IMPROVING SOCIAL WELFARE BY DEMAND RESPONSE GENERAL FRAMEWORK AND QUANTITATIVE CHARACTERIZATION

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ABSTRACT

One way to improve the efficiency of the electricity system is by including demand participation in the market, as opposed to the traditional supply-follow-demand approach. In this work, we build upon literature to model a general T-period electricity market consisting of price-taking prosumers with multiple types of appliances, an utility company and a social planner distribution system operator (DSO), where we remove the restricting assumptions that prosumer cannot be net-sellers and EVs cannot discharge back to the grid. We show that a social welfare maximizing market equilibrium exists. Furthermore, through a case study with quadratic utility functions, we characterize explicitly the social welfare improvement, resulting from allowing prosumers to be a net-seller and EVs to discharge, as a function of utility function parameters by using quadratic programming duality.

Index Terms— Electricity market, prosumer, net-seller, demand response, social welfare

1. INTRODUCTION

Active demand participation in electricity system, where consumers can actively participate in consuming/selling, has been proposed to help improve system efficiency over the conventional supply-follow-demand approach. [1, 2]. The active participation, often referred to as *demand response (DR)*, involves some time varying pricing signal encouraging the flexible part of demand to reduce consumption in high demand periods [3], [4], [5], [6], [7]. With introduction of energy storage and electric vehicles, the demand response system may also include consumers both providing/selling and consuming electricity, resulting in *prosumers* [8].

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Most of existing literature assumes that all prosumers are net-consumers. In addition, some assume that the benefit of having EV (Electric Vehicle)/PHEV (Plug-in Hybrid Electric Vehicle) (collectively referred to as EV) manifests in the ability to consume power over time, but does not allow EV to discharge [9], [10], [11], [12], [13]. In this work, we build upon the model in [9] and extend it to allow prosumers to be net-sellers and EVs to discharge. In particular, we propose a general market model for a T-period price-taking retail market with no uncertainty. The market is cleared by *Distribution System Operator (DSO)* whose goal is to maximize social welfare and balance supply and demand. We show that when prosumers and utility firm best respond to the market clearing price set by DSO locally, social welfare is maximized. For tracability, we will not consider strategic settings as in [14], [15].

The aforementioned existing literature falls short of understanding the connection between social welfare and consumer utilities. One of our main contribution in this work is to analyze this connection by performing a case study with simple quadratic utility function forms and characterize explicitly the dependency of social welfare improvement on utility function parameters. This work contributes to the understanding of benefits on social welfare of demand response.

Notations: Throughout the paper, we use bold font to represent vectors variables, and superscript T for transpose. Notation $\mathbf{x} = (x_i, \forall i)$ denotes $[x_1, x_2, ..., x_n]^T$. The operator $[\cdot]^+$ is given by $[a]^+ = \max\{a, 0\}$.

2. SYSTEM MODEL

In this section, we present our general market model, which is motivated by [9]. We use node 0 to denote the utility company; and set $\mathcal{N}^+ = \{1, 2, ..., N\}$ to denote the set of prosumers. We consider a day-ahead retail market with no uncertainty and T periods, indexed by t in $\mathcal{T} = \{1, 2, ..., T\}$. Prosumers and utility companies are all price taking.[1] DSO clears the market and maximizes social welfare.

For each prosumer $i \in \mathcal{N}^+$, let $\mathcal{A}_i$ denote the set of household appliances such as lights, AC, washer, energy storage and EV. For each appliance $a \in \mathcal{A}_i$, we define the power consumption schedule vector $\mathbf{q^D_{i,a}} = (q^D_{i,a}(t), \forall t \in \mathcal{T})$. We can concatenate these vectors to define for each i $\mathbf{q^D_i} = (\mathbf{q^D_{i,a}}, \forall a \in \mathcal{A}_i)$ and the aggregate vector $\mathbf{q^D} = (\mathbf{q^D_i}, \forall i \in \mathcal{N}^+)$. Vector $\mathbf{p} = (p(t), \forall t \in \mathcal{T})$ represents the price vector over time. Similarly, utility company's power supply schedule vector is defined as $\mathbf{q^S} = (\mathbf{q}(t), \forall t \in \mathcal{T})$. We use superscript S to denote the quantity associated with utility company (traditionally supply side) and superscript D for prosumers (traditionally demand side). We next describe the problems the prosumers, utility firm and DSO solve.

2.1. Prosumers' Problem

Each prosumer $i \in \mathcal{N}^+$, optimizes her power schedule to maximize her payoff by solving the following problem (P):

$$\max_{\mathbf{q^D_i}} \quad U_i\left(\mathbf{q^D_i}\right) - \mathbf{p}^T \mathbf{q^D_i} \qquad \text{s.t. } A_i \mathbf{q^D_i} \le h_i, \tag{P}$$

where, $U_i : \mathbb{R}^{T\times|\mathcal{A}_i|} \to \mathbb{R}$ is the net utility function, formed by $U_i(\mathbf{q^D_i}) = \sum_{a\in\mathcal{A}_i} U_{i,a}(\mathbf{q^D_{i,a}})$, where $U_{i,a}$ is the net utility function for appliance a, which could be either positive from consumption for appliances like AC, washers and EV when charging or negative representing cost of accessing storage and EV when discharging. Vector $\mathbf{p}$ is the market price announced by DSO. Thus the second term in the objective function represents payment.

This is a very general framework. We note that there is no sign restriction on $\mathbf{q^D_{i,a}}$, representing that the prosumers can either consume or produce/sell electricity (hence the name prosumer). The set of inequalities $A_i \mathbf{q^D_i} \le h_i$ describes different power consumption requirements for household appliances of prosumer i. For inflexible appliances such as light, computer, and TV, where their consumption level does not change in response to electricity price changes, their power consumption constraints are of the form $\underline{q}_{i,a}(t) \le q^D_{i,a}(t) \le \overline{q}_{i,a}(t)$, for all t, where $\underline{q}_{i,a}(t) = \overline{q}_{i,a}(t)$. If an appliance is turned off in period t, then we can set both bounds to be 0.

[1] For model tractability, we ignore power loss and congestion on lines.

For flexible appliances such as AC, washer, EV and storage, whose power consumption may change depending on price, their consumption constraints have the following form,

$$\underline{q}_{i,a}(t) \le q^D_{i,a}(t) \le \overline{q}_{i,a}(t), \quad \forall t \tag{1}$$

$$\underline{Q}_{i,a} \le \sum_{t\in\mathcal{T}_{i,a}} \alpha_{i,a}(t)\, q^D_{i,a}(t) \le \overline{Q}_{i,a}, \tag{2}$$

where the set $\mathcal{T}_{i,a} \subseteq \mathcal{T}$ denotes the pre-specified periods where these appliances should finish their task such as consume enough energy to maintain room temperature, finish laundry or (dis)charge EV to certain level, represented by Eq. (2). We call these periods *flexible periods*. The coefficient $\alpha_{i,a}(t)$ captures efficiency for appliances such as AC or storage. Constraint (1) represents the physical limits of the appliance, such as maximal charging/discharging speed for EV or storage. We also note that for flexible appliances, the associated utility $U_{i,a}$ may take the form of $U_{i,a}(\sum_{t\in\mathcal{T}_{i,a}} q^D_{i,a}(t))$ and may not be separable over time periods.

2.2. Utility Company's Model

Utility company optimizes its power supply schedule to maximize its profit by solving the following problem (U):

$$\max_{\mathbf{q^S}} \quad -C_0\left(\mathbf{q^S}\right) + \mathbf{p}^T \mathbf{q^S}, \tag{U}$$

where $C_0 : \mathbb{R}^T \to \mathbb{R}$ is the cost function of utility company.[2] Vector $\mathbf{p}$ is the market price announced by DSO.

2.3. Distribution System Operator's Model

DSO is a system planner, whose goal is to maximize the social welfare and to balance supply and demand. DSO solves the following problem (problem (D)), whose objective is referred to as the social welfare:

$$\max_{\mathbf{q^D}, \mathbf{q^S}, A_i \mathbf{q^D_i} \le h_i, \forall i} \quad U\left(\mathbf{q^D}\right) - C_0\left(\mathbf{q^S}\right) \tag{D1}$$

$$\text{s.t.} \quad \mathbf{q^S}(t) = \sum_{i\in\mathcal{N}^+} \sum_{a\in\mathcal{A}_i} \mathbf{q^D_{i,a}}(t), \tag{D2}$$

where $U : \mathbb{R}^{T\times\sum_{i\in\mathcal{N}^+}|\mathcal{A}_i|} \to \mathbb{R}$ is summation of all net utility functions of all prosumers. Constraint (D2) is the market clearing or supply and demand balance condition. We adopt the following standard assumptions:

[2] We left the cost function in general form and the cost of production can vary from one time period to another.

Assumption 1. *$U - C_0$ is concave, coecive[3] and continuously differentiable.*

Under these assumptions, we can show that under the additional constraint of market clearing (Eq. (D2)), problems (P) and (U) are equivalent to problem (D). This is a modified version of the well known fundamental theorems of welfare in economics, whose proofs follow from duality and we omit here due to space constraint.

Theorem 2.1. *Under Assumption 1, then the quantities $\boldsymbol{q^D}, \boldsymbol{q^S}$ at any market clearing equilibrium, where (P) and (U) are solved by prosumers and utility firm respectively and market clears, is efficient, i.e., is optimal with respect to problem (D). Conversely, any welfare maximizing allocation $\boldsymbol{q^D}, \boldsymbol{q^S}$ (solution to (D)) is a pair of optimal solutions to (P) and (U) with some equilibrium prices.*

3. CASE STUDY: QUADRATIC UTILITY

In this section, we perform a case study, where we specialize the net utility function to quadratic form following [9] and analyze the effect of allowing prosumers and EVs to discharge on social welfare. We specify the utility functions of prosumers and utility firm. In all the following, we use $\hat{a}$ to denote the quadratic term coefficient, $\hat{b}$ for the linear term coefficient and $\hat{c}$ for the constant term, i.e., a quadratic function of the form $\hat{a}x^2 + \hat{b}x + \hat{c}$.

1. Prosumer: For each prosumer i at time t, we assume all net utility functions are separable over time and present the coefficients of the quadratic utility function associated with each appliance in the following table, where we suppress the indices and write $\hat{a}_{i,a}(t)$ as $\hat{a}$.

	Coefficients of Net Utility Function
AC, Washer	$\hat{a} < 0, \hat{b} > 0$
EV/PHEV	$\hat{a} < 0, \hat{b} > 0, \hat{c} = 0$
Energy Storage	$\hat{a} < 0, \hat{b} = 0, \hat{c} = 0$

For flexible appliances such as AC and washers, coefficient $\hat{a}_{i,a} < 0$ represents concavity of the net utility function and can be viewed as the price-responsiveness of the appliance. We outline the intuition here: in a household with one appliance at price p, the optimal quantity demanded for one period is given by $\frac{p - \hat{b}_{i,a}}{\hat{a}_{i,a}}$. For a change of Δp in price, the quantity demanded changes by $\frac{\Delta p}{\hat{a}_{i,a}}$. A large absolute value of $\hat{a}_{i,a}$ represents less price-responsive appliances. The first order coefficient $\hat{b}_{i,a}(t) > 0$ is the utility associated with consuming the first unit of energy. For EV and energy storage type of appliances, $\hat{a}_{i,a} < 0$ reflects depreciation cost of charging and discharging associated with non-ideal batteries. The negative utility (or cost associated with using battery) is higher as the battery is used more by either positive or negative quantity. As $\hat{a}_{i,a}$ goes to 0, the battery cost approaches 0. Coefficient $\hat{b}_{i,a}(t)$ is positive for EV representing the utility of having one unit of charge when EV battery is close to empty. The same coefficient $\hat{b}_{i,a}(t)$ associated with energy storage is 0, since the prosumer does not derive any net utility from storing energy in the battery (other than possibly using/selling it later).

2. Utility Firm: We assume utility firm's cost of production is linear, i.e., for some $\mathbf{b_0} \in \mathbb{R}_+^T$, $C_0(\mathbf{q^S}) = \mathbf{b_0}^T \mathbf{q^S}$.

With these quadratic utility functions and some algebraic manipulation, we can express (D) as a quadratic problem and by duality of quadratic problems, we can find dual variables associated with various constraints in closed form.

We note that our model does not include restriction on prosumers being net-buyer or the ability of EVs to discharge, i.e., constraints

$$\sum_{a \in \mathcal{A}_i} q_{i,a}^D(t) \geq -M, \quad (3) \qquad q_{i,a}^D(t) \geq -W, \quad (4)$$

for a in $\mathcal{A}_i^{EV}$, t in $\mathcal{T}$. For both of these constraints, existing literature often assumes M and W to be 0: no prosumer can be a net-seller and EVs can only charge but not discharge. We may view the absence of these constraints as effectively having very large M and W. To understand their effect, we first introduce each of these two constraints with $M = W = 0$ into problem (D) and then relax them by allowing one prosumer to be net-seller (or one EV to discharge) at one time period and quantify the relationship between welfare improvement of the relaxed problem and utility function parameters. Our approach here is conservative and the improvement of social welfare by allowing multiple net-sellers (EVs to discharge) over many time periods could be even higher.

3.1. Effects of allowing a prosumer to be net-seller

In this section, we analyze the effect of allowing prosumers to be net-sellers on social welfare. We do so by focusing on

[3] A function f is coercive iff $f(x) \to +\infty$, as $||x|| \to +\infty$.

one prosumer i at one time period t and analyze the effect of relaxing constraint (3) on the objective value of problem (D), while all other inequality constraints are assumed to be not tight at optimality. We use λ_{NS}^* to denote the optimal dual variable corresponding to constraint (3) for prosumer i at time t. The dual variable λ_{NS}^* can be interpreted as a shadow price reflecting the improvement in social welfare (objective function value of (D)) associated with allowing prosumer i to sell one unit, i.e., decreasing the right hand side of (3) by one unit. We can further show that the social welfare improvement associated with allowing prosumer to sell up to M unit, i.e., relaxing constraint (3) to $\sum_{a\in\mathcal{A}_i} q_{i,a}^D(t) \geq -M$, is upper bounded by $M\lambda_{NS}^*$. Hence the dual variable λ_{NS}^* plays an essential role in quantify social welfare improvement.

By duality and KKT optimality conditions, we obtain a closed representation of λ_{NS}^* as $\lambda_{NS}^* = \left[\frac{\sum_{a\in\mathcal{A}_i} \frac{b_0(t)-\hat{b}_{i,a}(t)}{\hat{a}_{i,a}}}{\sum_{a\in\mathcal{A}_i} \frac{1}{\hat{a}_{i,a}}}\right]^+$, where scalars $\hat{a}_{i,a} < 0$ and $\hat{b}_{i,a}(t) \geq 0$ are the quadratic and linear coefficients of the net utility functions of prosumer i associated with appliance a and scalar $b_0(t) > 0$ represents the marginal cost of production at time t as defined in Eq. (3). We make the following observations:

• If we fix the net utility function associated with appliances other than storage, then as energy storage becomes ideal, i.e., $\hat{a}_{i,a} \to 0$, by L'Hospital's rule and the fact that for energy storage $\hat{b}_{i,a}(t) = 0$, we have $\lambda_{NS}^* = b_0(t) - \hat{b}_{i,a}(t) = b_0(t) > 0$. The dual variable λ_{NS}^* equals the utility company's marginal cost of production. Hence the benefit of net-selling can be interpreted as production cost saving due to the availability of ideal storage.

• If the net utility function is much flatter for one appliance (very price-responsive) compared with the rest, i.e., there exists an a_i with $0 > \hat{a}_{i,a_1} >> \hat{a}_{i,a_2}, ..., \hat{a}_{i,a_{|\mathcal{A}_i|}}$, and if all appliances have similar $\hat{b}_{i,a_1}(t)$, then $\lambda_{NS}^* = \left[b_0(t) - \hat{b}_{i,a_1}(t)\right]^+$. The dual variable calculation is dominated by this particular appliance. The welfare improvement is determined by how much change in quantity there is, which is in turn dominated by the most price-responsive appliance.

• In general, the dual variable λ_{NS}^* may either increase or decrease as the number of appliances increases.

3.2. Effects of allowing EVs to discharge

We next analyze the effect of allowing EV to discharge, by analyzing the effect of relaxing constraint (4) for one prosumer at one time period. We denote the corresponding optimal dual variable by λ_{EV}^*. Similar to the previous section, we consider the case that only constraint (4) is tight at optimality. By using the same analysis as above, we obtain that the improvement in the social welfare by relaxing constraint to $q_{i,a}^D(t) \geq -W$ for some $a \in \mathcal{A}_i^{EV}$ is upper bounded by $\lambda_{EV}^* W$.

By using KKT condition one more time, we can get a closed form representation of λ_{EV}^* as $\lambda_{EV}^* = \left[b_0(t) - \hat{b}_{i,a}(t)\right]^+$. where scalar $b_0(t) > 0$ is the marginal cost of production at that period and scalar $\hat{b}_{i,a}(t) > 0$ represents increasing rate of utility when batteries in EV is close to empty. We make the following observations:

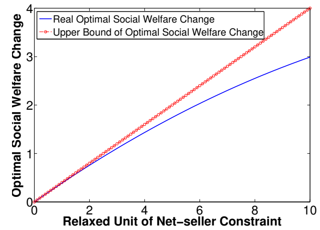

Fig. 1. Optimal social welfare improvement with relaxation of net-selling constraint, M.

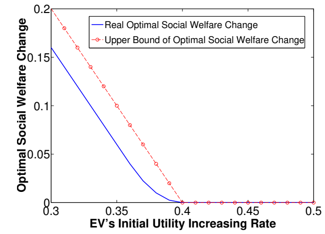

Fig. 2. Optimal social welfare improvement with rise of EV's initial utility increasing rate, $\hat{b}_{i,a}(t)$.

• If utility company's marginal cost of production is no more than EV's initial utility increasing rate, i.e. $b_0(t) \leq \hat{b}_{i,a}(t)$. Then $\lambda_{EV}^* = 0$, and social welfare does not improve by allowing EV selling electricity, since it costs less to generate electricity than to access storage.

• On the other hand, if utility company's marginal cost of production is more than EV's initial utility increasing rate, i.e. $b_0(t) > \hat{b}_{i,a}(t)$, then $\lambda_{EV}^* \geq 0$ and the social welfare increases as EVs are allowed to provide vehicle to grid service, since it is cheaper to use energy there than to generate.

• The dual variable is not affected by the prosumer i's EV battery depreciation $\hat{a}_{i,a}$, or other appliances.

3.3. Numerical Study

We simulate a simple case with 2 prosumers, 1 utility firm and 1 DSO. Each prosumer has three appliances: fixed load, energy storage and EV. Both utility and cost function have quadratic form with coefficients generated according to Section 3. Figure 1 shows the actual improvement of optimal social welfare and the theoretical upper bound against the allowed net-selling amount M in (3). Figure 2 shows that the improvement of optimal social welfare is bounded by the theoretical upper bound (when only (4) is tight with $W = 2$) as EV's initial utility increasing rate $\hat{b}_{i,a}(t)$ becomes larger.

4. CONCLUSION

In this paper, we propose and analyze a model of day-ahead electricity market with one distribution system operator, one utility company and many price-taking prosumers (with energy storage devices) who can both buy and sell electricity. We establish that there exists a market equilibrium, at which a market clearing price emerges. We analyze the effect on social welfare of allowing prosumers to be net-seller and EVs to discharge. We characterize the relationship between increase in welfare and quadratic utility function properties.

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