

EECS 311: Data Structures and Data Management Homework 5 Assigned: 10/21/10 Due: 10/28/10
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For this week only, there will be no late penalty for homeworks turned in by Monday, 5pm at the start of the TA section. After this deadline, late homeworks will not be accepted.

1 Reading.

Chapter 5 and the supplemental reading on hash tables. (If you didn't get one in class, you can find it on Blackboard.)

2 Problems.

1. Problem 5.1.
2. In the balls-and-bins example in class, two balls collide when if they are tossed into the same bin. This problem concerns counting the total number of collisions. If three distinct balls a , b , and c , collide at the same location it counts as $\binom{3}{2} = 3$ collisions (as a collides with b , b collides with c , and a collides with c ; if four balls collide at the same location it counts as $\binom{4}{2}$ collisions, etc. Consider the following question about tossing n balls in to s bins. Express your answers terms of s and n , as needed.
 - (a) For an arbitrary (i.e., not random) tossing of the balls into bins, what is the worst case number of collisions on n keys? Give an example that achieves this worst case.
 - (b) Suppose we toss the balls uniformly at random. What is the expected total number of collisions? Prove your bound. (Hint: define the appropriate set of indicator variables and use linearity of expectation.)
3. Suppose we were to implement hashtables via the bit array representation discussed in class, i.e., where `bitarray[k] = 1` when key k is in the table and `bitarray[k] = 0` when the k is not in the table. Such a hashtable can be used to keep track of a dictionary that contains any subset of the N possible keys $\{0, \dots, N - 1\}$. Describe how to implement such a table with the operations: create, insert, delete, and find. All operations should be $\Theta(1)$. In particular create should be $\Theta(1)$. You may assume that allocating an uninitialized array can be done in $\Theta(1)$; however, this array is uninitialized. Clearly, you cannot zero all the elements in such an uninitialized array in $\Theta(1)$. Hint: you may maintain auxiliary data structures in addition to a length N array of bits.