Northwestern University
Department of Electrical and Computer Engineering

ECE 428: Information Theory — Spring 2004

Problem Set 5 — **Date issued: April 29, 2004**
Date Due: May 11, 2004

Reading Assignment: Chapter 8

Do the following problems:

1. Problem 8.1 in C&T.

2. Problem 8.4 in C&T.

3. Problem 8.6 in C&T.

4. Problem 8.9 in C&T.

5. Problem 8.12 in C&T.

6. **Random Source Coding:** This problem develops the idea of using a random coding approach to prove a coding theorem for lossless fixed-to-fixed length source codes. Let $X_1, X_2, \ldots$ be an i.i.d. source with p.m.f. $p(x)$. For any $\varepsilon > 0$, let $A_\varepsilon^{(n)}$ be the set of typical sequences of length n. Let $x^n \in A_\varepsilon^n$ be a particular typical sequence.

- **a.** Let Y^n be a length n i.i.d. sequence with p.m.f. $p(x)$. Find a good lower bound on the probability q that $x^n = Y^n$.
- **b.** Let C be a random codebook containing M i.i.d. sequences drawn according to $p(x)$. Find an exact expression for the probability that $x^n \notin C$. Convert this to an upper bound using (a).
- **c.** Find the least rate R^* such that for a random codebook $C^{(n)}$ with $M = 2^{nR^*}$ codewords, the probability that $x^n \notin C$ approaches 0 as $n \to \infty$.
- **d.** Argue that for sufficiently large n there exists a codebook with $M = 2^{nR^*}$ codewords that noiselessly encodes the i.i.d. source with arbitrarily low probability of failure.