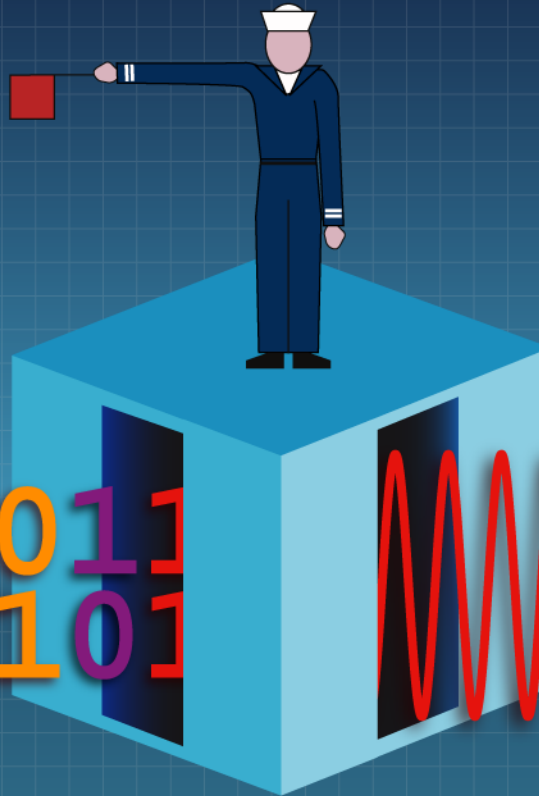
Modulator



1000 Symbols
(Phases) Per Second

1000 Symbols Per Second x 2 Bits Per Symbol = 2000 Bits Per Second

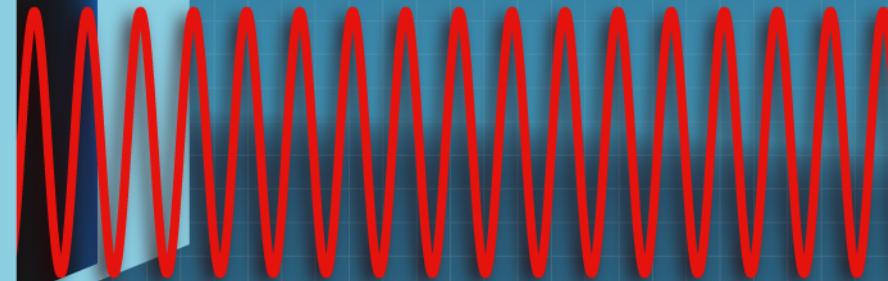
QPSK



101001010011
100011001101

2000 Bits
Per Second

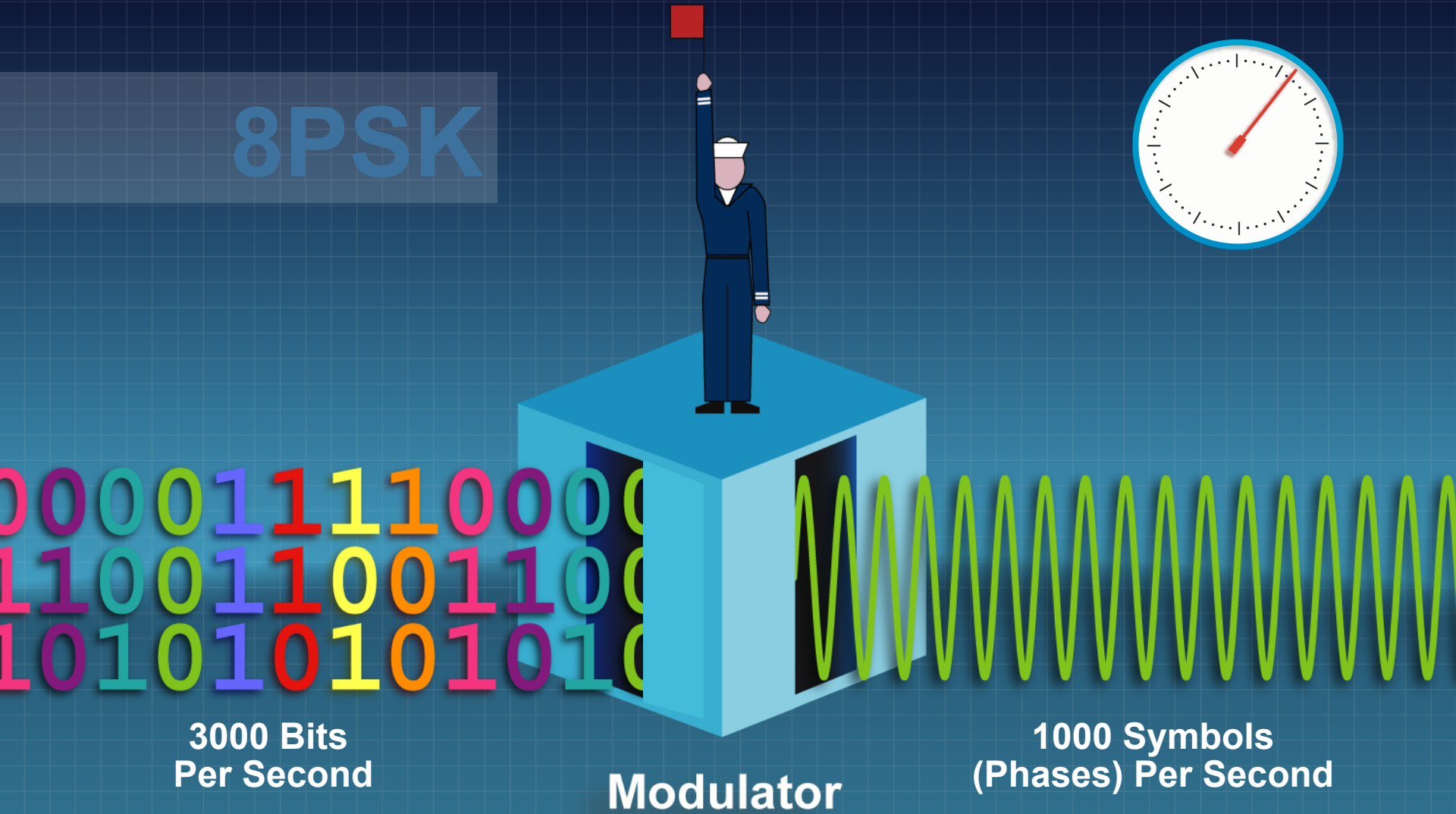
Modulator



1000 Symbols
(Phases) Per Second

1000 Symbols Per Second x 2 Bits Per Symbol = 2000 Bits Per Second

8PSK



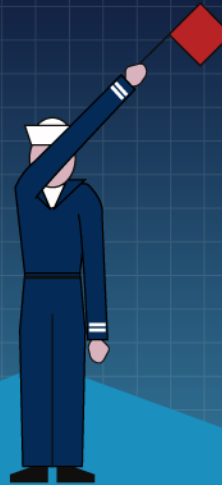
3000 Bits
Per Second

Modulator

1000 Symbols
(Phases) Per Second

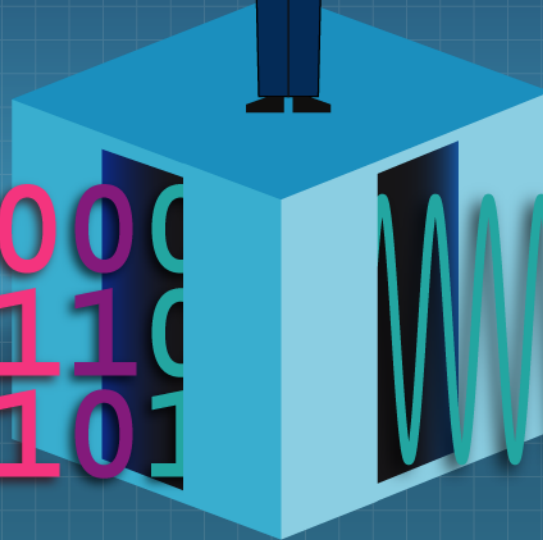
$$1000 \text{ Symbols Per Second} \times 3 \text{ Bits Per Symbol} = 3000 \text{ Bits Per Second}$$

8PSK

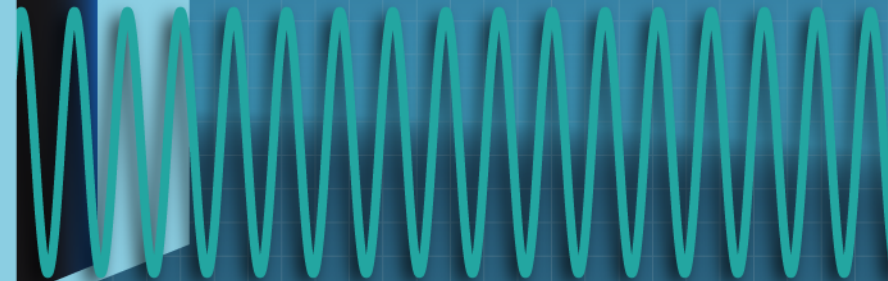


100001111000
011001100110
010101010101

3000 Bits
Per Second



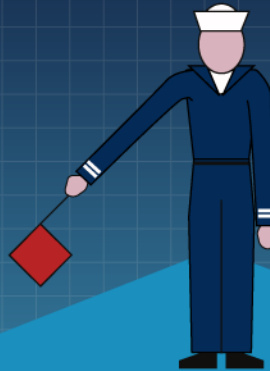
Modulator



1000 Symbols
(Phases) Per Second

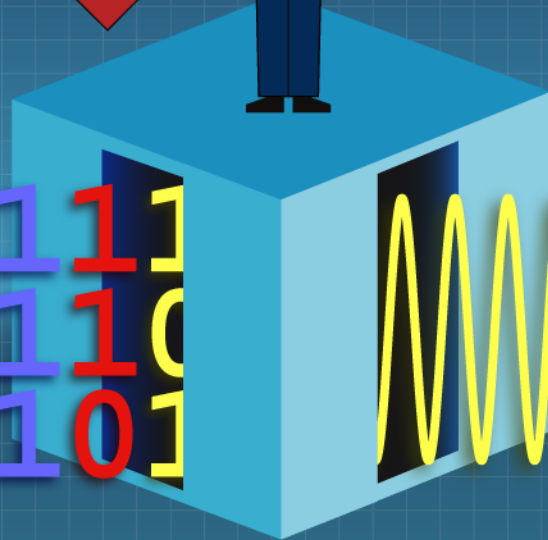
1000 Symbols Per Second x 3 Bits Per Symbol = 3000 Bits Per Second

8PSK

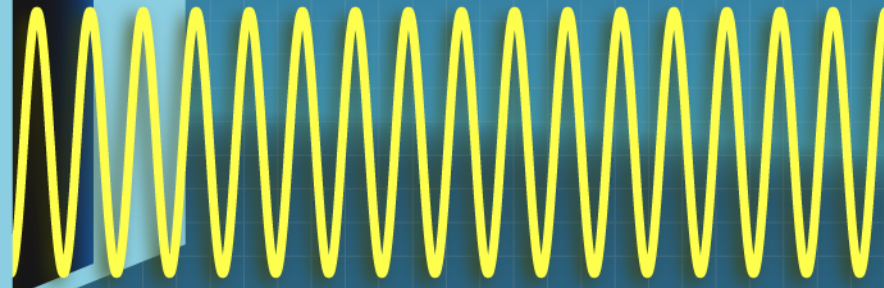


011110000111
011001100110
010101010101

3000 Bits
Per Second



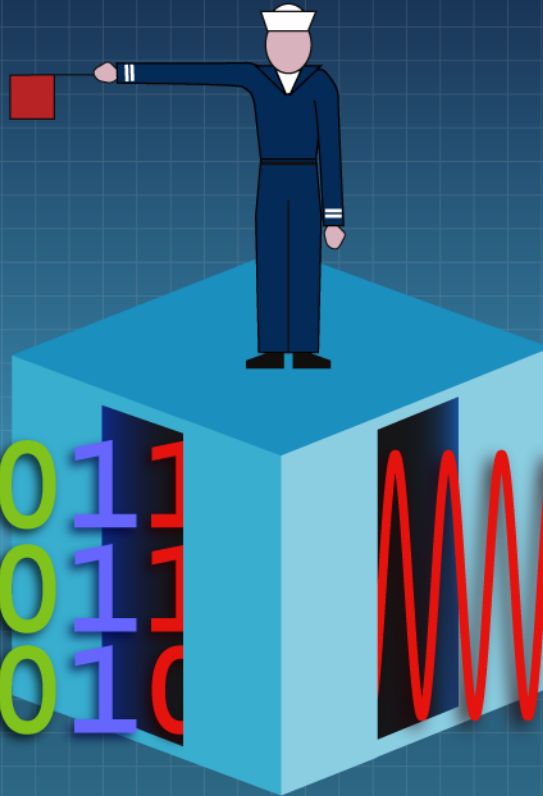
Modulator



1000 Symbols
(Phases) Per Second

$$1000 \text{ Symbols Per Second} \times 3 \text{ Bits Per Symbol} = 3000 \text{ Bits Per Second}$$

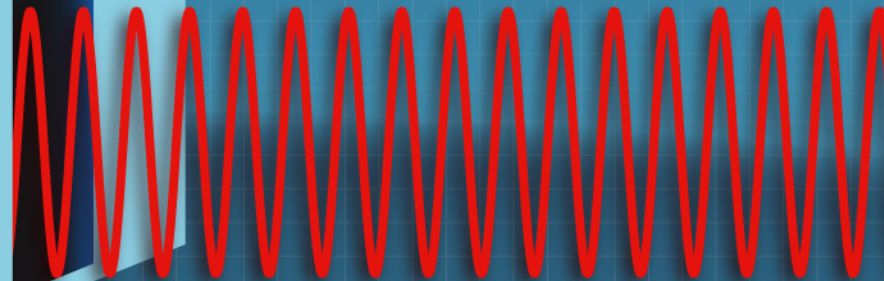
8PSK



001111000011
001100110011
101010101010

3000 Bits
Per Second

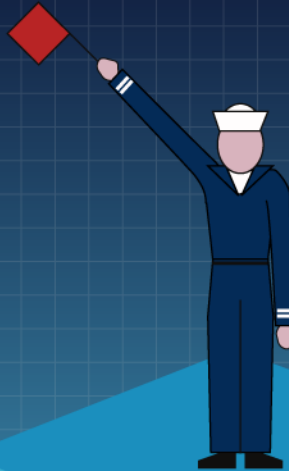
Modulator



1000 Symbols
(Phases) Per Second

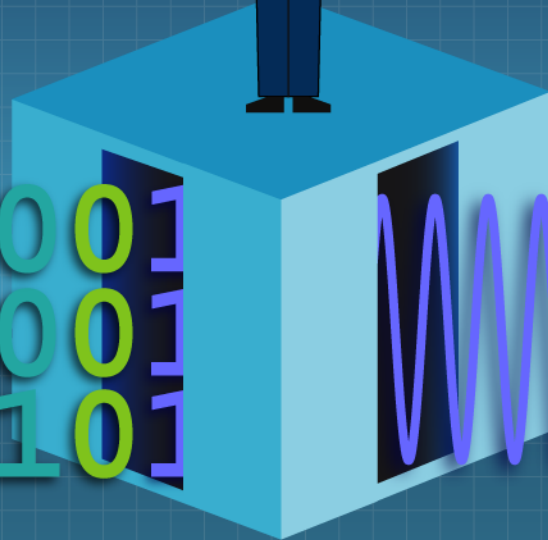
$$1000 \text{ Symbols Per Second} \times 3 \text{ Bits Per Symbol} = 3000 \text{ Bits Per Second}$$

8PSK

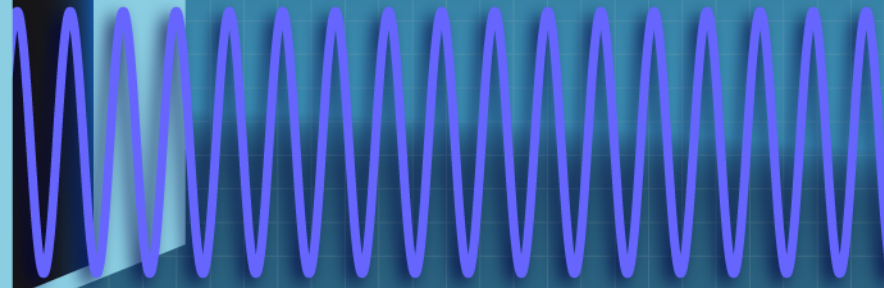


0001110000
10011001100
01010101010

3000 Bits
Per Second

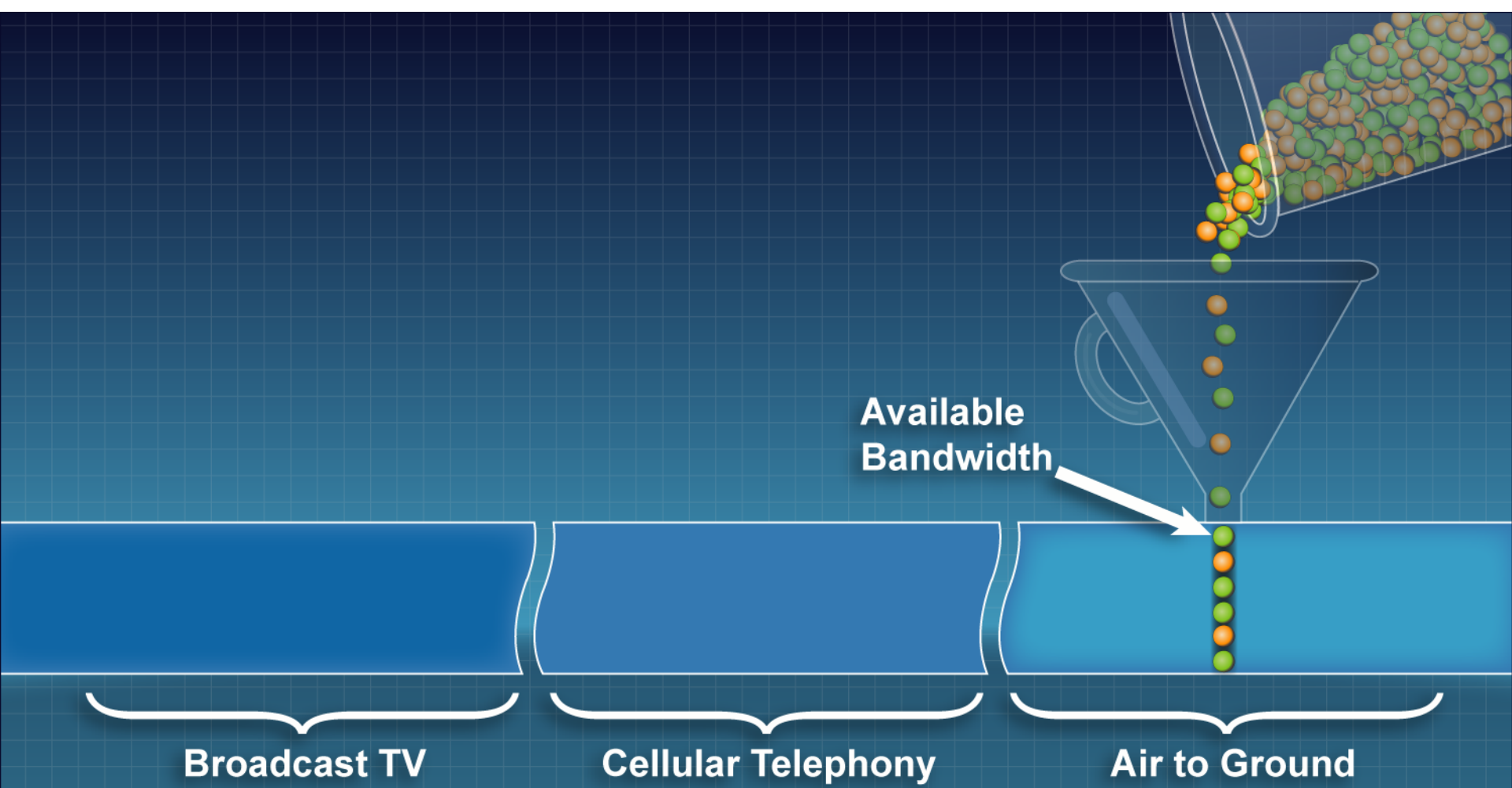


Modulator



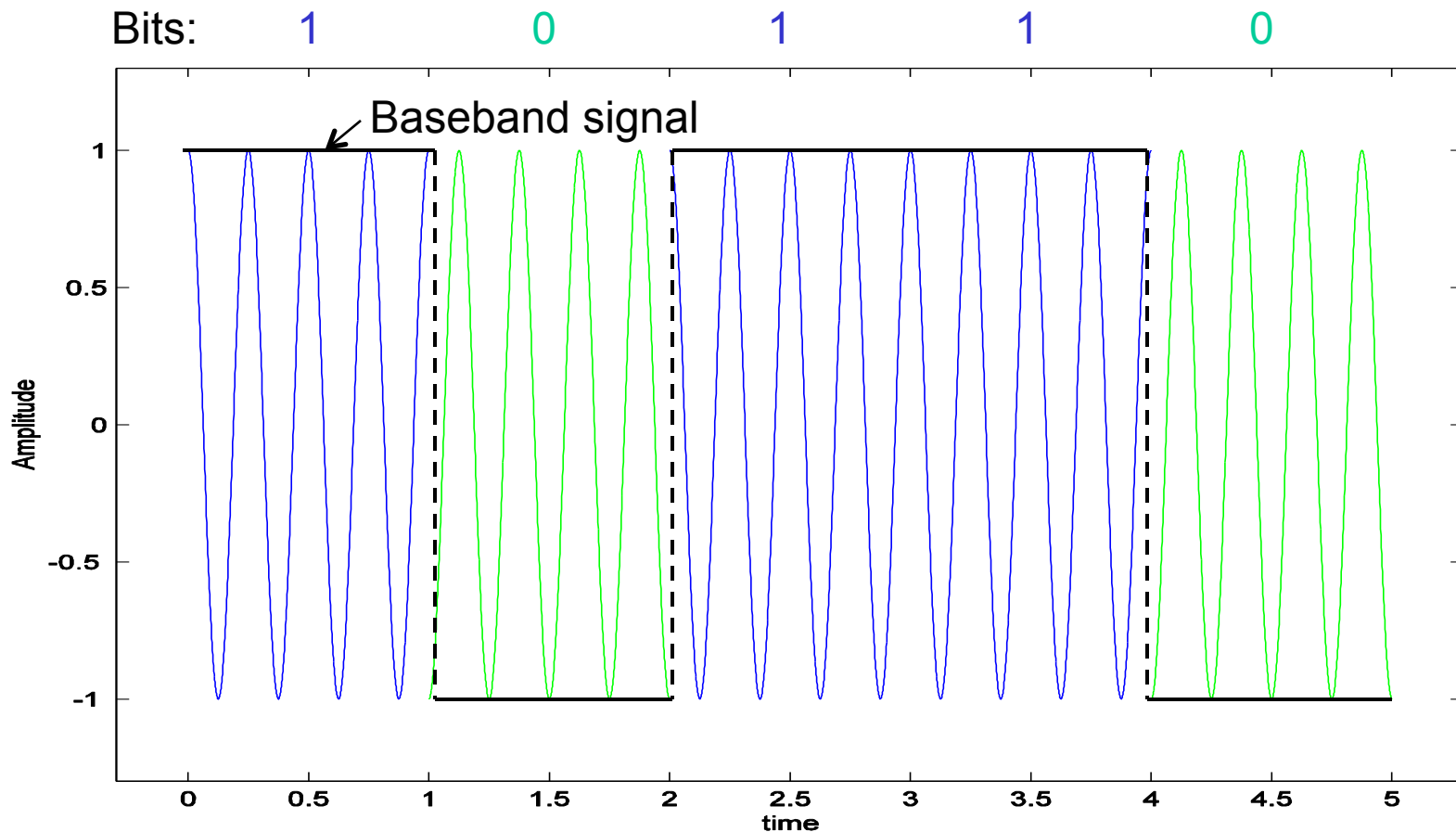
1000 Symbols
(Phases) Per Second

$$1000 \text{ Symbols Per Second} \times 3 \text{ Bits Per Symbol} = 3000 \text{ Bits Per Second}$$



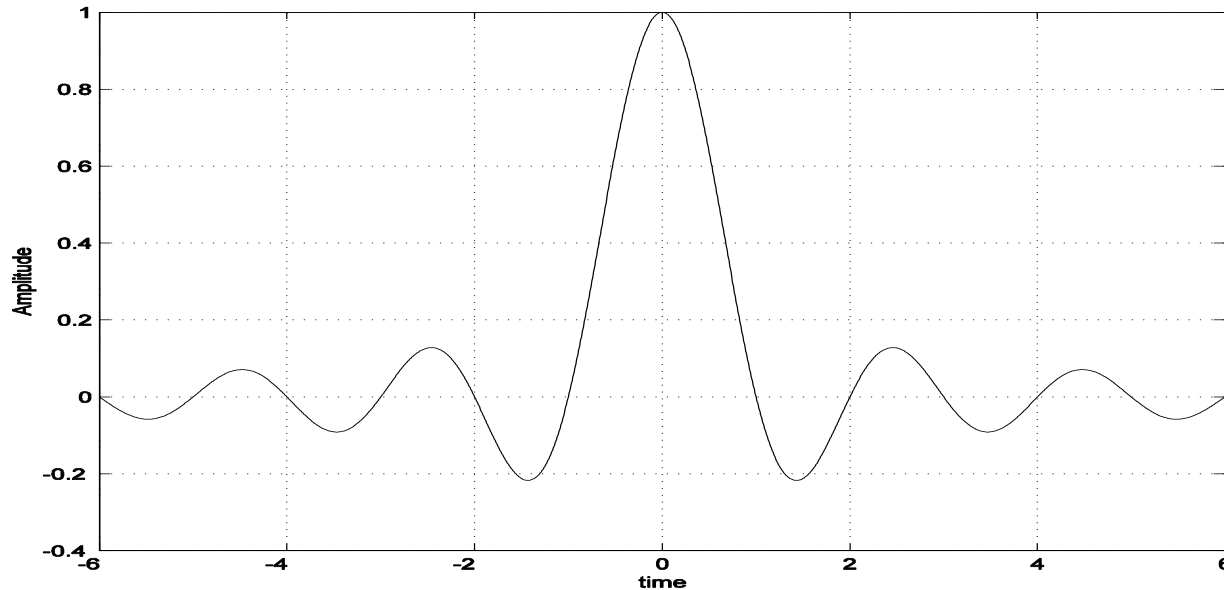


Binary Phase Shift Keying (BPSK)





Minimum Bandwidth (Nyquist) Pulse Shape

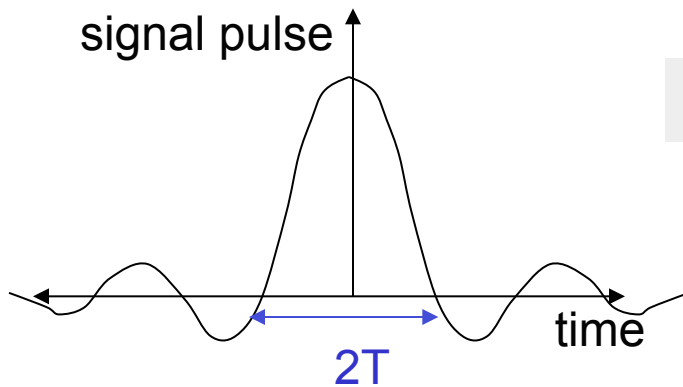


This pulse has the **minimum** bandwidth for a given symbol rate.

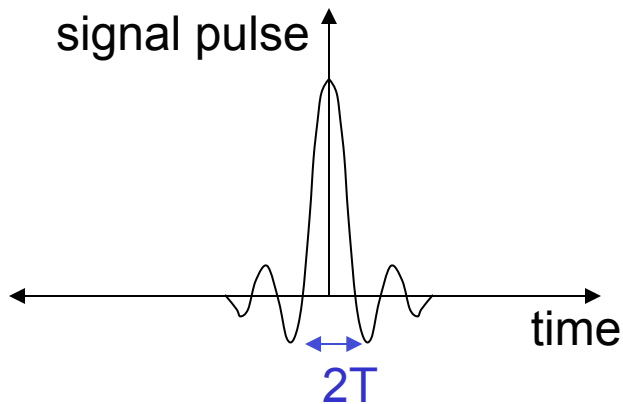
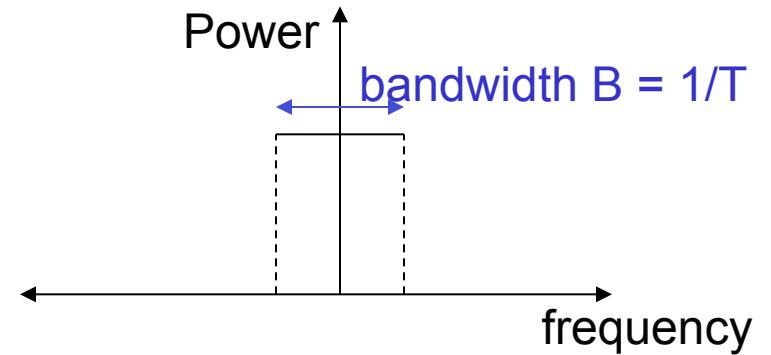
Given bandwidth B , the maximum symbol rate without intersymbol interference (ISI) is B , the “Nyquist rate”.



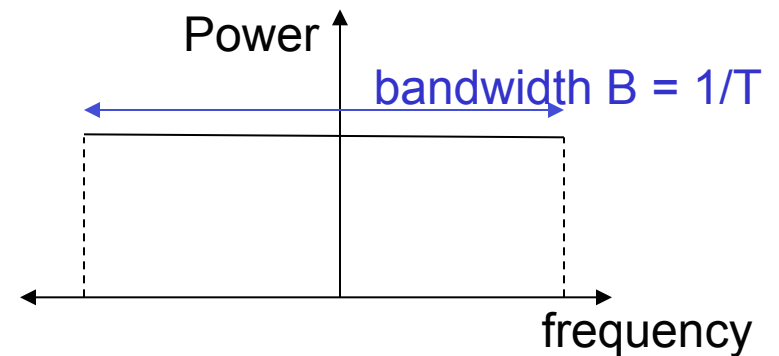
Pulse Width vs. Bandwidth



Narrowband



Wideband

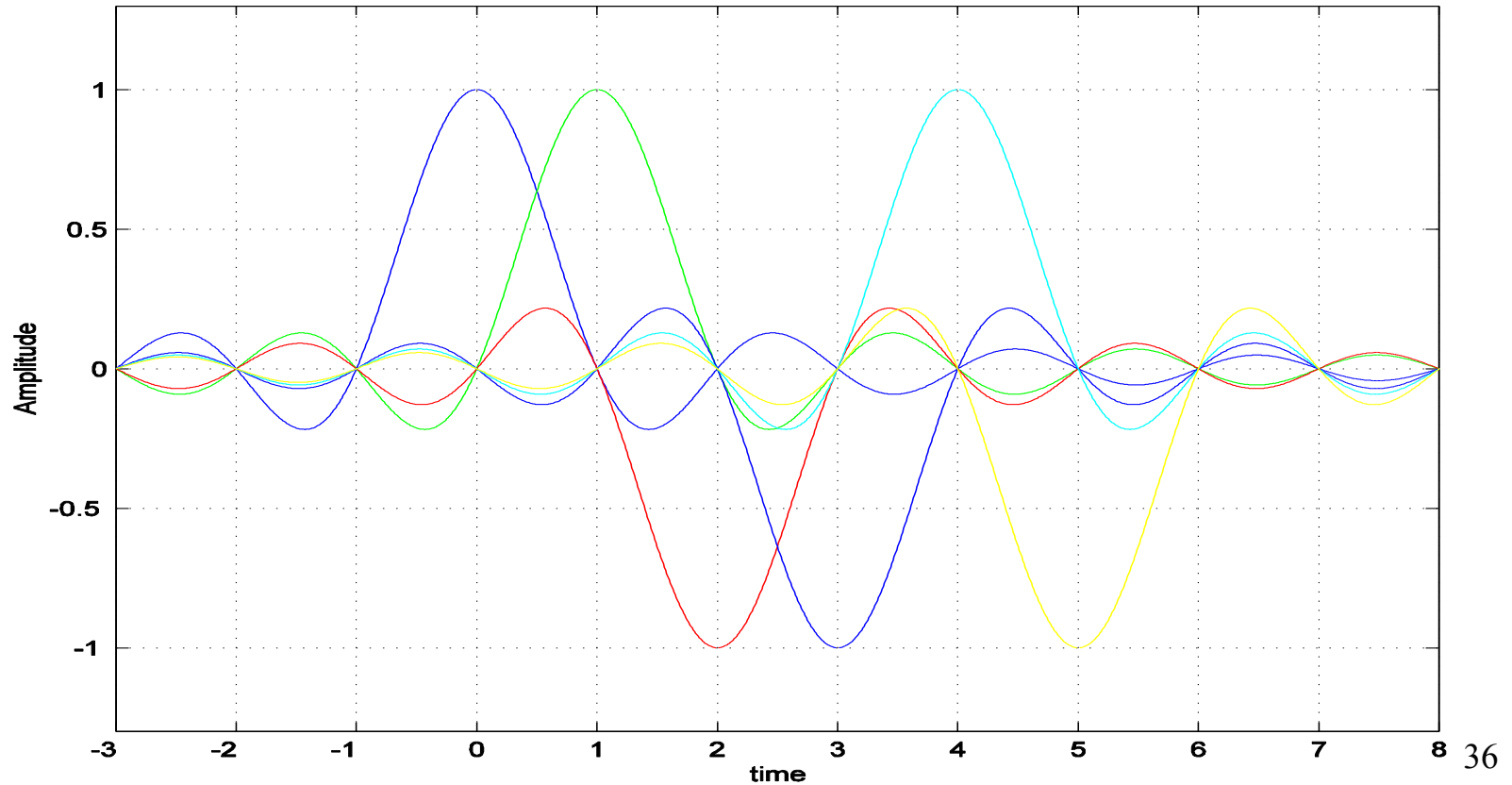


The bandwidth $B = 1/T$ (symbol rate) is often called the **Nyquist bandwidth**. ³⁵



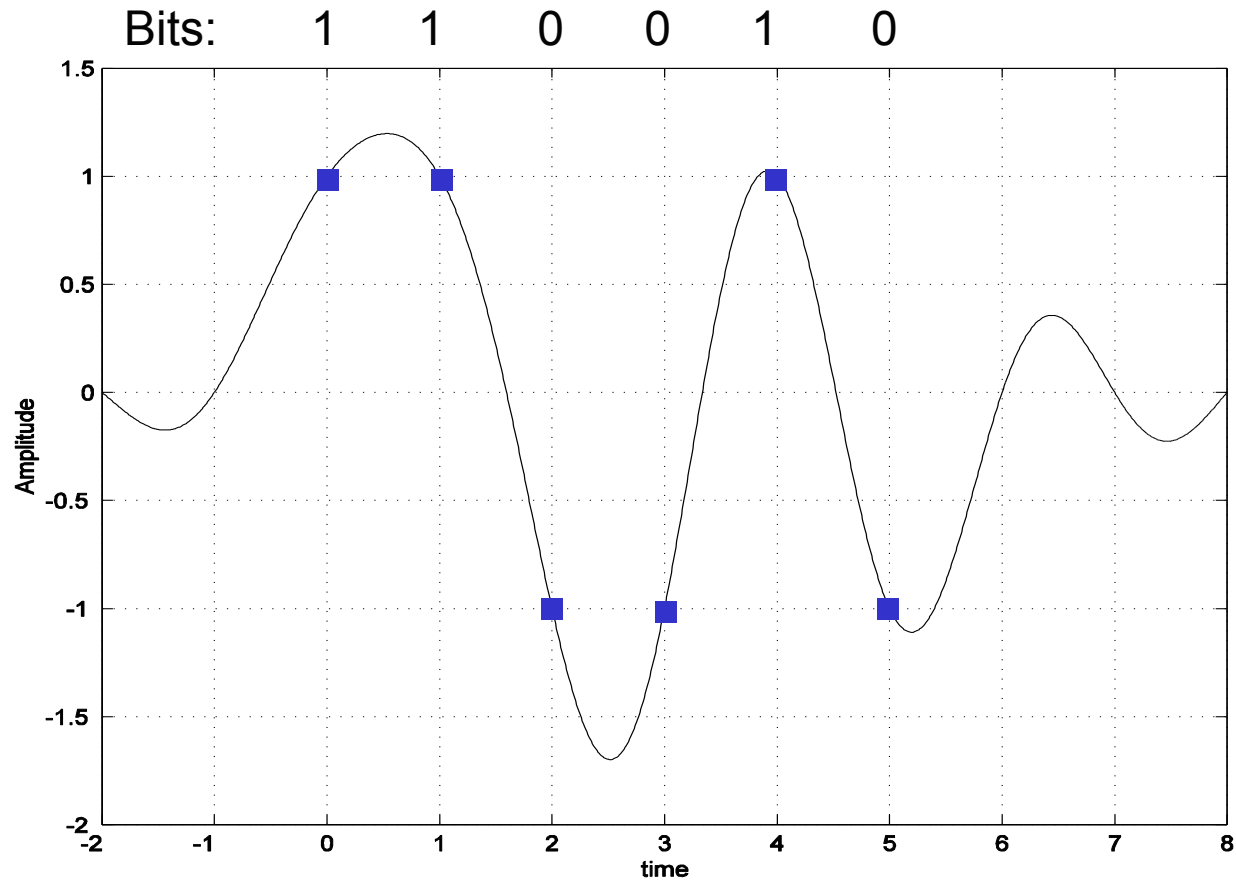
Shifted Nyquist Pulses

Bits: 1 1 0 0 1 0



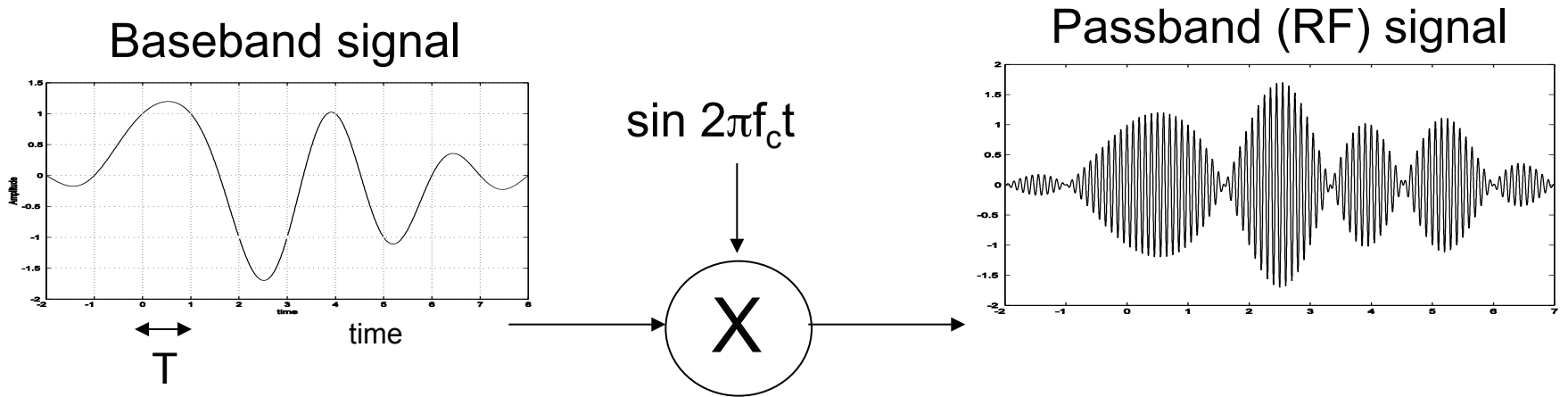


Baseband Waveform (Nyquist Signaling)

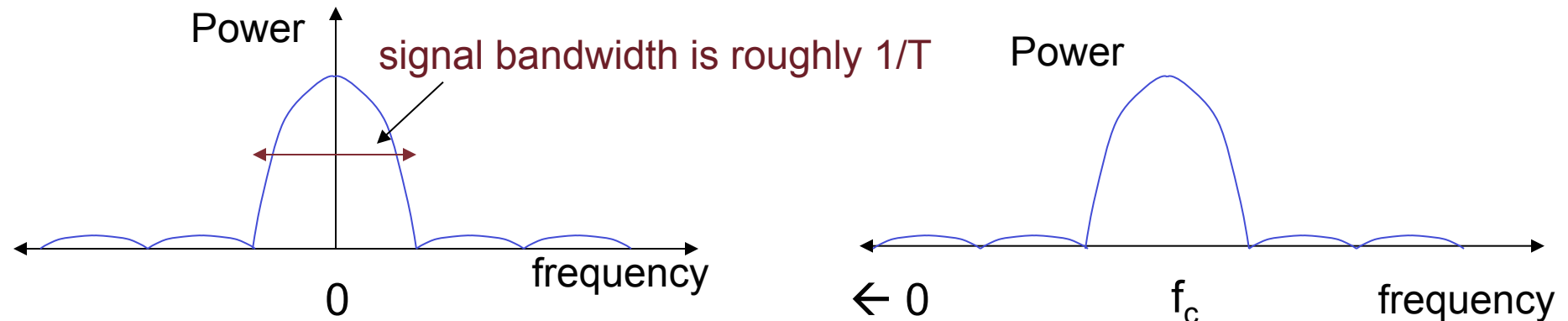




Baseband \rightarrow RF Conversion

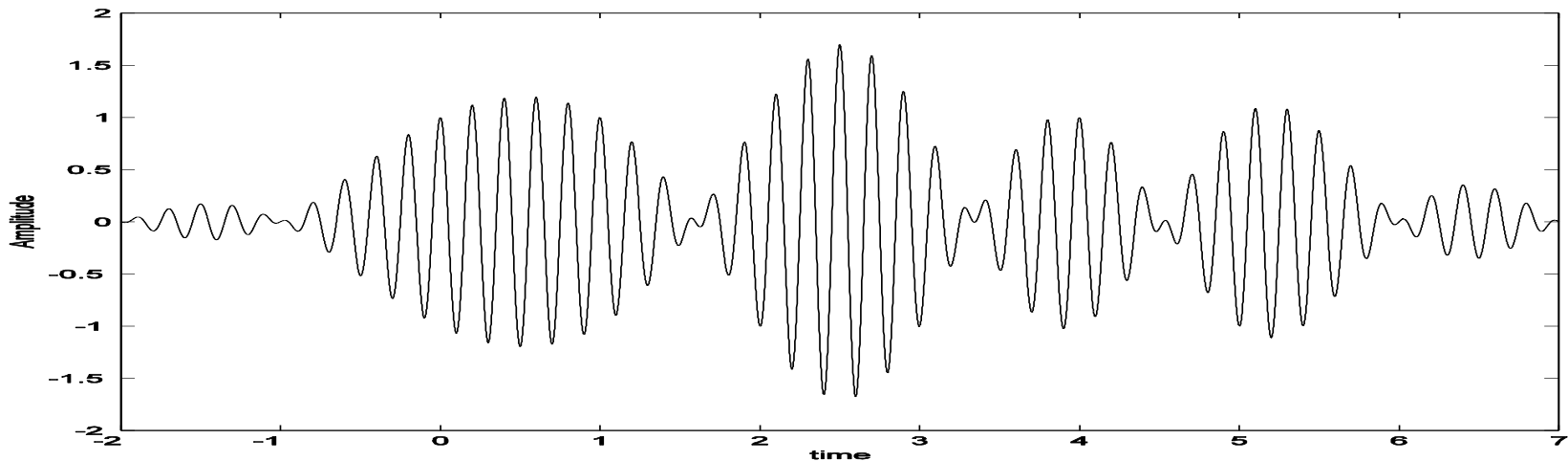
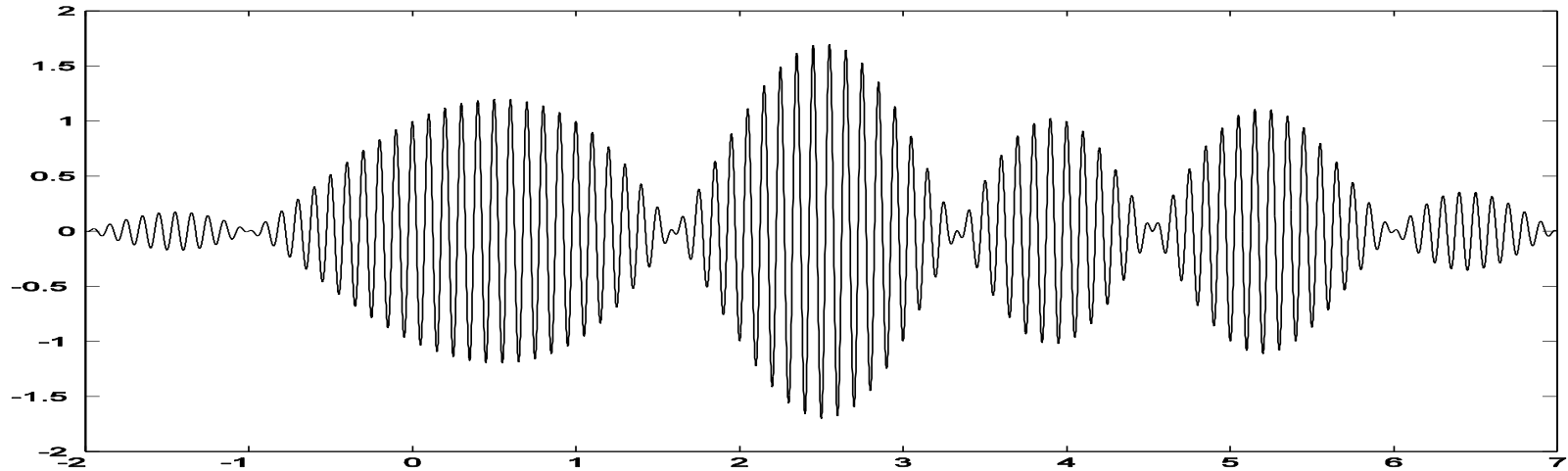


Modulate to the **carrier** frequency f_c



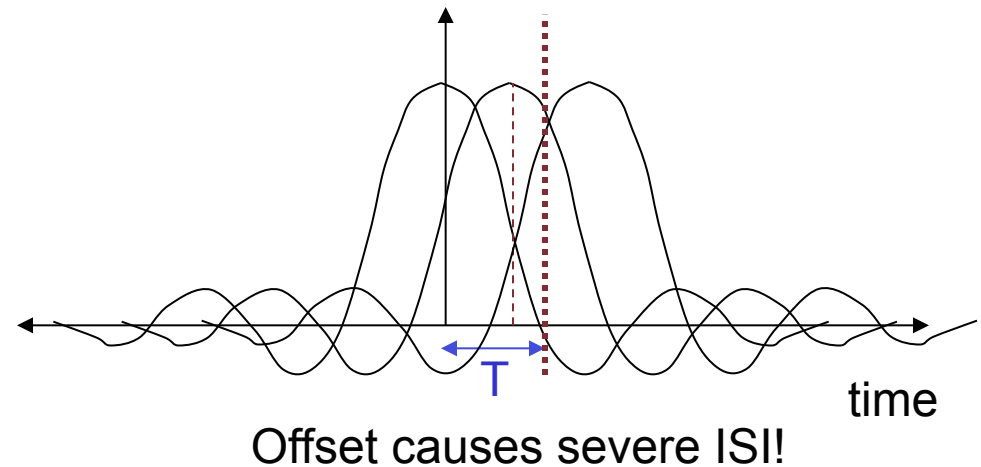
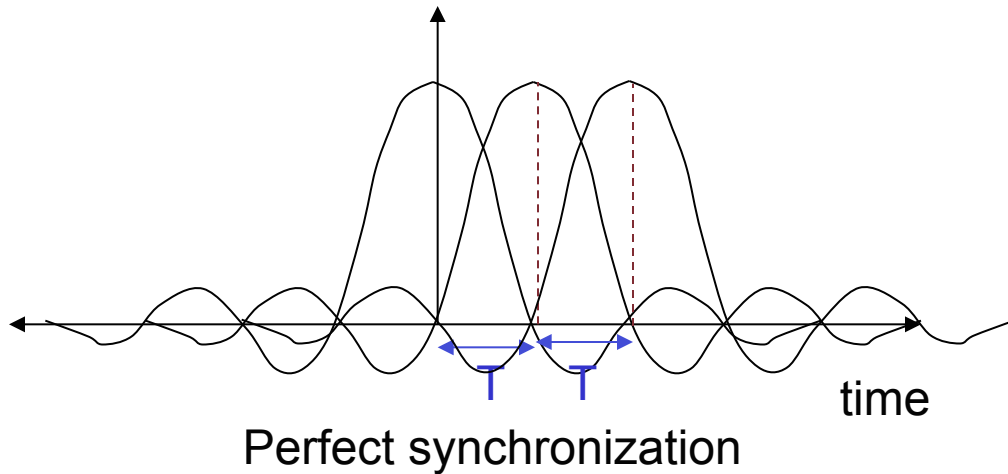


Passband Signal with Different Carrier Frequencies





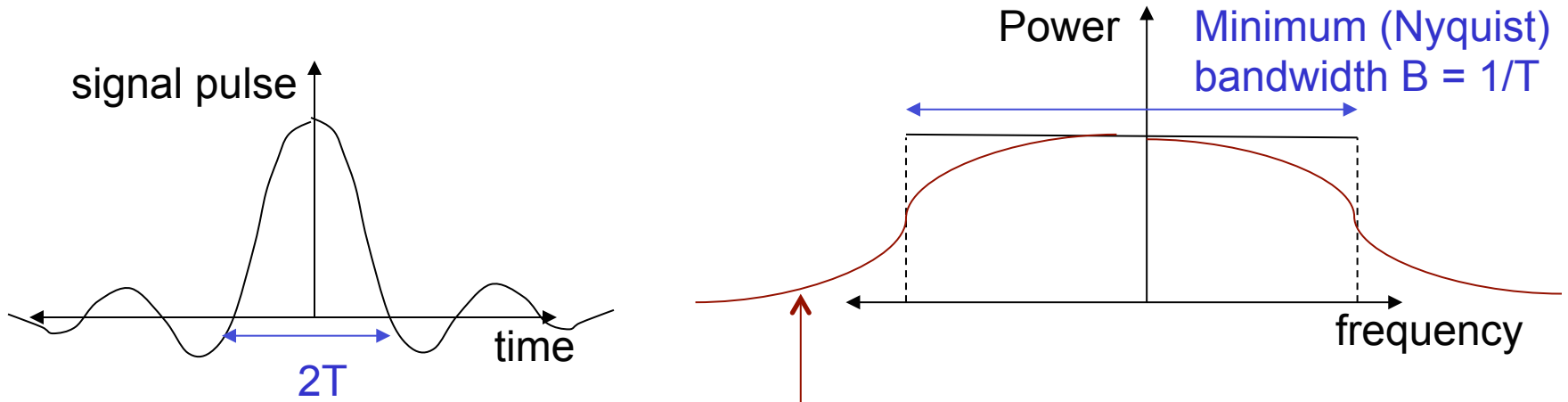
Importance of Synchronization



Need perfect synchronization to avoid severe interference!



Excess Bandwidth

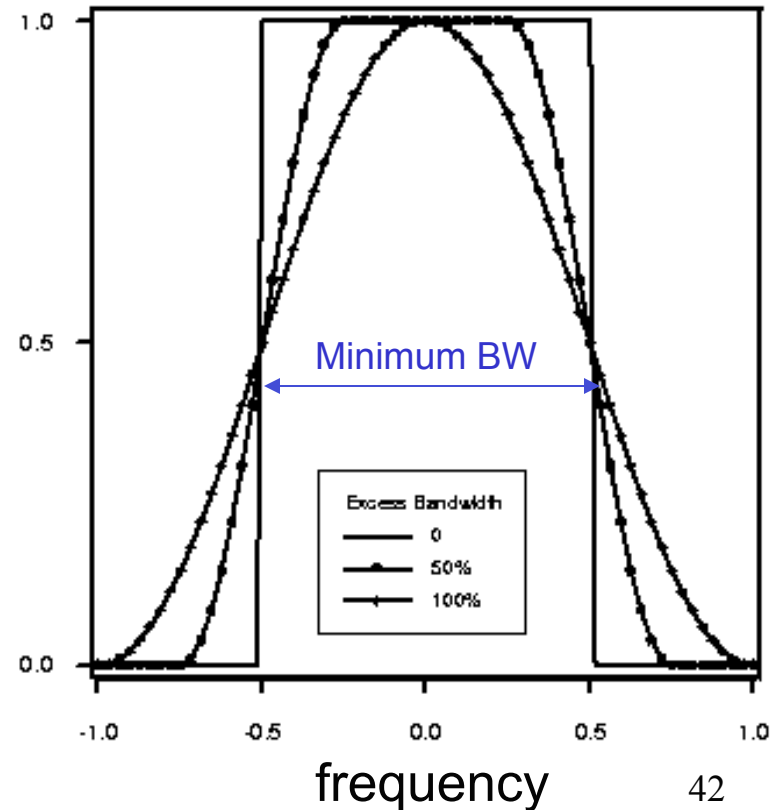
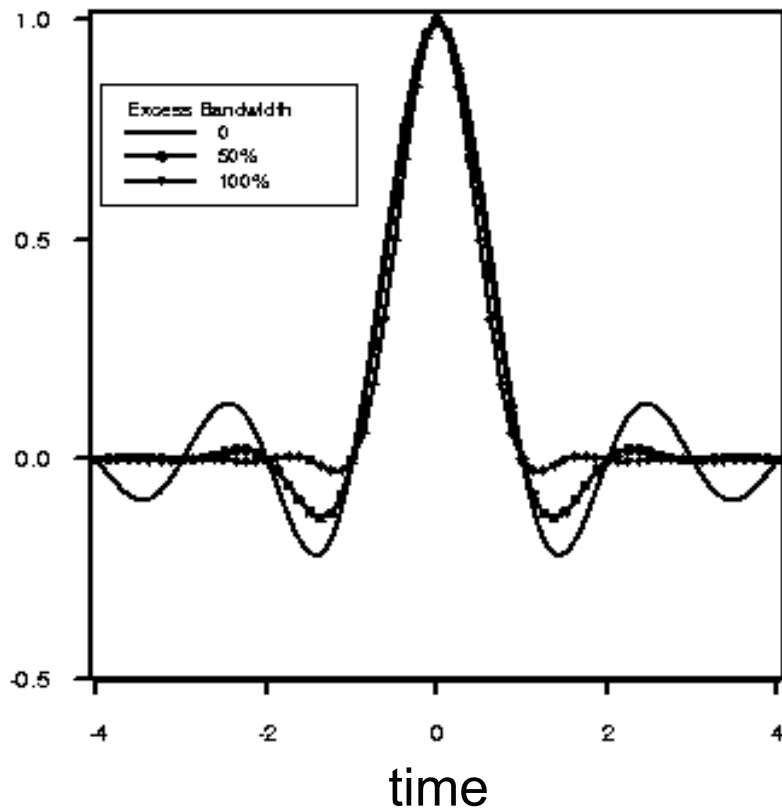


Adding "excess" bandwidth eases filtering and synchronization.



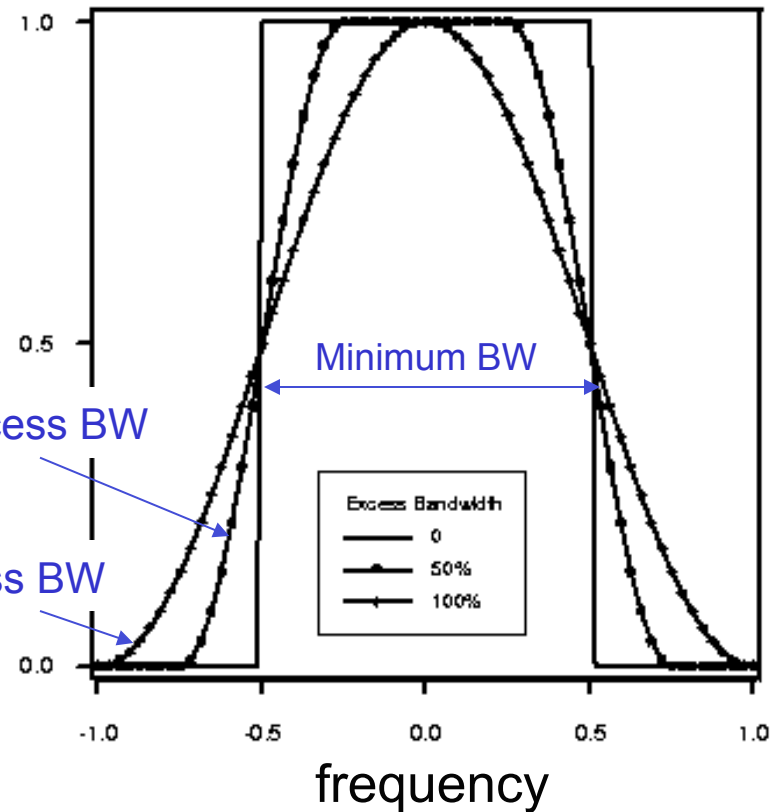
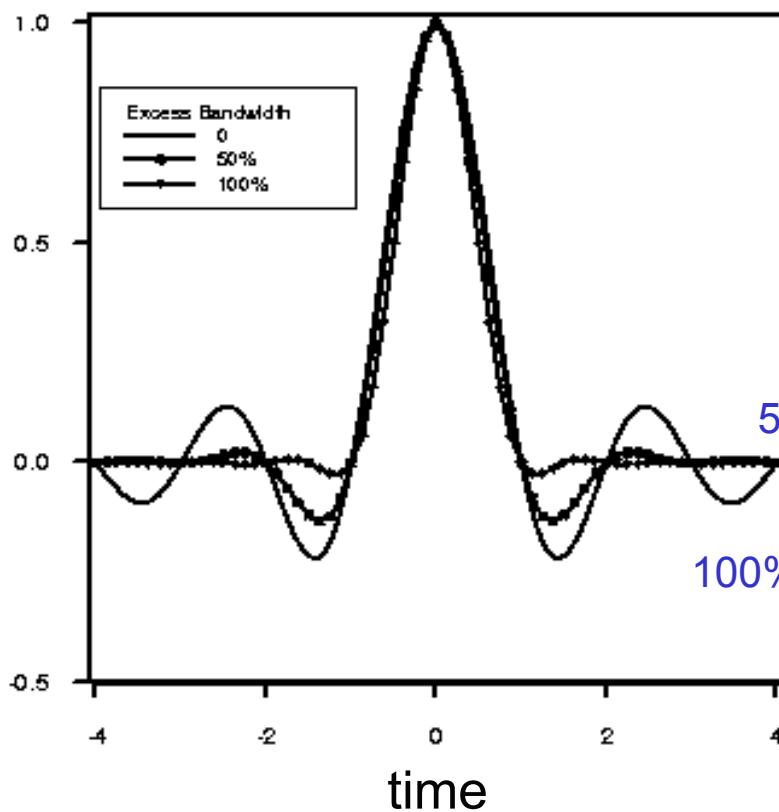
“Excess” Bandwidth (Raised Cosine Pulses)

Add “excess” bandwidth to reduce self-interference:





Excess Bandwidth: Definition

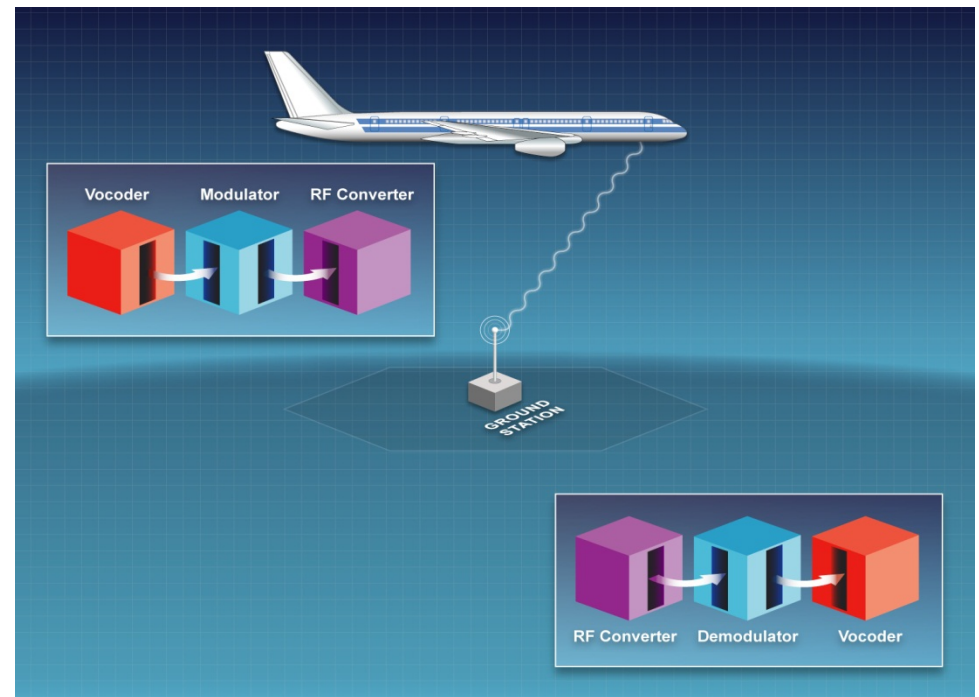


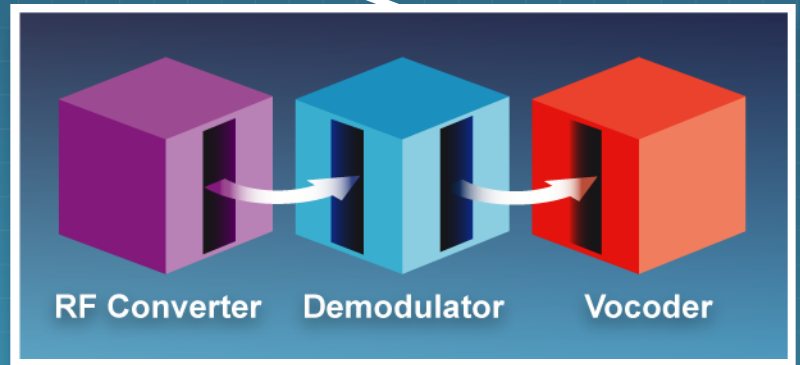
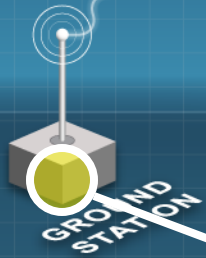
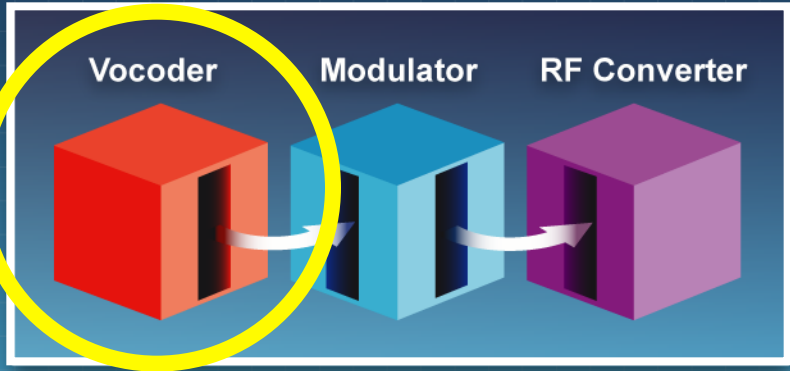
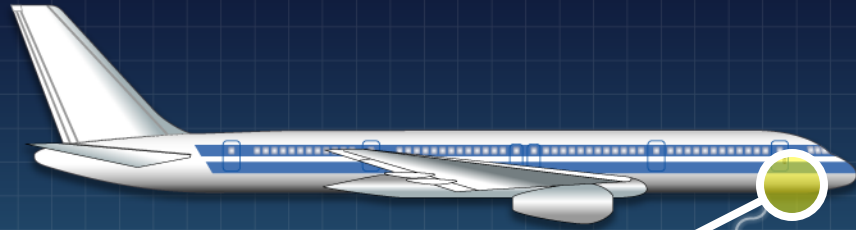
$$\text{Excess bandwidth} = (\text{Total bandwidth} - \text{Nyquist bandwidth}) / \text{Nyquist bandwidth}$$



“Case” Study: In-Flight vs. AT&T (2003)

- AT&T accused of stealing design for air-to-ground communication system
- System features:
 - 12 kbps for voice service
 - 8 PSK
 - 6 kHz bandwidth





Vocoder

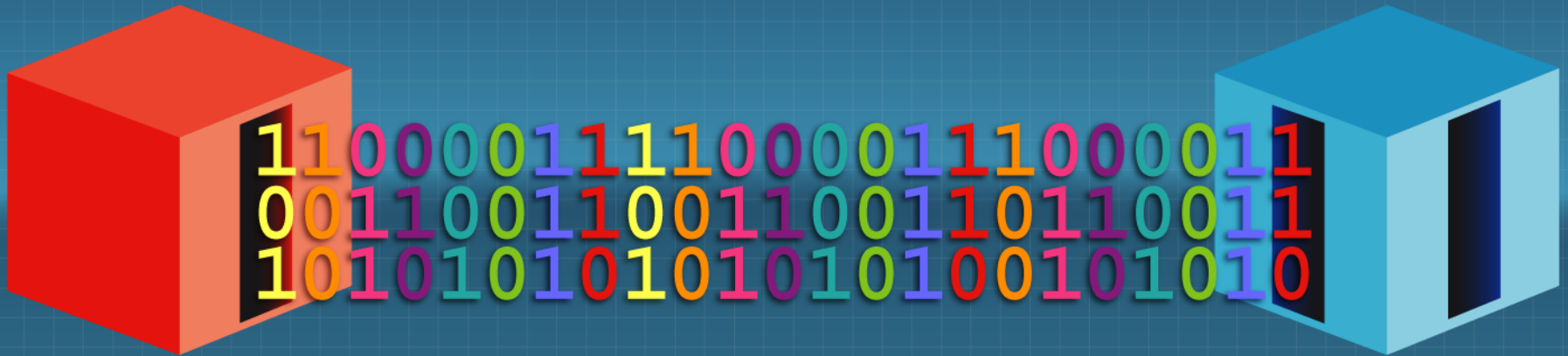
Modulator

RF Converter



Vocoder

Modulator

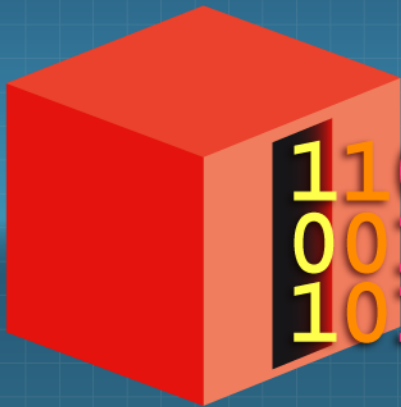


8000 Bits
Per Second

Additional Bits

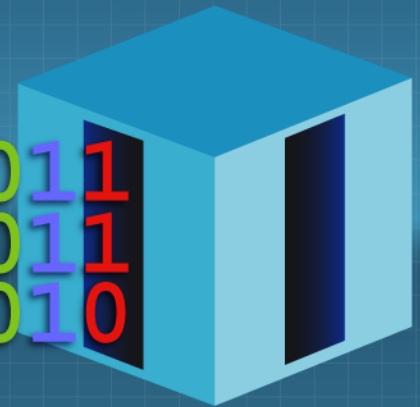


Vocoder

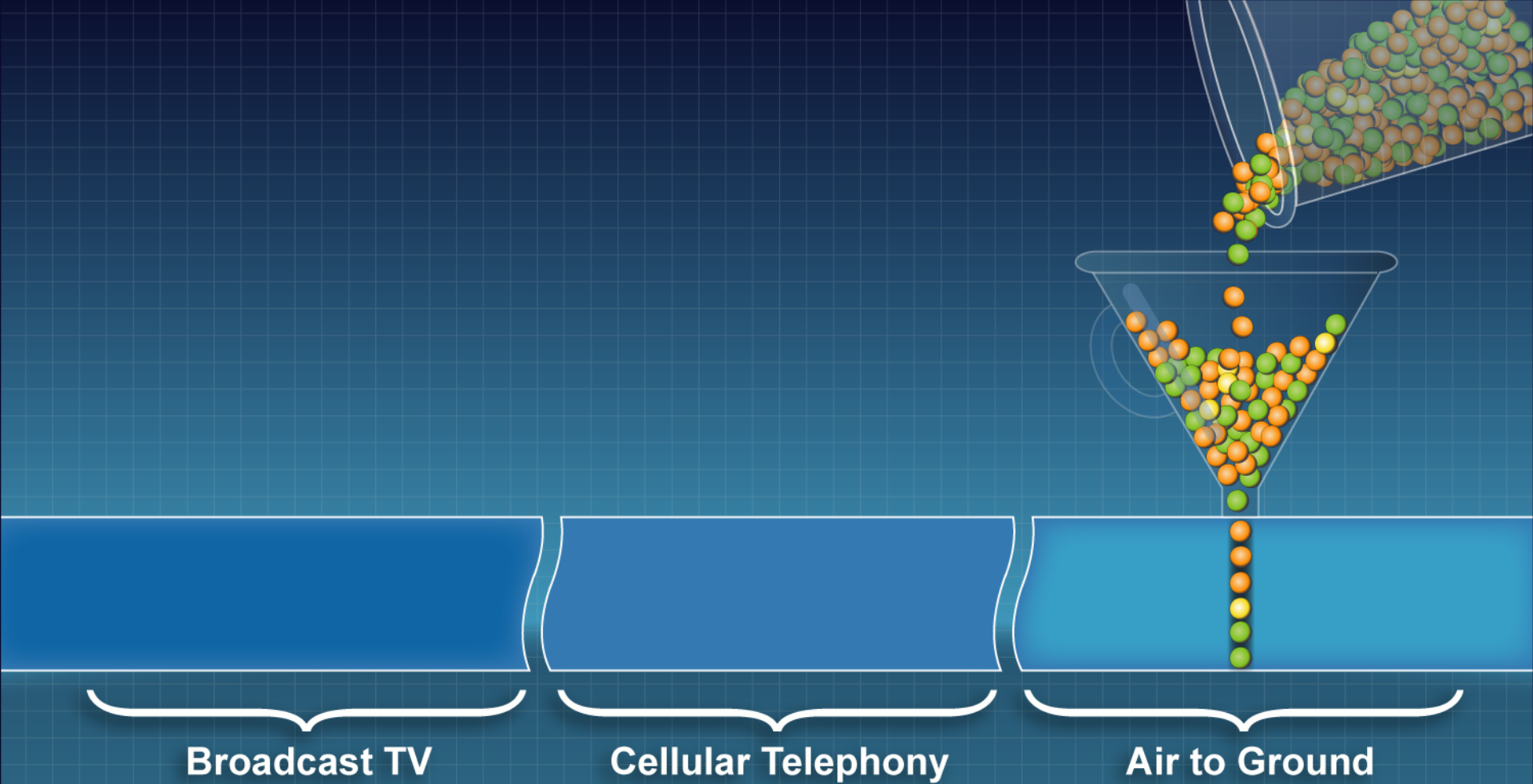


110000111100001110000011
001100110011001100110110110011
10101010101010101000101010

Modulator

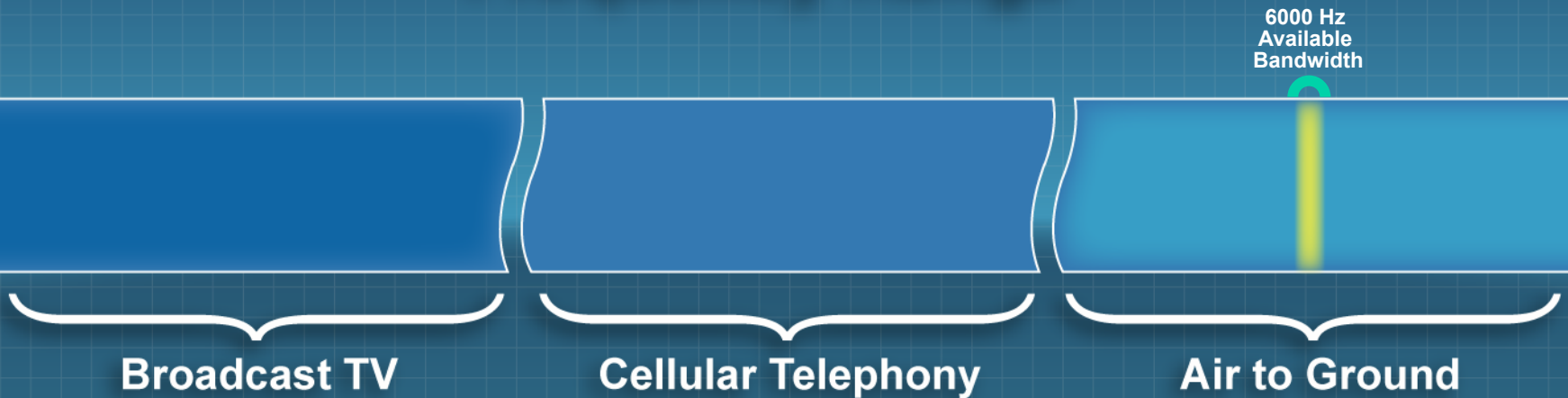


1. Error Correction
2. Control Information
 - Channel
 - Ground Station
 - Power
 - Identity
 - Billing

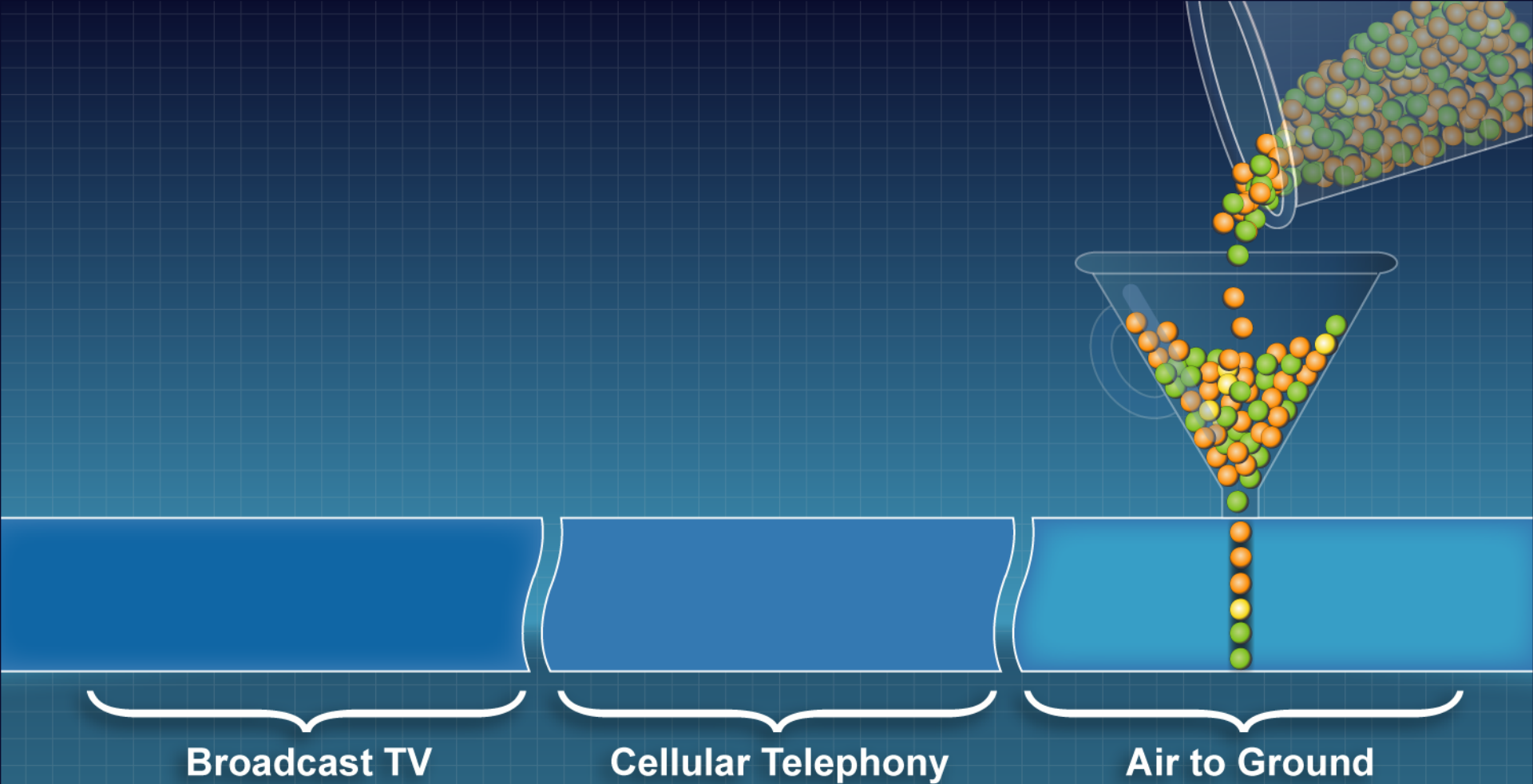


Nyquist's Theorem: Need at least B Hz to transmit B symbols per second.

Frequency Range



Nyquist's Theorem: Can transmit 4000 symbols per second through a 6000 Hz channel



Nyquist's Theorem: Can transmit 4000 symbols per second through a 6000 Hz channel

- **BPSK: 1 Bit Per Symbol**
- **4000 Bits Per Second (bps)**
- **4000 bps < 8000 bps**
(Vocoder rate)

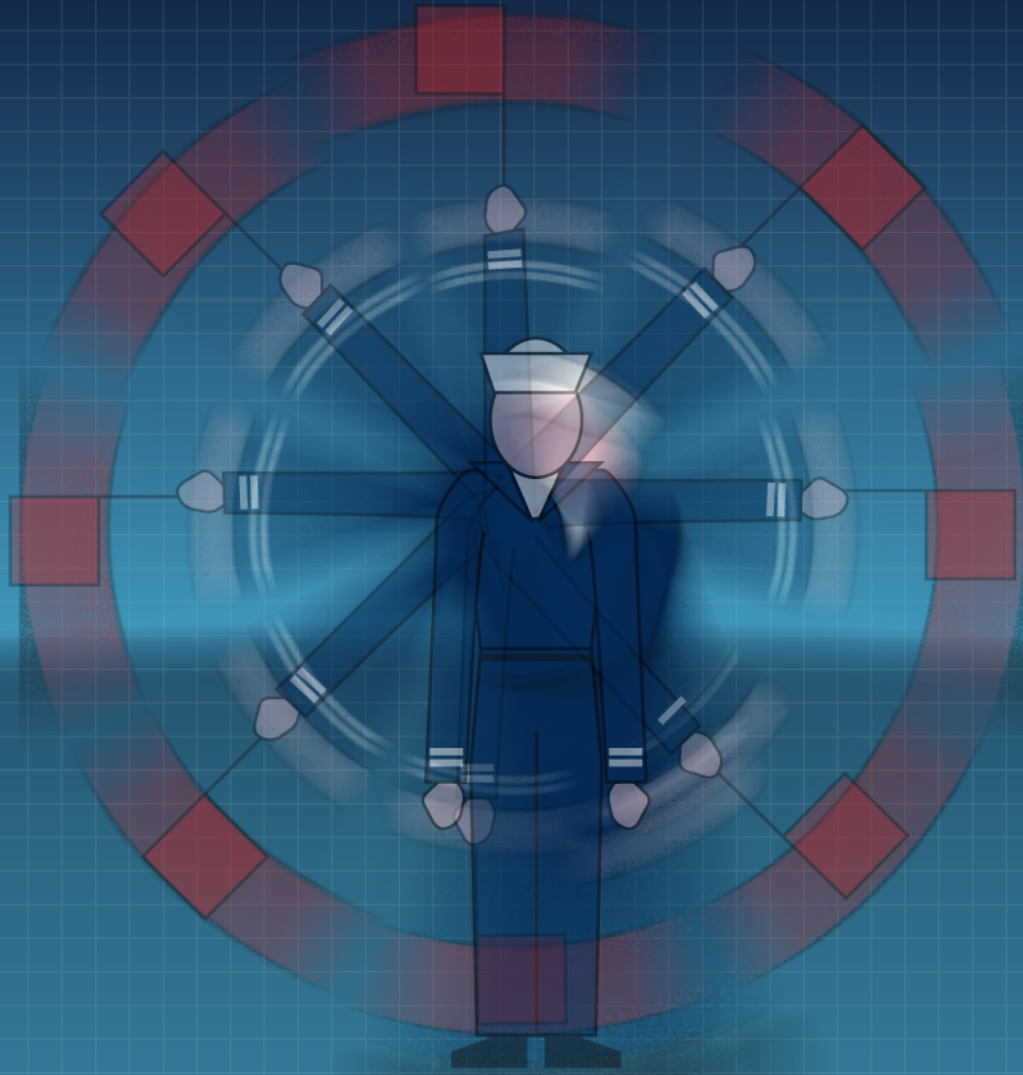
~~• BPSK~~

- **QPSK: 2 Bits Per Symbol**
- **$2 \times 4000 = 8000$ Bits Per Second (bps)**
- **8000 bps = Vocoder rate**
- **Need more bits for error correction and control!**
- ~~**QPSK**~~

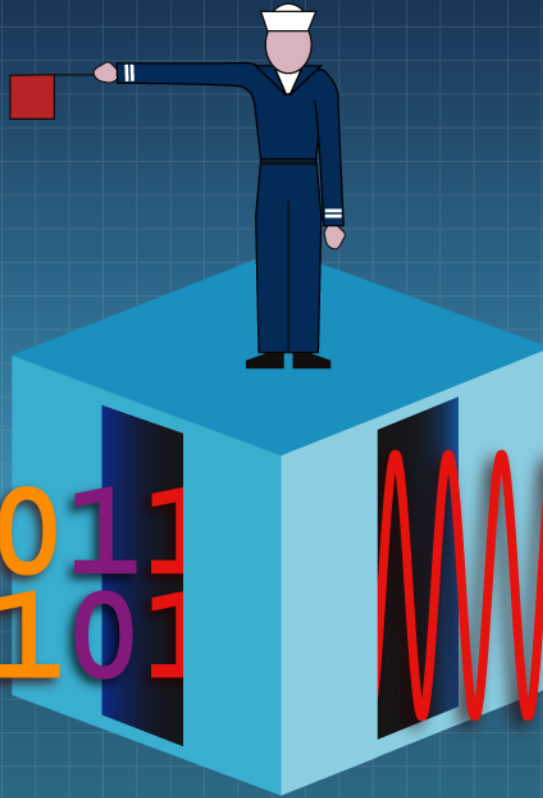
- **8PSK: 3 Bits Per Symbol**
- **3 X 4000 = 12,000 Bits Per Second (bps)**
- **8000 bps + 4000 bps**
Vocoder rate + Error Correction and Control

• **8PSK**

- **16 phases: 4 Bits Per Symbol**
- **$4 \times 4000 = 16,000$ bps**
- **More than enough for vocoder rate + overhead**



QPSK



101001010011
100011001101

2000 Bits
Per Second

Modulator

1000 Symbols
(Phases) Per Second

1000 Symbols Per Second x 2 Bits Per Symbol = 2000 Bits Per Second

Quadrature Phase Shift Keying

(QPSK)

90°

0°

180°

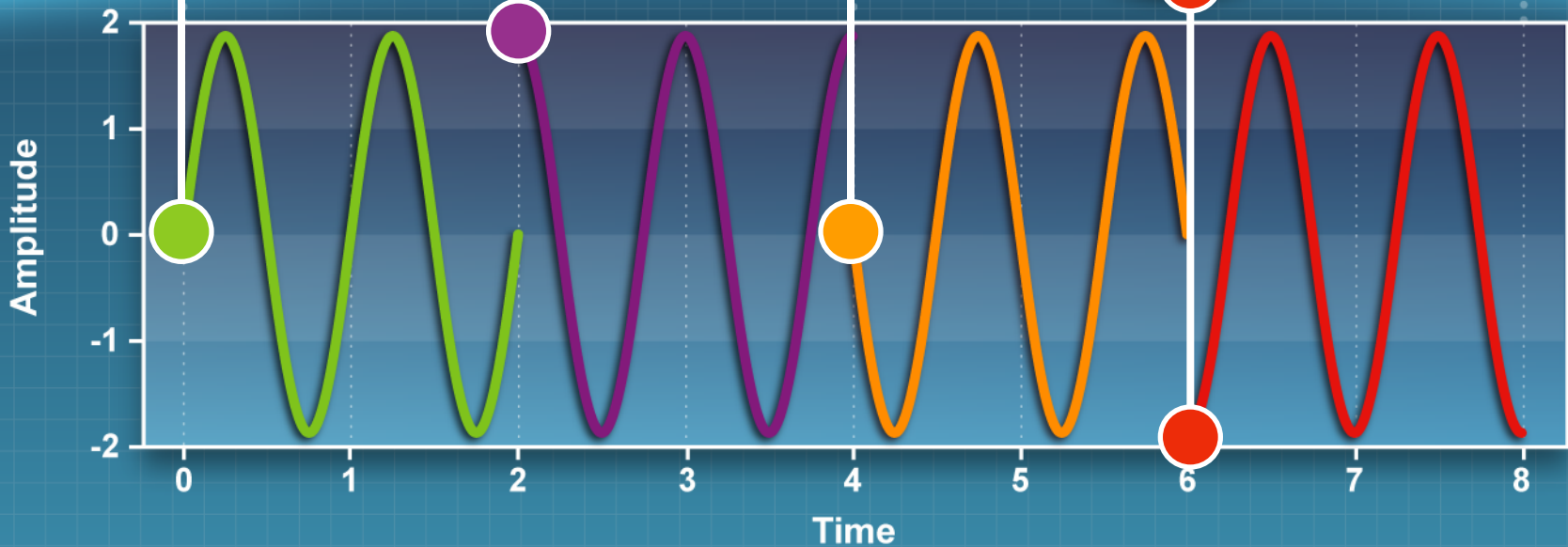
270°

00

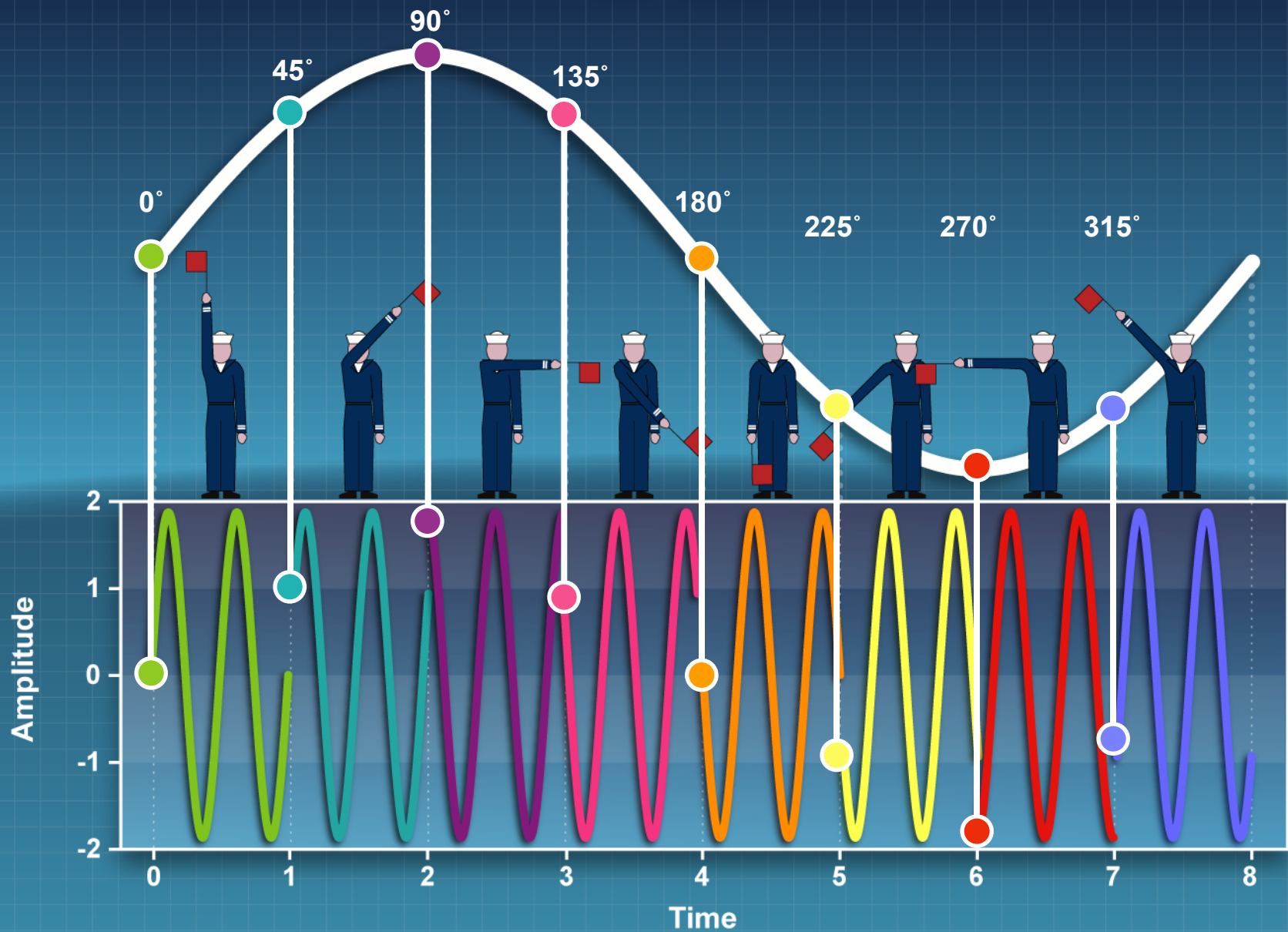
10

01

11



8PSK



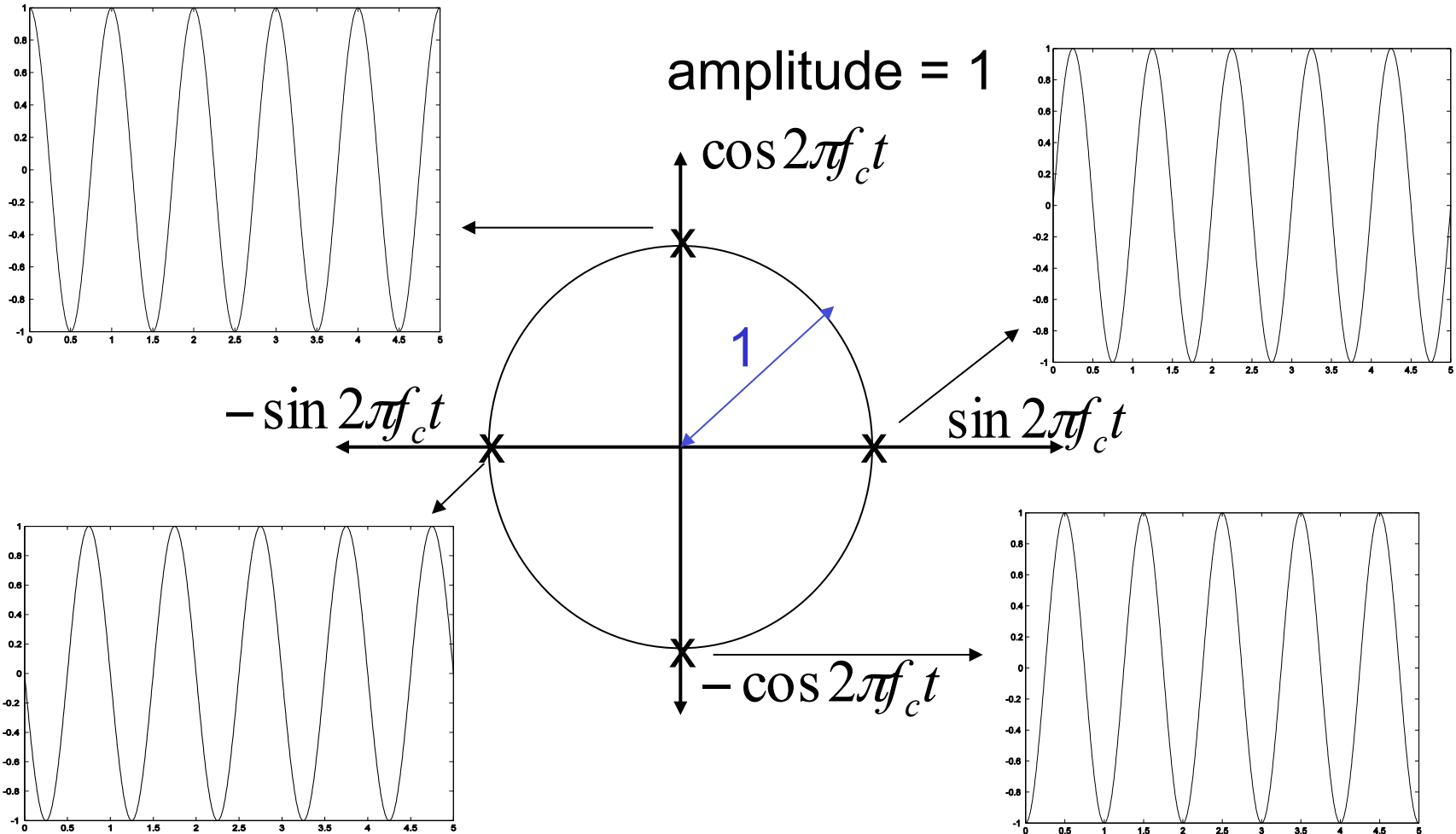


Sine Wave Animation

<http://www.rkm.com.au/ANIMATIONS/animation-sine-wave.html>



QPSK Signal Constellation

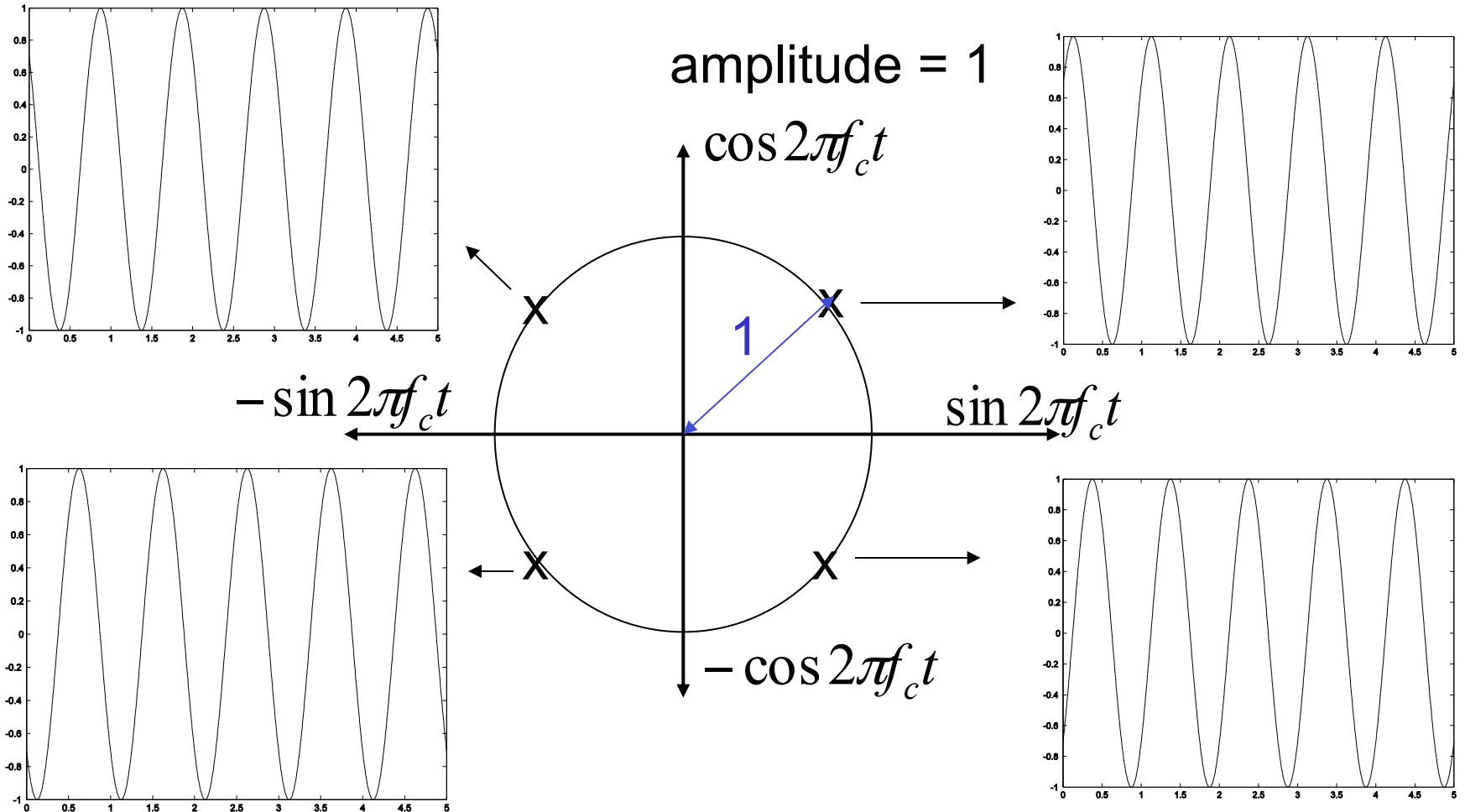




“Rotated” QPSK Signal Constellation

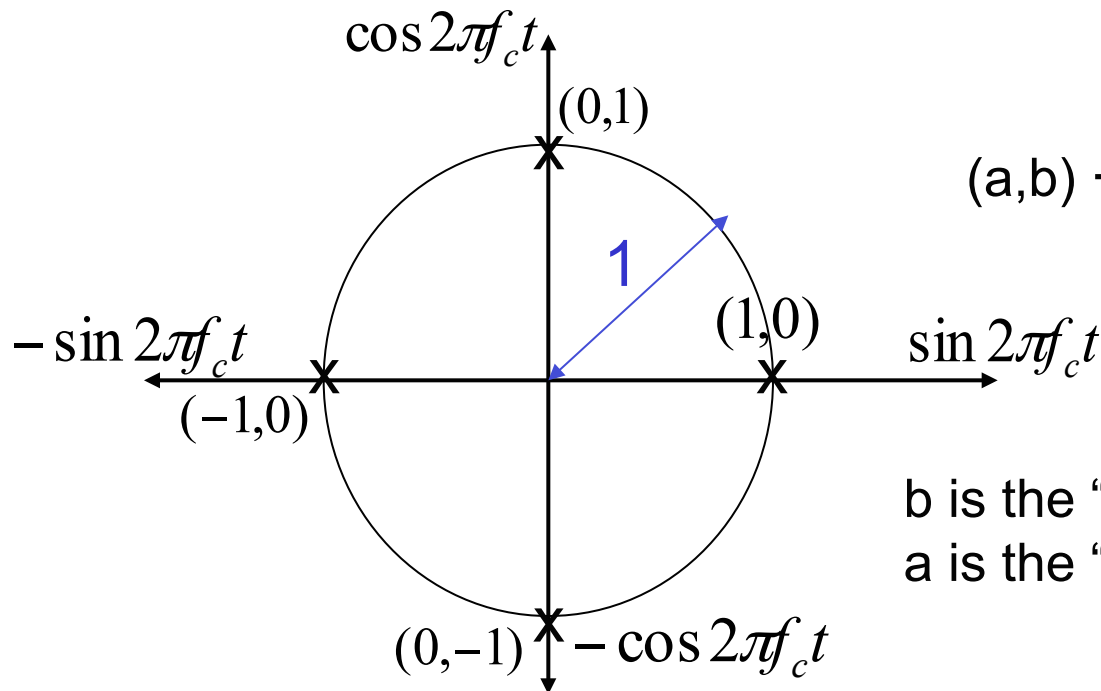


“Rotated” QPSK Signal Constellation





In-Phase/Quadrature Components

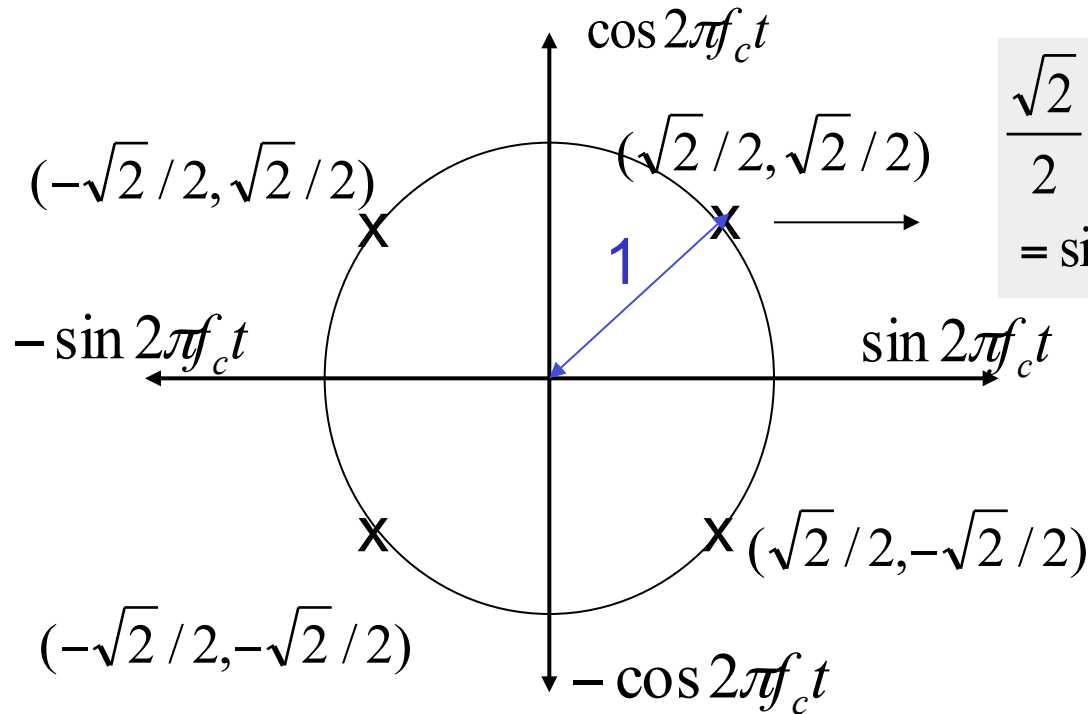


$$(a,b) \rightarrow a \sin 2\pi f_c t + b \cos 2\pi f_c t$$

b is the “in-phase” signal component
a is the “quadrature” signal component



In-Phase/Quadrature Components

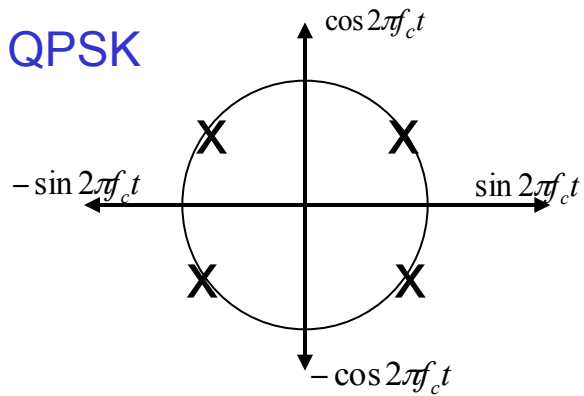


$$\begin{aligned} & \frac{\sqrt{2}}{2} \sin(2\pi f_c t) + \frac{\sqrt{2}}{2} \cos(2\pi f_c t) \\ &= \sin(2\pi f_c t + \pi/4) \end{aligned}$$



Example Constellations

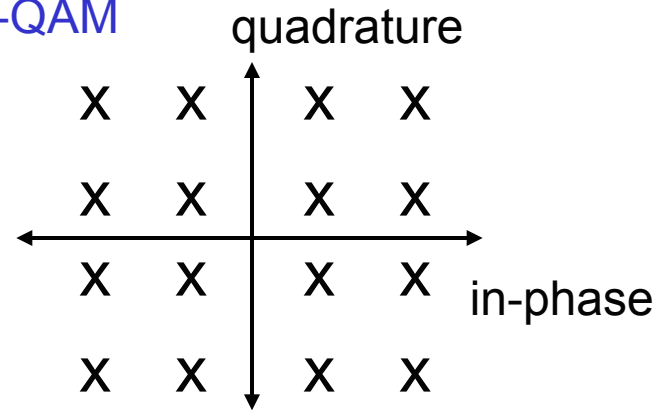
QPSK



BPSK

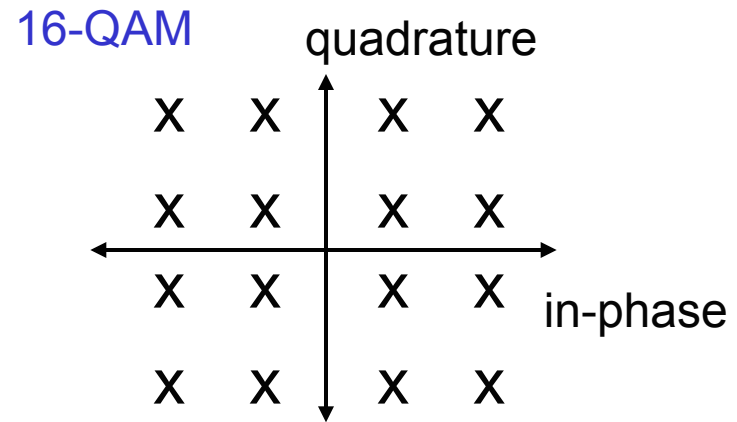
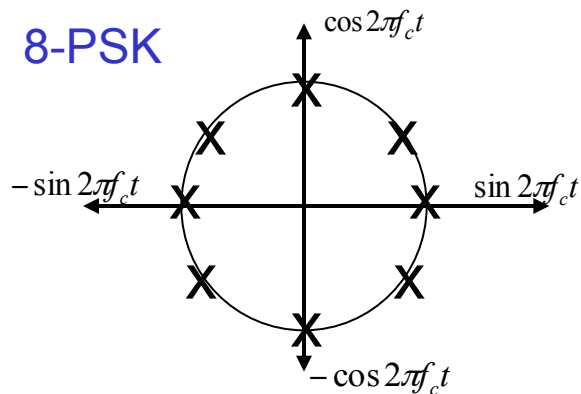
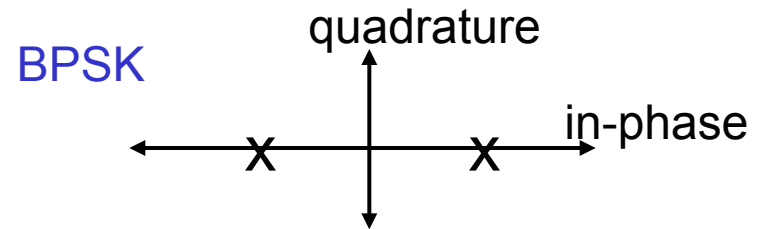
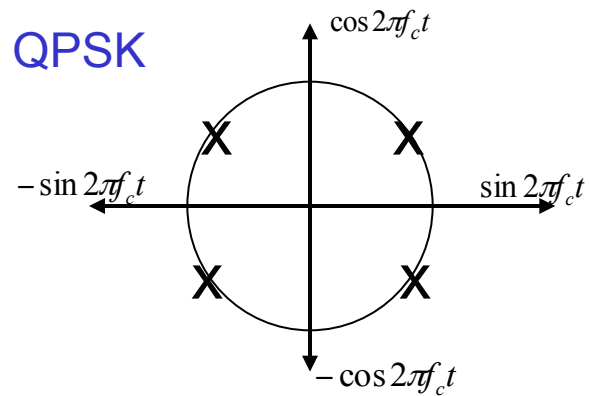
8-PSK

16-QAM



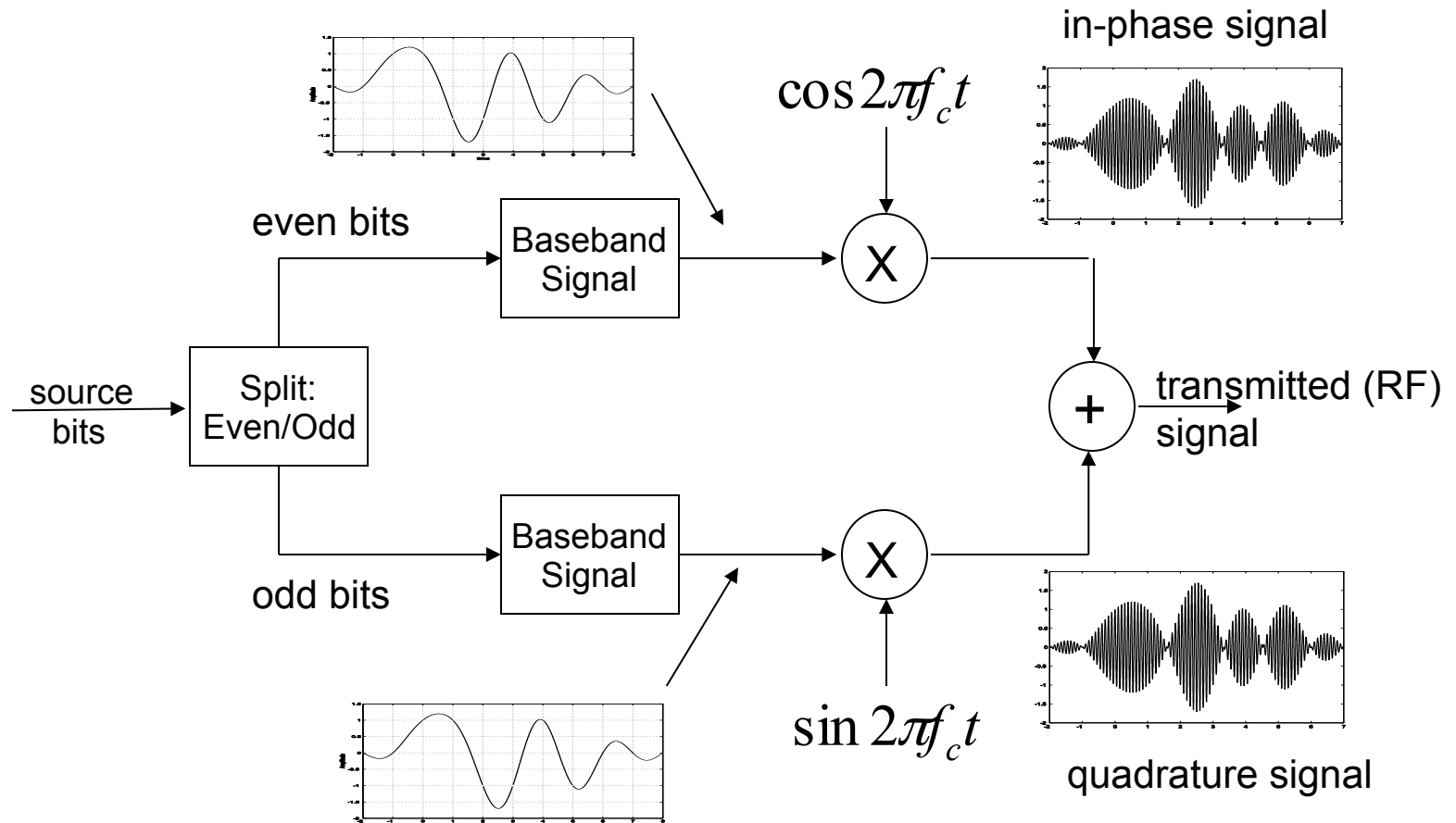


Example Constellations





Quadrature Modulation



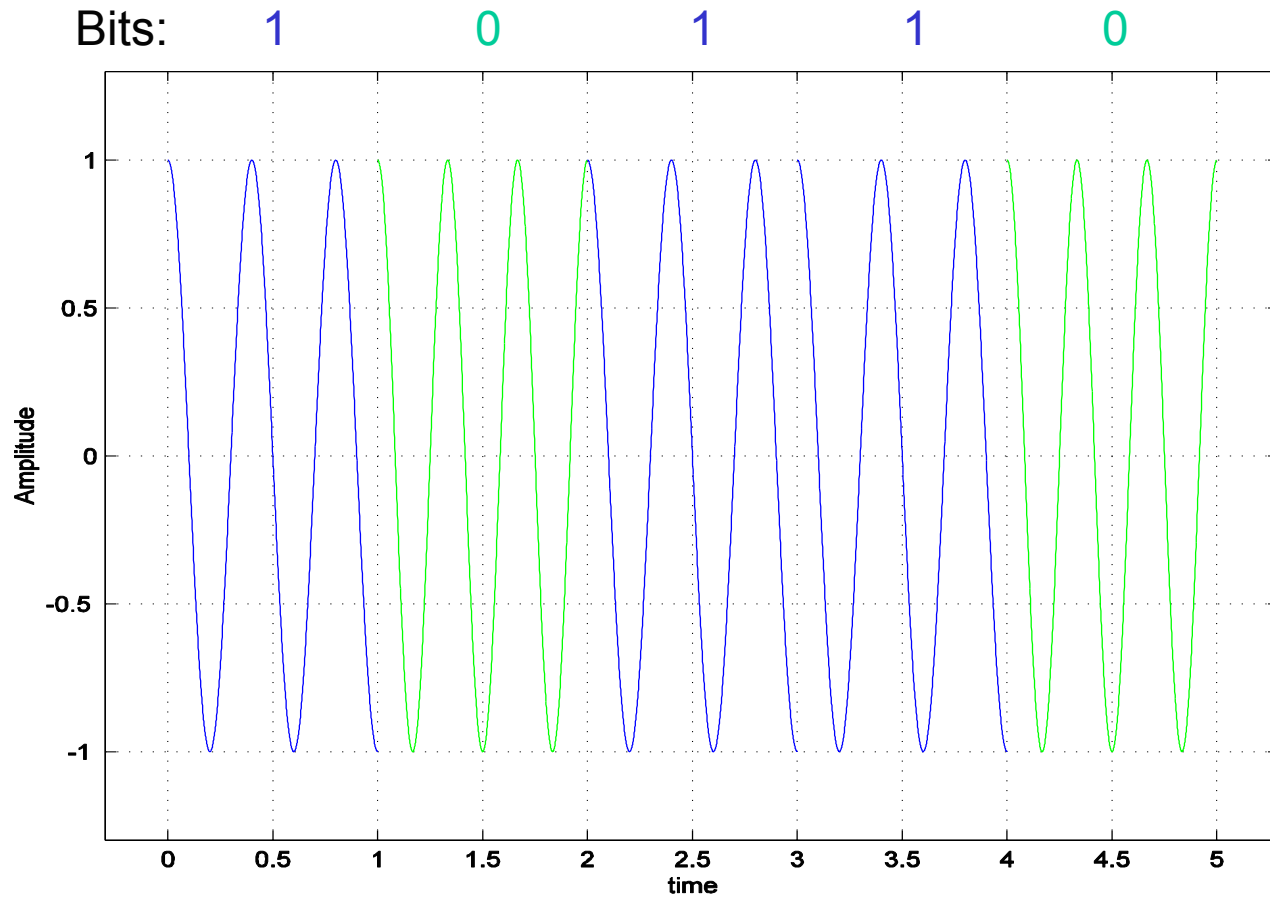


Modulation for Fading Channels

- Problems:
 1. Amplitude variations (shadowing, distance, multipath)
 2. Phase variations
 3. Frequency variations (Doppler)
- Solution to 1:
 1. Avoid amplitude modulation
 2. Power control
- Solution to 2 & 3:
 1. Avoid phase modulation (use FSK)
 2. “Noncoherent” demodulation: does not use phase reference
Differential coding/decoding
 3. “Coherent” demodulation:
Estimate phase shifts caused by channel.
 4. Increase data rate/Doppler shift ratio



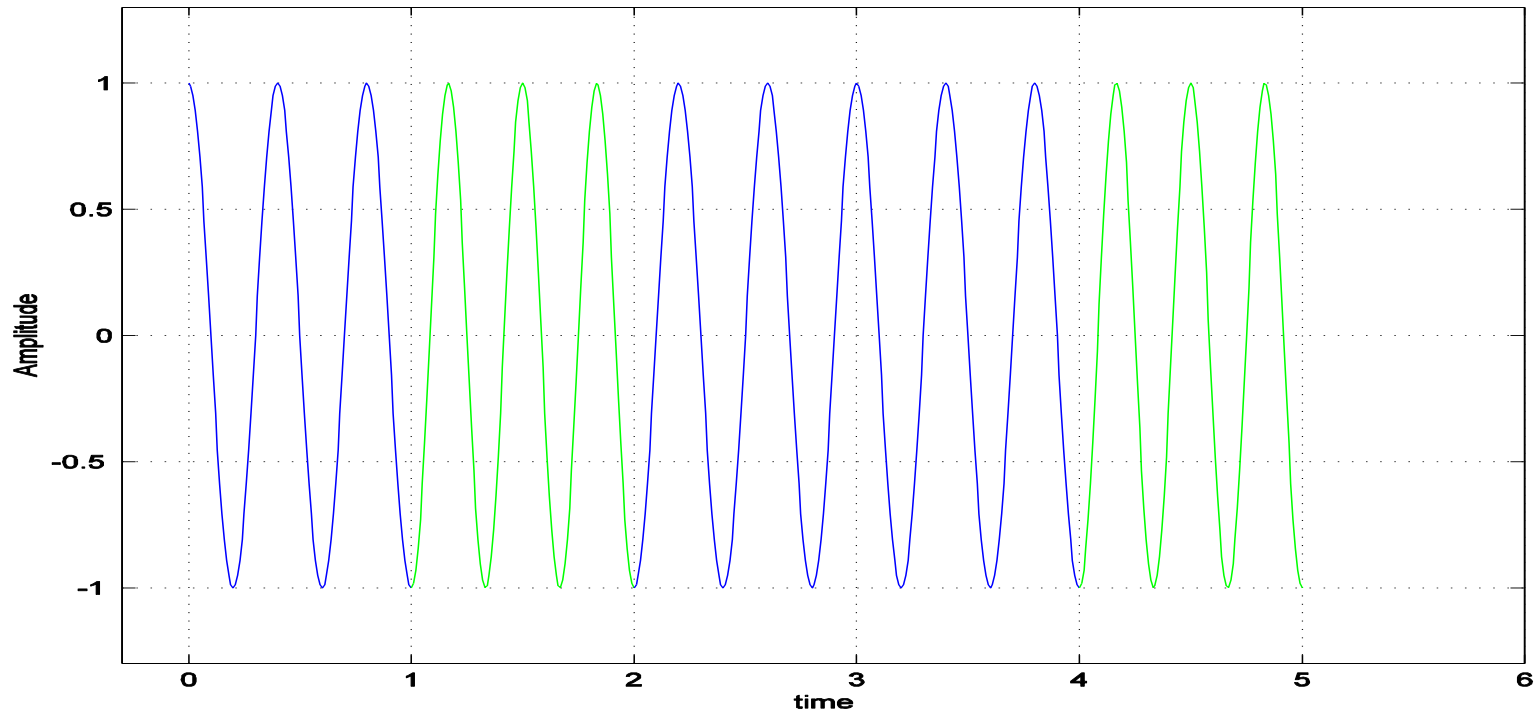
Binary Frequency-Shift Keying (FSK)





Minimum Shift Keying (MSK)

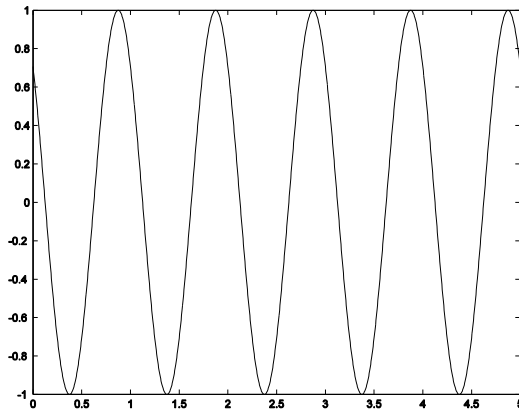
Bits: 1 0 1 1 0



- Frequencies differ by $\frac{1}{2}$ cycle
- Used in GSM

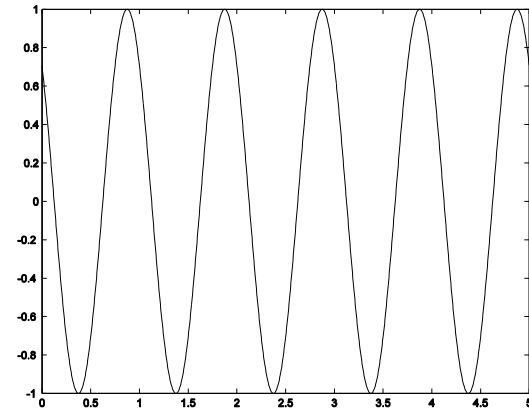
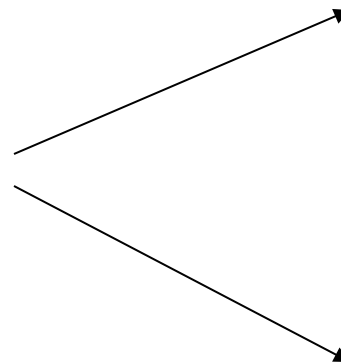


Binary Differential Modulation

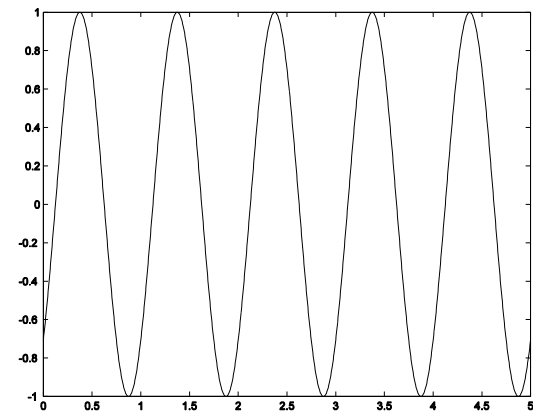


waveform for i th symbol

$(i+1)$ st bit = 0:
 0° phase shift

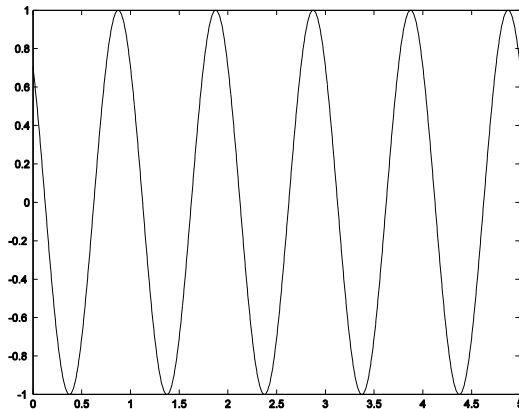


$(i+1)$ st bit = 1:
 180° phase shift



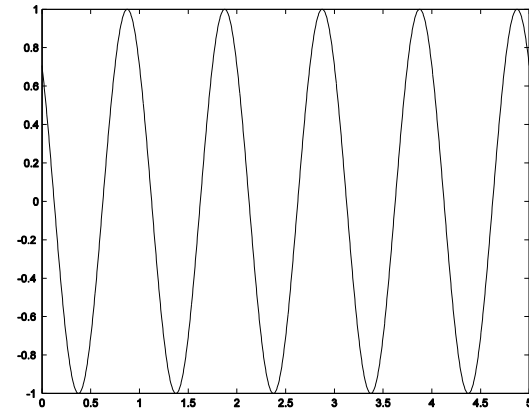


Binary Differential Modulation

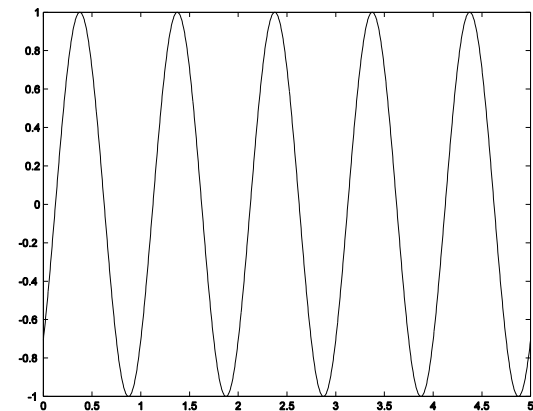


waveform for i th symbol

$(i+1)$ st bit = 0:
 0° phase shift



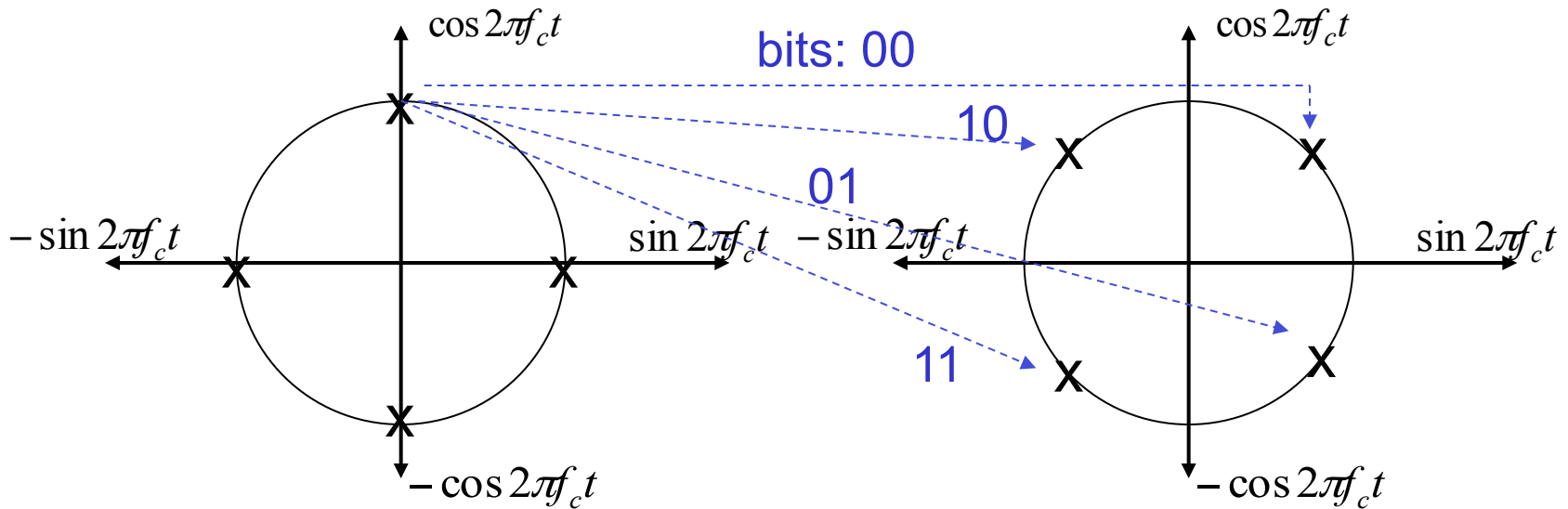
$(i+1)$ st bit = 1:
 180° phase shift



Drawback: a detection error for the i th bit propagates to the $(i+1)$ st bit.



Example: DQPSK



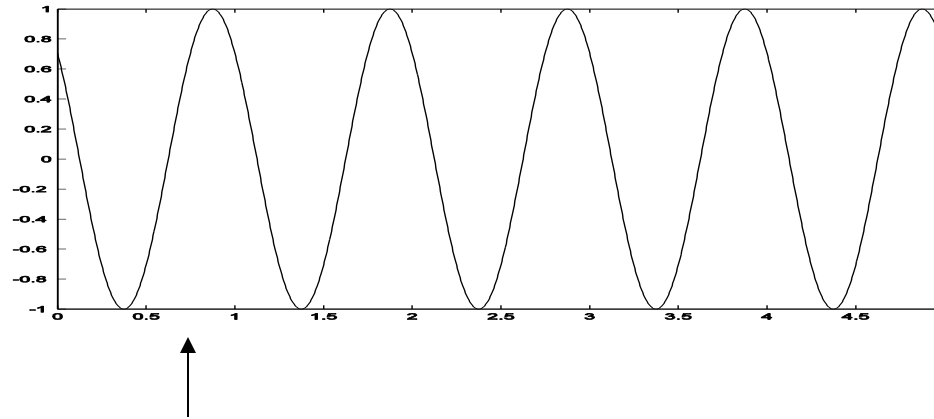
constellation for i th symbol

constellation for $(i+1)$ st symbol

Used in IS-136



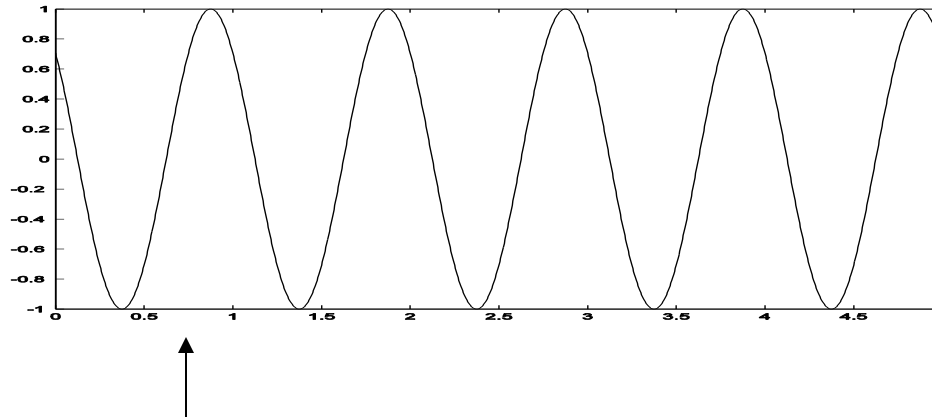
Coherent Phase Detection



Receiver must determine where symbol begins
(need phase reference)



Coherent Phase Detection

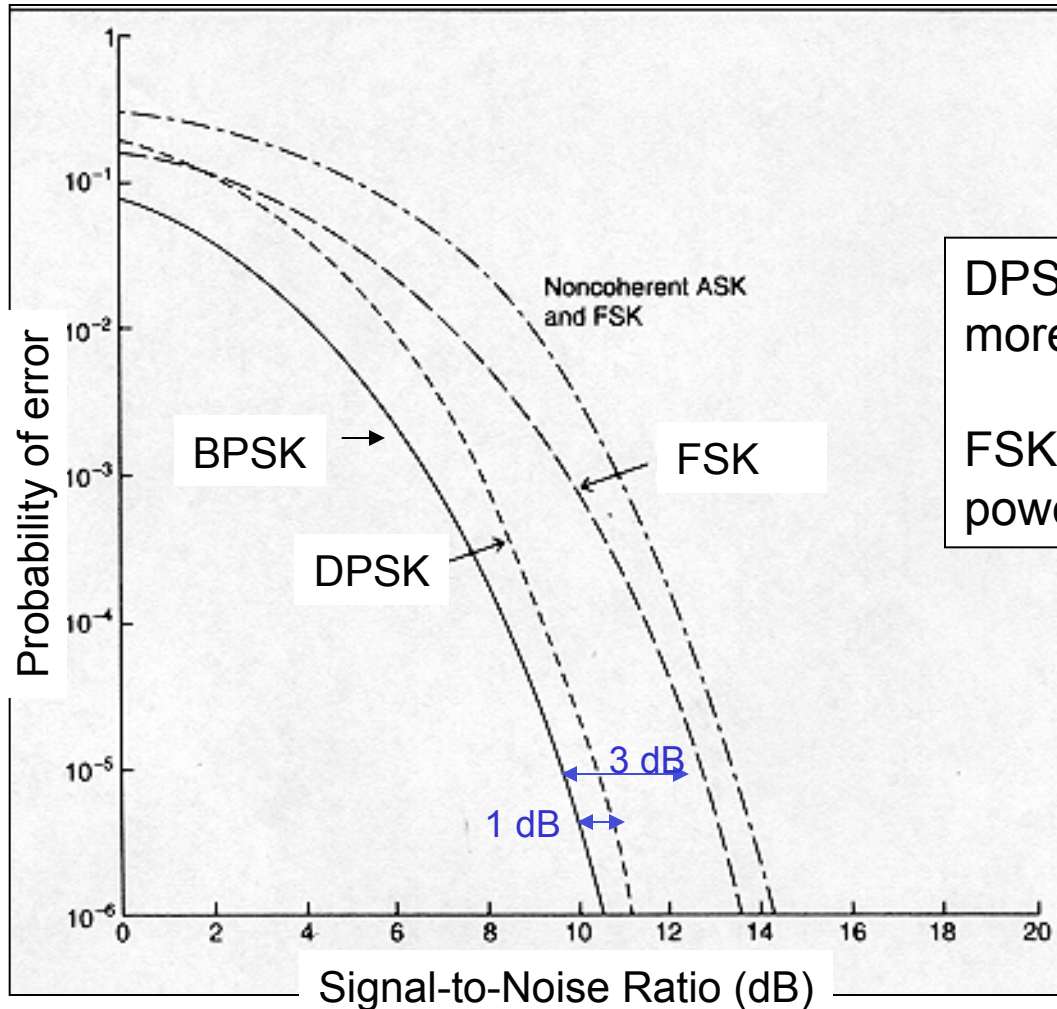


Receiver must determine where symbol begins

- More complicated than noncoherent (e.g., differential) modulation.
- Transmitter must send **pilot symbols**.
 - Known to receiver, used to measure phase.
 - Pilot symbols are overhead (not information bits).



Probability of Error (no fading)

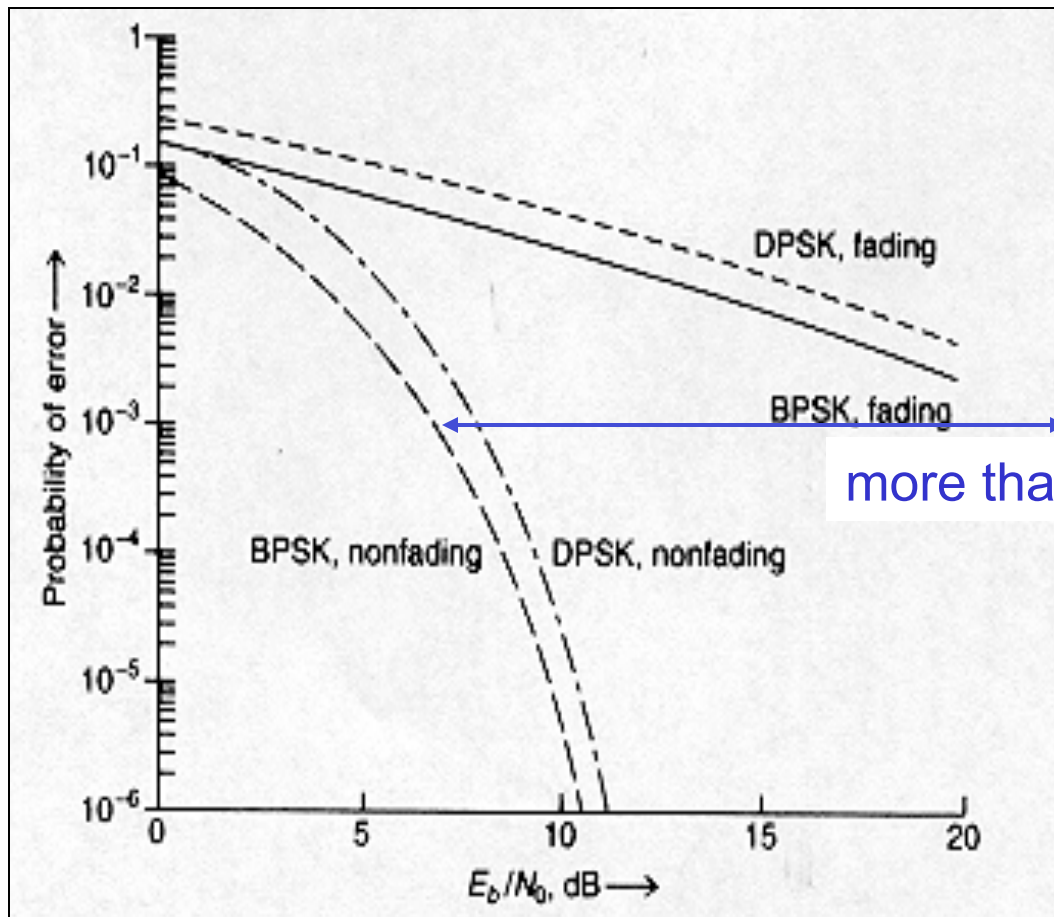


DPSK requires about 1 dB more power than BPSK

FSK requires about 3 dB more power

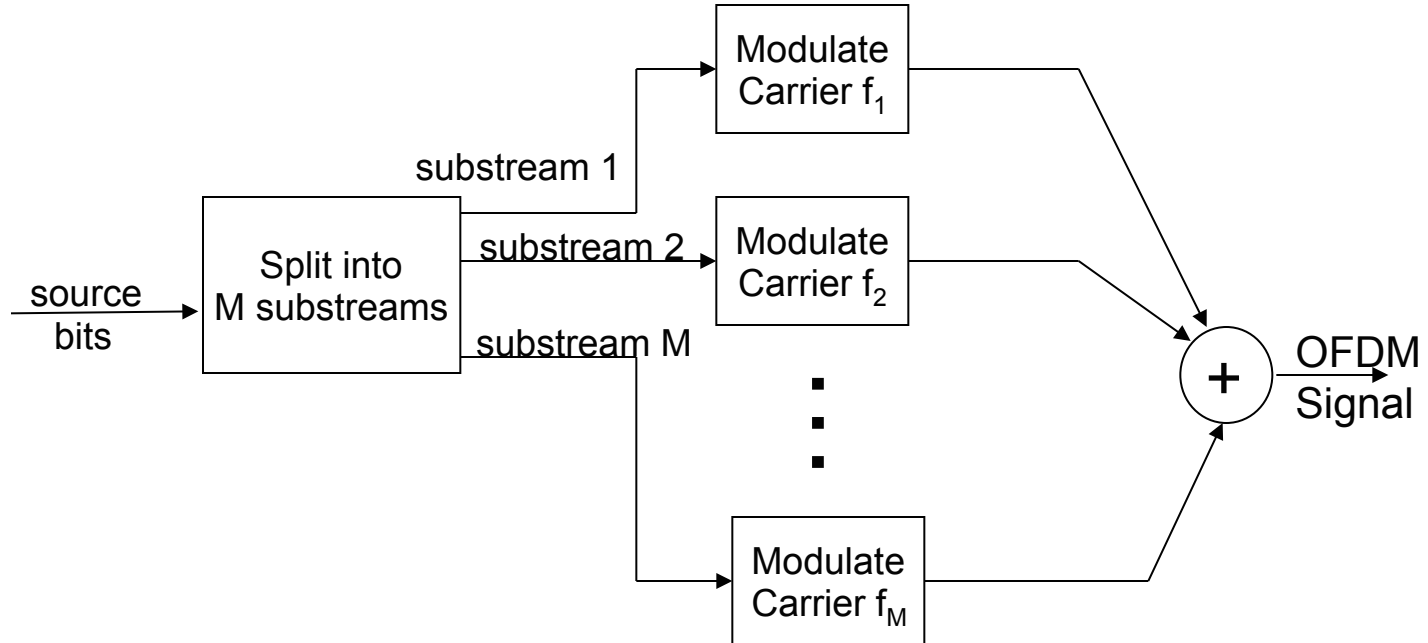


Probability of Error with Fading



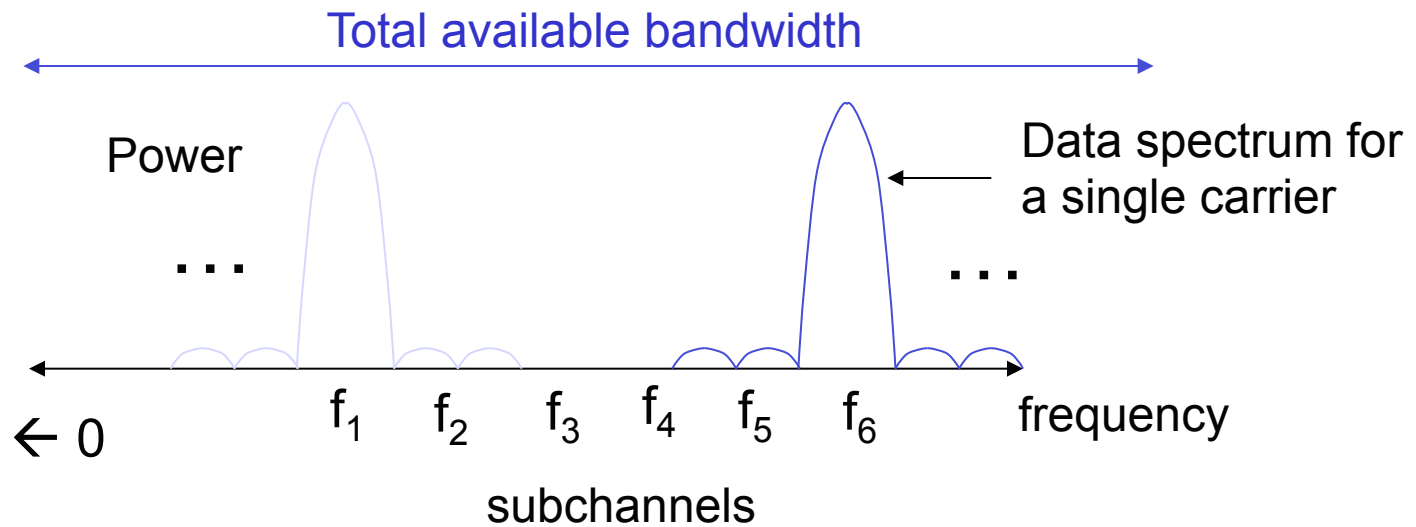


Orthogonal Frequency Division Multiplexing (OFDM)





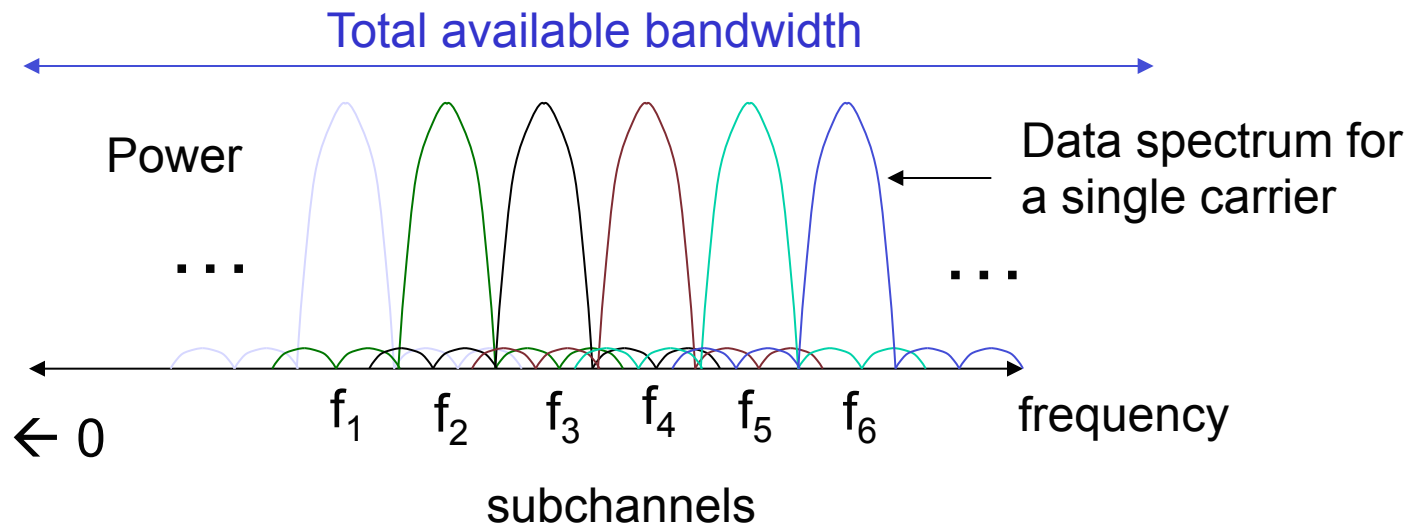
OFDM Spectrum



M "subcarriers, or subchannels, or tones"



OFDM Spectrum

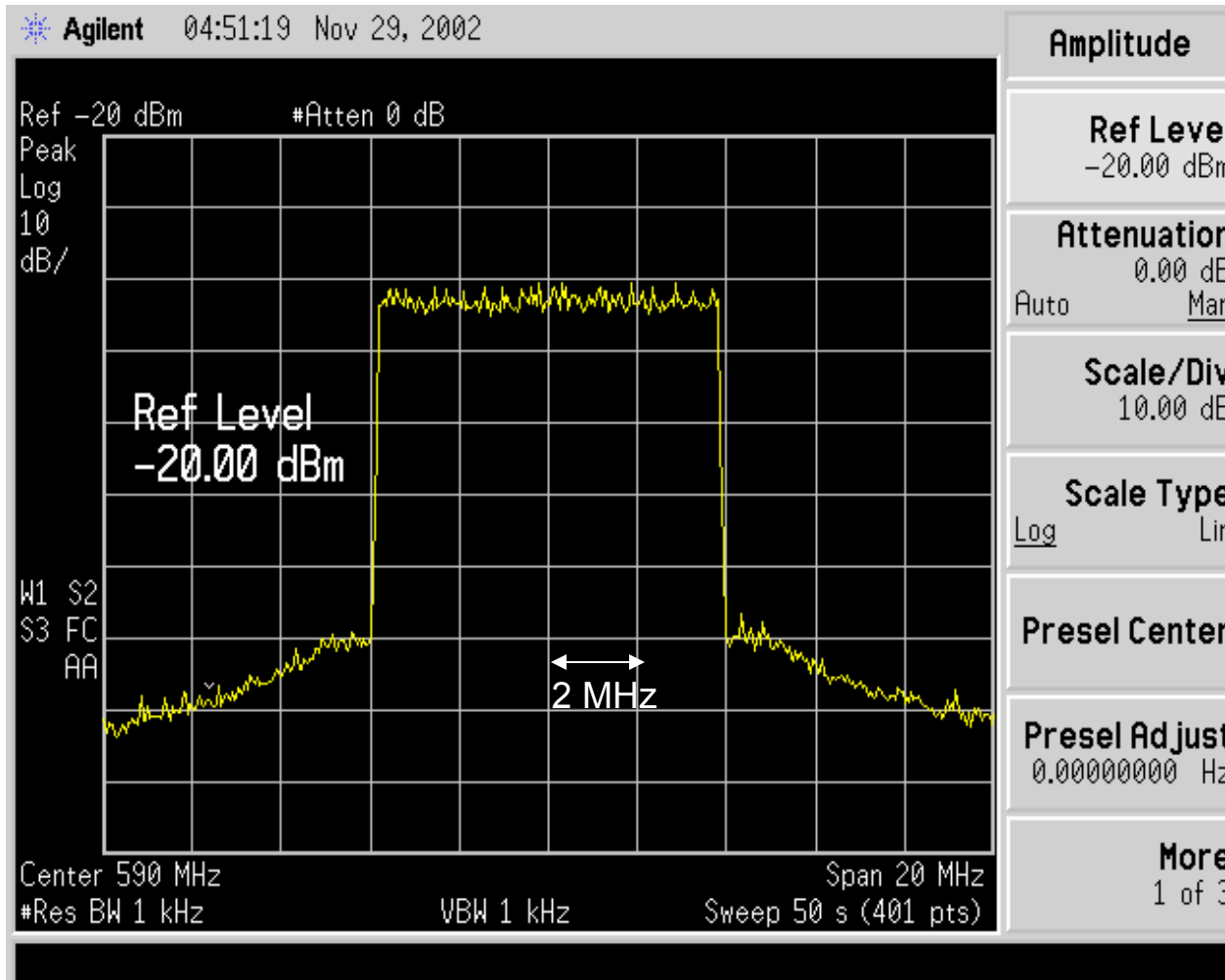


M “subcarriers, or subchannels, or tones”

“Orthogonal” subcarriers → no cross-channel interference



Measured OFDM Spectrum





OFDM Example: 802.11a

- 20 MHz bandwidth, $M=64$ (48 for data payload)
- Subchannel bandwidth = $20 \text{ MHz} / 64 = 312.5 \text{ kHz}$
- Symbol rate / subchannel = 250 kilosymbols/sec
- Total symbol rate = $64 \times 250 \times 10^3 = 16 \text{ Msymbols/sec}$
- Bit rate?
 - 16 QAM/subchannel
 - ➔ $4 \text{ bits/symbol} \times 250 \times 10^3 = 1 \text{ Mbps/subchannel}$,
or 64 Mbps total
 - 64 QAM/subchannel
 - ➔ $6 \text{ bits/symbol} \times 250 \times 10^3 = 1.5 \text{ Mbps/subchannel}$,
or 96 Mbps total
 - Includes overhead (synchronization, error correction, control)
Actual data rate: 36 / 54 Mbps

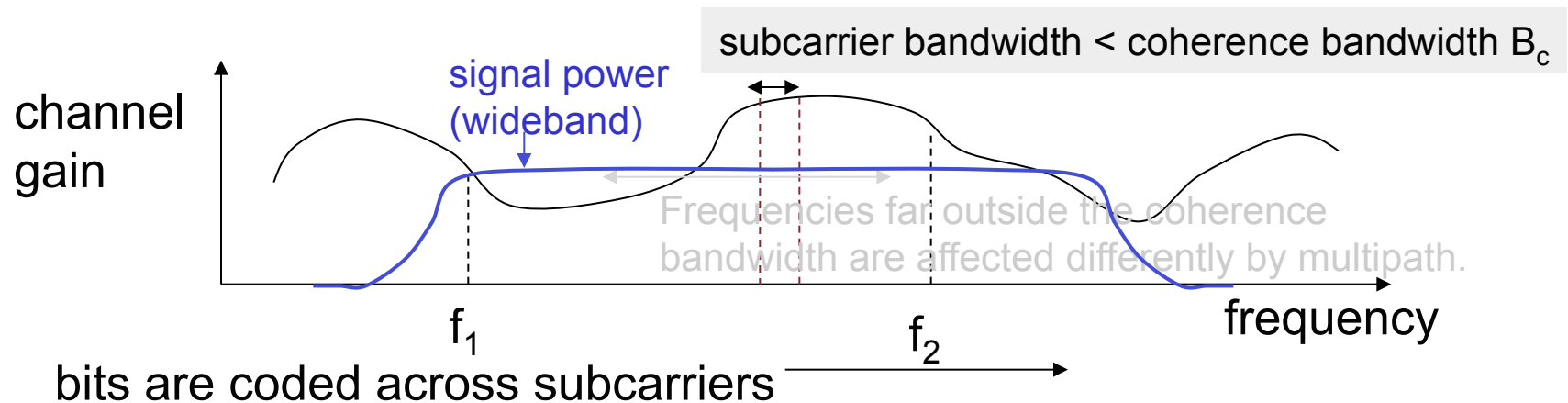


Why OFDM?



Why OFDM?

- Exploits frequency diversity



- Flat fading on each subchannel simplifies receiver (no multipath/ISI)
- Slower symbol rate on each subchannel simplifies signal processing.
- Drawback: signal amplitude varies a lot; high peak-to-average power.



Diversity

- Idea: Obtain multiple **independent** copies of the received signal.
 - Improves the chances that at least one is not faded.
- **Macroscopic (space)**: copies of signal are received over distances spanning many wavelengths.
- **Microscopic (space)**: copies of signal are received over distances spanning a fraction of a wavelength
- Different types...



Macroscopic Diversity

Copies of signal are separated by many wavelengths.



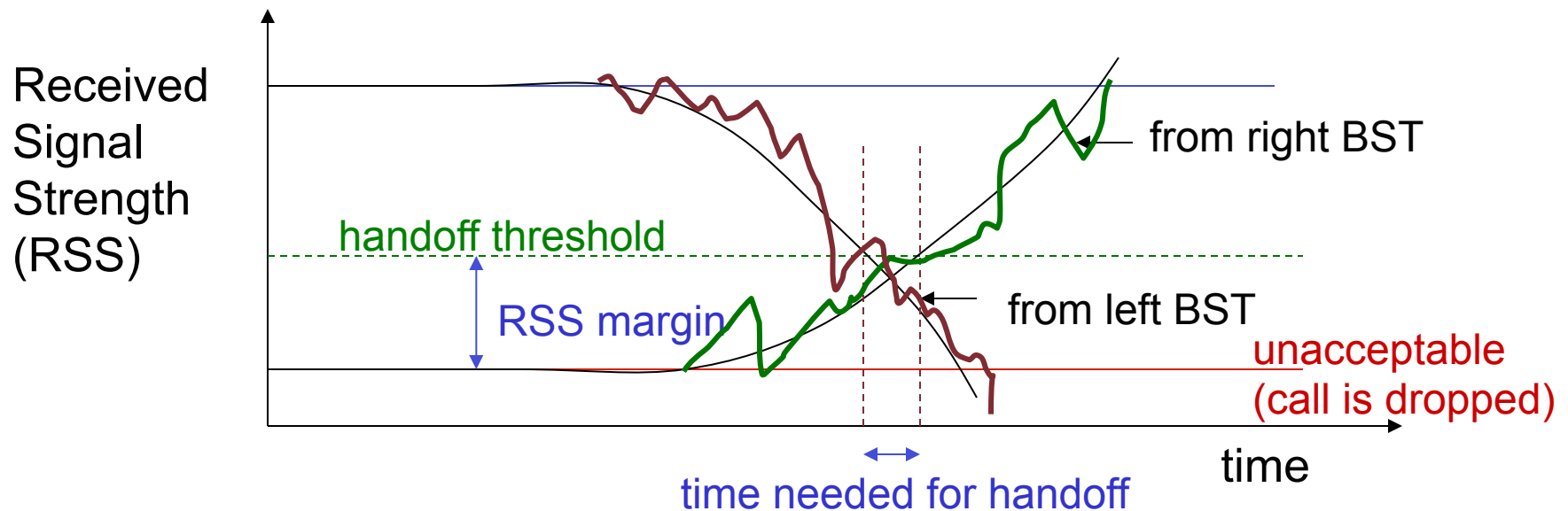
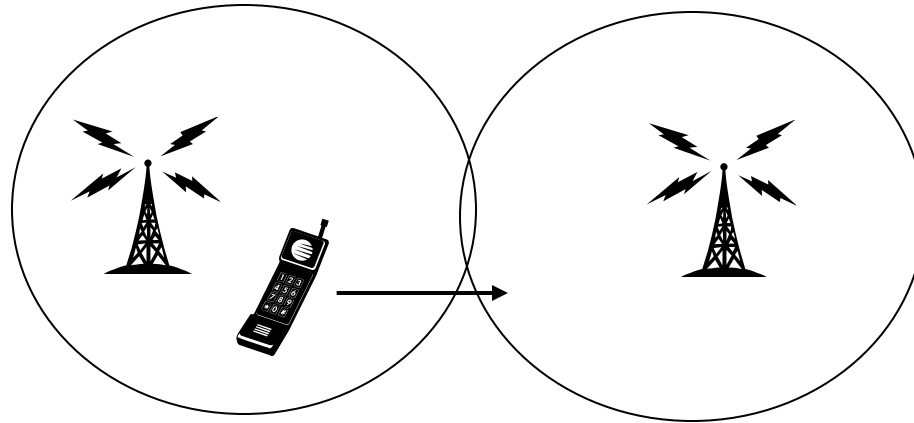
Macroscopic Diversity



Copies of signal are separated by many wavelengths.

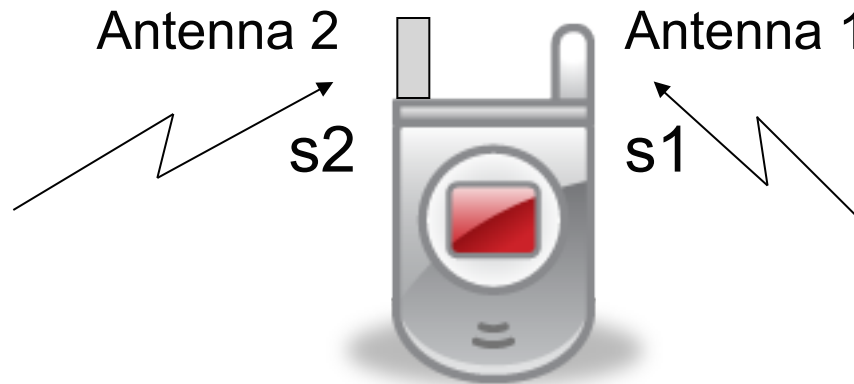


Macroscopic Diversity: Handoff





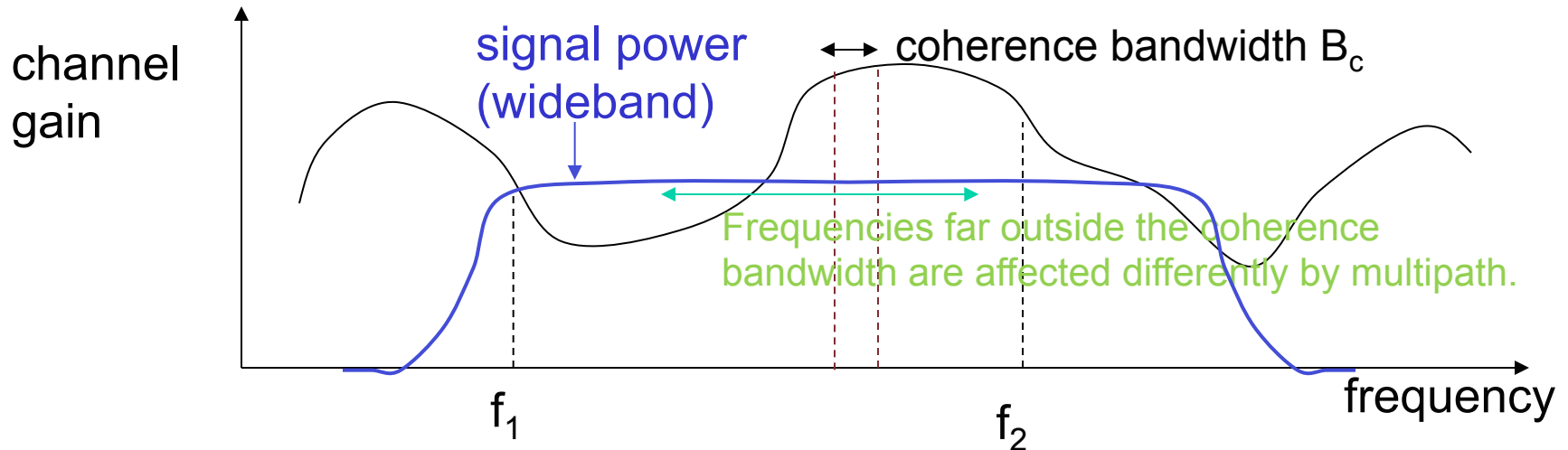
Microscopic Space Diversity



- Want signals s_1 and s_2 to experience independent fading (why?).
 - distance between antennas should be $\geq \frac{1}{2}$ wavelength.
 - Ex: 900 MHz, $\lambda = c/f \approx 1/3$ meter
2 GHz, $\lambda \approx 0.15$ meter



Frequency Diversity



Wideband signals exploit **frequency diversity**.
Spreading power across many coherence bands reduces the chances of severe fading.

Wideband signals are distorted by the channel fading (distortion causes intersymbol interference).



N O R T H W E S T E R N
U N I V E R S I T Y

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Time Diversity



Time Diversity

- Transmit multiple copies of the signal in time.
 - Error control coding: add redundant bits
- Problem: slow fading
 - Combine with power control

Path Diversity

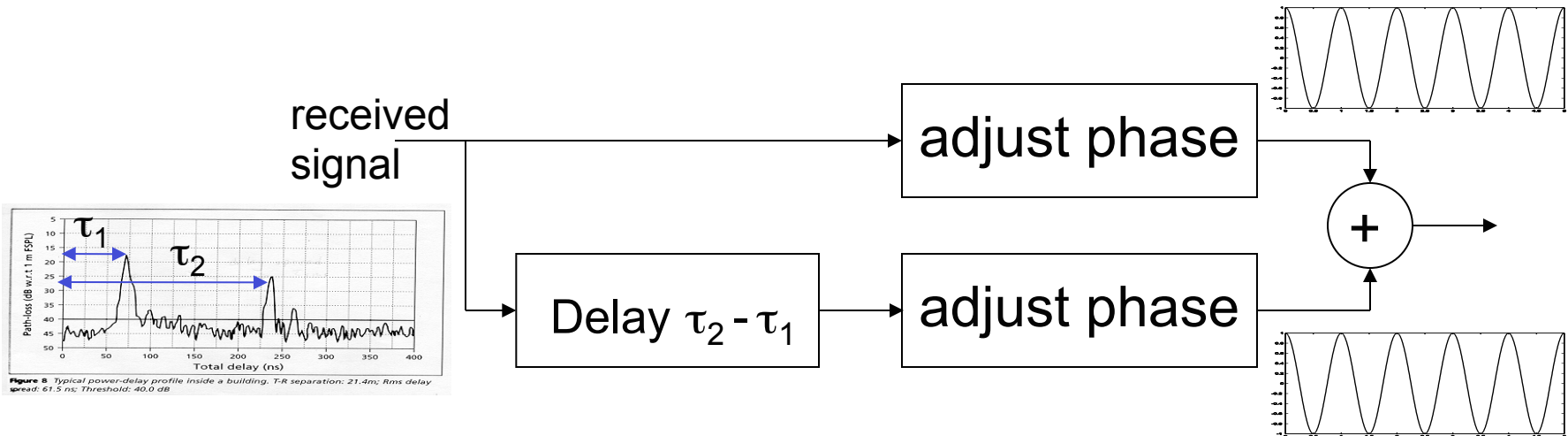


Figure 8 Typical power-delay profile inside a building. T-R separation: 21.4m; Rms delay spread: 61.5 ns; Threshold: 40.0 dB

- Called a “RAKE” receiver, since it “rakes” up (combines) the energy in the different paths.
 - Can substantially increase the S/I.
- An important component of CDMA receivers.
 - Each branch in the Rake is typically referred to as a “finger”.



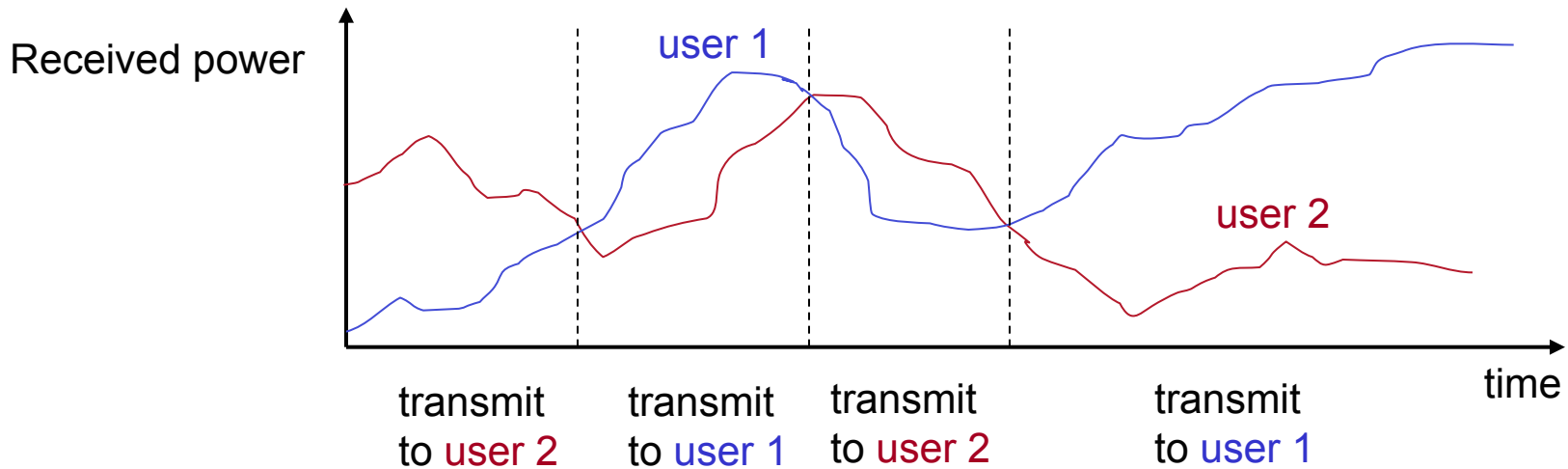
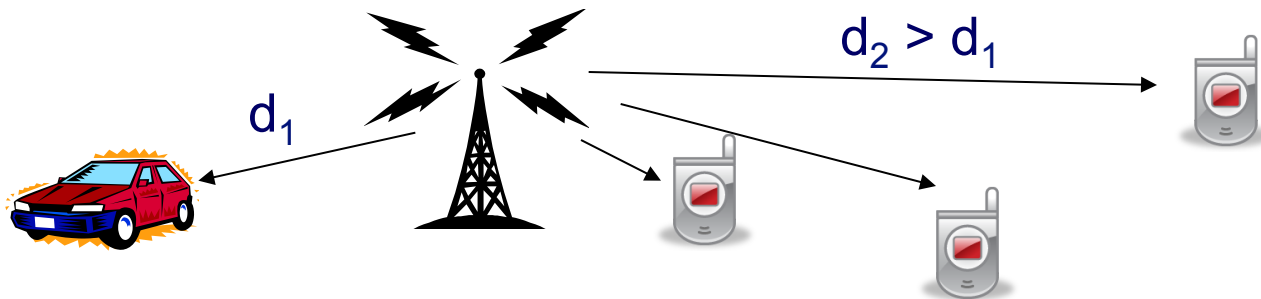
N O R T H W E S T E R N
U N I V E R S I T Y

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Multuser Diversity



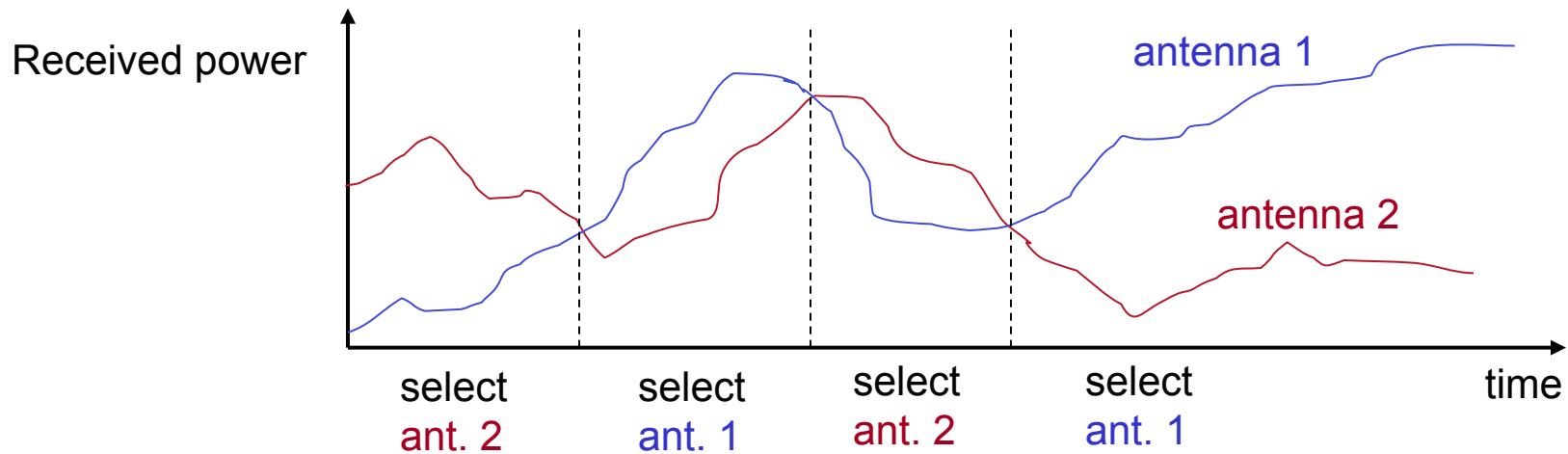
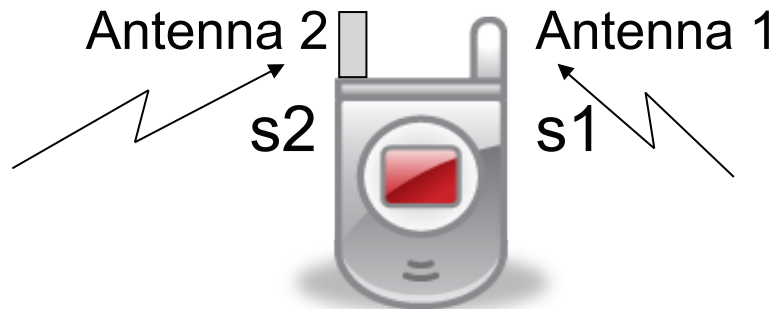
Multuser Diversity



The BST can choose to transmit to the user with the best channel. Exploits variations in signal strength across users.



Selection Diversity



Choose the best signal (highest instantaneous SNR).
Easy to implement (antenna switch).



Benefit of Selection Diversity (Example)

- Suppose that the signal on each antenna experiences independent Rayleigh fading.
- Determine the probability that the received signal is faded:

Recall Rayleigh fading formula:

Probability that the signal power is less than

$a \times P_0$ (average received power) = $1 - e^{-a}$

Hence the probability that the signals on **both** antennas are less

than $a \times P_0 = (1 - e^{-a})^2$

Without diversity, probability of a signal fade = $1 - e^{-1} = 0.63$

With 2-branch diversity, probability of a signal fade = $0.63^2 = 0.39$



Benefit of Selection Diversity (cont.)

- Suppose that there are N copies of the signal (e.g., N antennas, paths, coherence bands, etc.)

Probability that the signal power is less than $a \times P_0$ (average received power) = $1 - e^{-a}$

Hence the probability that **all** N signals are less than $a \times P_0$
= $(1 - e^{-a})^N$

Without diversity, probability of a signal fade = $1 - e^{-1} = 0.63$

With **4-branch** diversity, probability of a signal fade = $0.63^4 = 0.16$

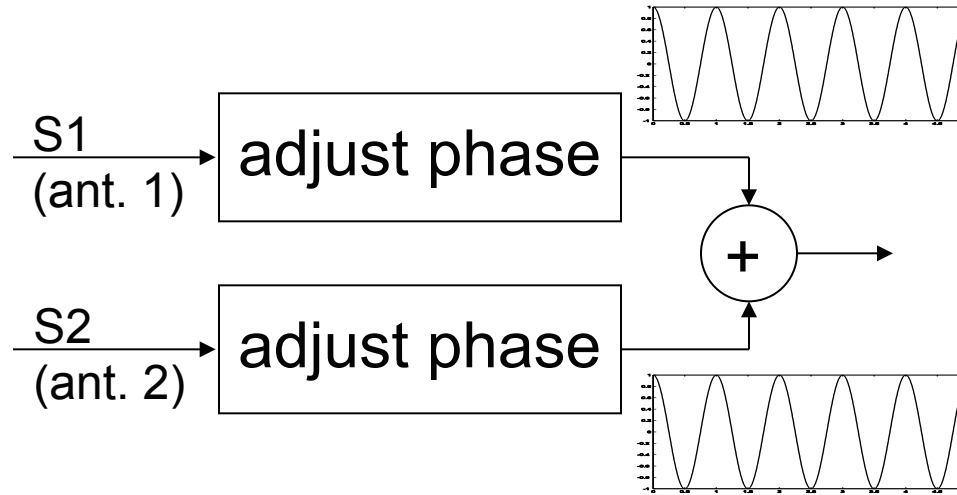
Without diversity,

Prob(signal is faded by more than 10 dB) = $1 - e^{-0.1} \approx 0.1$

With diversity this probability is $(1 - e^{-0.1})^4 \approx 0.0001$!

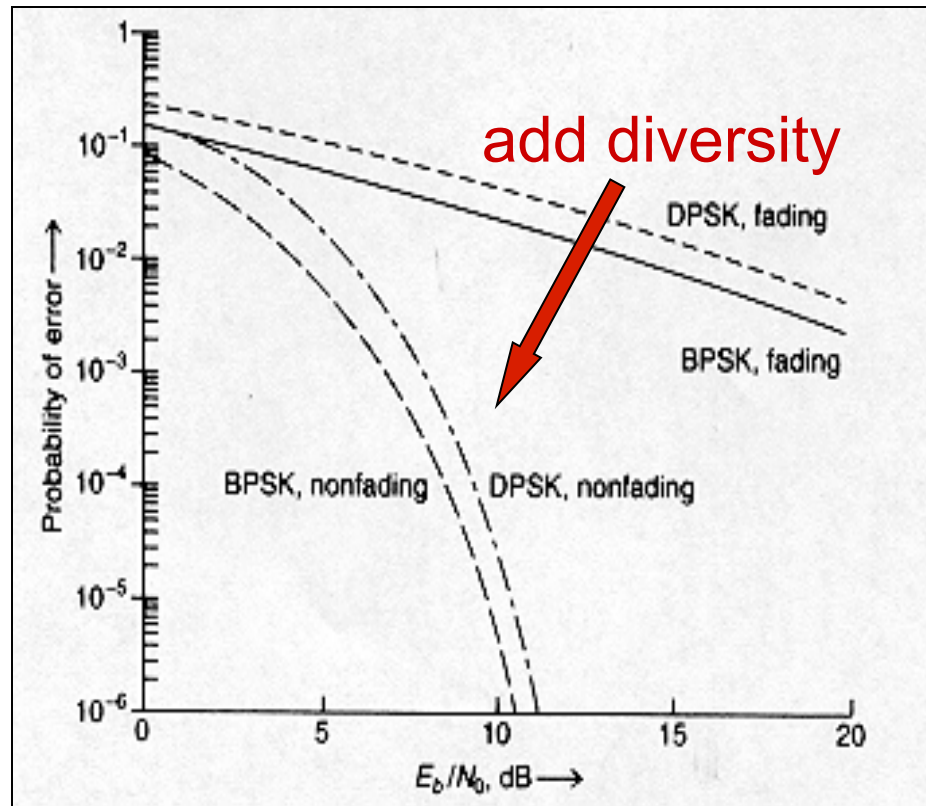


Coherent Combining



- “Coherent” means that the phases of the two signals are estimated at the receiver and aligned.
- Performs better than selection combining (why?).
- Example: RAKE receiver
- Can weight the combined signals to maximize the received SNR. (How should the weights depend on the signal levels?)

Probability of Error with Fading



- Diversity can transform a fading channel back to a non-fading (additive noise) channel.
- Essential for mobile wireless communications.