**Northwestern University**
Department of Electrical and Computer Engineering

ECE 510 Spring 2005

**Problem set 3:** Due: May 3, 2005

**Problem 1:** In lecture it was claimed that in a slow fading channel, transmitter CSI can in some cases greatly reduce the required SNR for a given outage probability. In this problem you will numerically investigate this. Specifically, consider a slow fading channel with flat Rayleigh fading as in Sect. 5.4.1 of Tse and Viswanath. Plot the outage probability, $p_{out}(R)$ (on a log scale) vs. the average received SNR (in dB) for the following two cases:

1. CSI available only at the receiver.
2. CSI available at both the transmitter and recever with only a long-term average power constraint.

Include plots for a rate of 1 bit per channel use and 0.1 bits per channel use. Note you will need to numerically evaluate the optimal power allocation for the second case. For each rate what is the reduction in required SNR with CSI at the transmitter for an error probability of $10^-3$?

**Problem 2:** In class we gave a heuristic argument to explain why at low SNR the water-filling stategy results in a larger capacity than that of an AWGN channel with the same average received SNR (see also pg. 241 in Tse and Viswanath). In this problem, you will make this more precise for a Rayleigh fading channel.

Consider a fast fading channel with flat Rayleigh fading and assume that $\mathbb{E}(|H|^2) = 1$ and $N_0 = 1$. Let $\bar{P}$ be the average power constraint, so that the average received SNR is $\bar{P}$.

a.) Show that the capacity of this channel with full CSI satisfies

$$C \geq e^{-\lambda} \log(1 + \lambda \bar{P} e^{\lambda}),$$

where $\frac{1}{\lambda}$ is the "water-level" that satisfies the average power constraint.

b.) Now for all $\lambda$, let $\bar{P}(\lambda)$ denote the average power constraint that results in a water-level of $\frac{1}{\lambda}$. Show that as $\lambda \to \infty$, $\lambda \bar{P}(\lambda) e^{\lambda} \to 0$. (*Hint: use the expression for the optimal power allocation and the fact that this must satisfy the average power contraint.*)

c.) Finally combine these two results to show that as $\bar{P} \to 0$ (or equivalently $\lambda \to \infty$) that the capacity $C(\bar{P})$ satisfies

$$\lim_{\lambda \to \infty} \frac{C(\bar{P})}{\lambda \bar{P}(\lambda) \log_2 e} \geq 1.$$

**Problem 3:** In class we discussed a model for the case where the receiver has imperfect knowledge of the channel gain - this model can be found in the paper by Medard listed in the references.

a. Look at the derivation of the lower bound on mutual information in this paper (given by equation (46)). In this derivation explain why is $\alpha$ choosen as in (43).

b. For several different values of the received SNR, $\frac{\bar{F}\sigma_S^2}{\sigma_N^2}$, plot the lower and upper bounds on the mutual information as a function of $\frac{\sigma_F^2}{\bar{F}^2}$. Your plots should be over a large enough range to see the convergence of these bounds. Comment on the results.

c. Assume that with $K$ bits of training, $\sigma_F^2 = \frac{1}{K}$, the received SNR $=$ 0db, and $\bar{F}^2 = 1$. Again plot the upper and lower bounds, this time as a function of the training $K$.