Northwestern University

Department of Electrical and Computer Engineering

ECE 428: Information Theory Spring 2004

Problem Set 6 **Date issued: May 11, 2004**
Date Due: May 18, 2004

Reading Assignment: Finish reading Chapter 8 and look over supplemental notes.

Do the following problems:

1. Problem 8.10 in C&T.

2. Exceeding Capacity: A Bernoulli(1/2) source produces an output letter once every two seconds. The source is encoded and transmitted over a binary symmetric channel with crossover probability ε. The channel may be used once every second.

a. Find the largest value of ε such that the source can be recovered at the receiver with arbitrarily low probability of error.

b. If ε = 0.4, find a good lower bound on the minimum achievable per-bit probability of error.

c. Argue that the bound in (b) can be asymptotically achieved.

3. Binary Erasure Channel Union Bound: Consider a binary erasure channel with erasure probability ε. Let $a^n = a_1, \ldots, a_n$ and $b^n = b_1, \ldots, b_n$ be two random codewords drawn according to a Bernoulli(1/2) distribution.

a. Assume that the codeword $x^n = a^n$ is transmitted over the channel. Let y^n be the received sequence (which will equal x^n except for erasures). The receiver uses a maximum likelihood decoder to decide whether a^n or b^n was transmitted. Compute the exact probability of decoding error as a function of n and ε. You may assume that decoding ties are resolved with a fair coin flip. Express you answer in closed form.

b. Using the union bound and the result in part a, upper bound the probability or error for a random codebook with 2^{nR} codewords. As a function of of ε, find the largest value of R such that the probability of decoding error goes to zero as $n \to \infty$.

c. Are the values of R found in part (c) an upper bound or lower bound on the capacity of the channel, or are they merely an approximation to capacity?