Northwestern University
Department of Electrical and Computer Engineering

ECE 454: Advanced Communication Networks

Supplemental Notes on Loss Networks

A *loss network* refers to a network model such as a circuit switched network that can accommodate a fixed number of "connections" on each link. Once a link is full a new connection is blocked from using that link. For a fixed route, a common performance measure is to study the blocking probability for requests that use that route. If requests arrive according to a Poisson process, from PASTA, this is the steady-state probability that at least one link along the route is full. Note that when a route consists of a single link, requests arrive according to a Poisson process and all requests are identical, the blocking probability is given by the Erlang (B) loss formula (assuming that block requests leave). The models for loss networks can be viewed as a generalization of this for multiple classes of customers and/or multiple links.

As an example consider a single link with multiclass traffic (An example with two classes of customers is given in Example 3.12 in Bertsekas and Gallager). We assume that the link has a capacity of C units and each class requires some integer number of these units. Assuming there are K classes, each class $k \in \{1, 2, \ldots, K\}$ arrives according to a Poisson process with rate λ_k, has an exponential holding time with mean $1/\mu_k$, and requires b_k units of bandwidth. If there are n_k class k calls using the link at a given time, then the capacity constraint implies that

$$\sum_{k=1}^{K} n_k b_k \leq C. \tag{1}$$

Hence, if a class l call arrives and

$$b_l \geq C - \sum_{k=1}^{K} n_k b_k, \tag{2}$$

it will be blocked. Note here we are assuming that all calls will be accepted unless there is not enough bandwidth available. These models can also accommodate various preferential admission policies (see example 3.13 in Bertsekas and Gallager).

In lecture we discussed modeling such loss networks using multi-dimensional Markov chains and then using techniques based on the *truncation* of products of independent Markov Chains for calculating the steady-state probabilities (see also Sect. 3.4.4 in Bertsekas and Gallager). After truncation, the steady-state probabilities still have a "product-form" expression with a suitable normalization constant. Once, we have the steady-state probabilities (using PASTA), the blocking probability for each class is just given by summing the steady-state probabilities over the set of states for which an arriving packet from that class will be blocked (i.e. the states for which (2) holds). Therefore, using these techniques, it is straightforward to give the form of the blocking probability. However, for large networks, actually calculating these probabilities can be computationally difficult (for example, for a given capacity the number of states in the Markov chain is exponential in the number of classes; which means an exponential number of terms must be summed to directly calculate the normalizing constant). A number of different techniques have been studied for simplifying these calculations. One common technique for network models is based on the so-called "reduced load approximations;" a discussion of this can be found in:

- F.P. Kelly, "Blocking probabilities in large circuit-switched networks," *Advances in Applied Probability*, vol. 18, 1986.

Some of the examples from the lectures were based on material from the following paper:

- Frank Kelly, "Modeling Communication Networks, Present and Future," *Phil. Trans. Roy. Soc. Lond.*, pp. 437-463, 1996.

Both of these papers can be found at:

`http://www.statslab.cam.ac.uk/~frank/PAPERS/`

Several other papers on loss networks can also be found at this site.

Loss networks were originally motivated as models of circuit switched networks, however extensions of these ideas have also been considered for packet switched networks, in this setting the "circuits" needed for a connection are not physical quantities but are based on a stochastic description of the traffic (see, for example, page 7 of the 2nd paper above). These ideas have also been applied to optical networks, where a "circuit" is a wavelength path set-up between two points in the network.