

# **ECE 333: Introduction to Communication Networks**

## **Fall 2002**

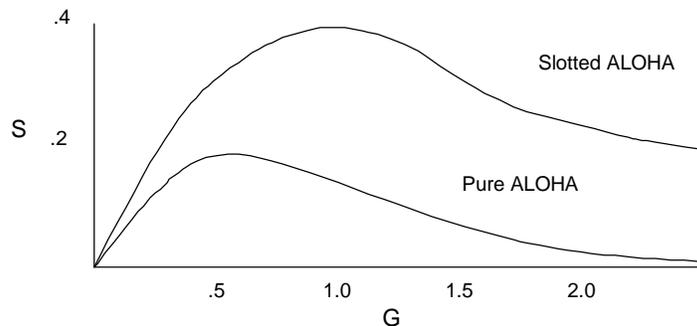
### **Lecture 15: Medium Access Control III**

- Slotted Aloha
- Stability

In the previous lecture we introduced the ALOHA medium access control protocol. This protocol is a contention based or random access protocol in which there is some probability that multiple users transmit at the same time resulting in a collision. Using a simplified analysis for a network with many users we showed that the maximum throughput of ALOHA was  $1/(2e)$  packets/ per transmission time. In this lecture we first look at an improvement on ALOHA, called "slotted ALOHA". We then consider the stability of these protocols.

## Slotted Aloha (Roberts '72)

Slotted Aloha, as the name implies, changes the protocol from continuous time to slotted time operation. Specifically, we view the time axis a sequence of slots of length  $T$ , where one frame can be sent in each slot. The transmitters are assumed to all be synchronized so that all transmissions start at the beginning of a slot. When a frame arrives to be transmitted during a slot, it is queued until the beginning of the next slot. Thus a frame only contends with frames generated during the same slot; this reduces the contention period from 2 frame times to 1 frame time. Repeating the argument from last lecture, we now find the probability of a successful transmission,  $P_0 = e^{-G}$ , and the throughput vs. offered load is given by  $S = Ge^{-G}$ , which is plotted below. In this case, the maximum throughput is  $1/e$  (0.36) and occurs at  $G = 1$ . This is twice the maximum throughput of pure Aloha.



3

## Stability of Slotted Aloha

For either slotted or pure Aloha, we have not yet taken into account the dynamics of the system, *i.e.* how  $G$  changes with time. Notice as the number of backlogged packets increases,  $G$  will increase, which will generate further changes in the number of backlogged packets.

We next take a closer look at these dynamics for a slotted Aloha system. In the following we still assume a model with an infinite number of nodes and that the average arrival rate per slot is denoted by  $S$ . We also make the following additional assumptions:

- Assume that when a packet arrives, it is transmitted in the next slot.
- If a transmission has a collision, the node becomes **backlogged**.
- Backlogged nodes transmit in each slot with probability  $q$  until successful.

4

Let  $n$  denote the number of backlogged packets.

Let  $G(n)$  = the average transmission attempt rate given that there are  $n$  backlogged packets. This is the average number of new arrivals plus the average number of retransmission attempts, i.e.,

$$G(n) = S + nq$$

As we assumed in lecture 14, the number of attempted transmissions per slot when  $n$  packets are backlogged can be shown to be approximately a Poisson random variable of mean  $G(n)$ .

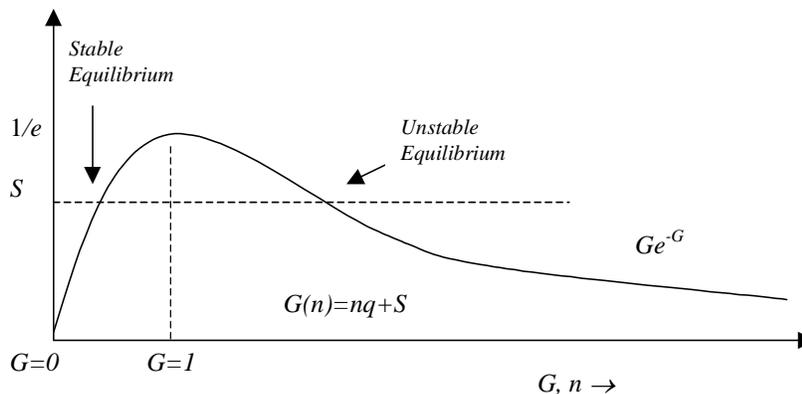
The probability of a successful transmission in a slot with  $n$  packets backlogged is the probability one transmission occurs. Using the Poisson assumption, this is given by:

$$P_{suc}(n) \approx G(n)e^{-G(n)}.$$

In a slot, either 1 packets departs the system with probability  $P_{suc}(n)$ , or 0 packets depart with probability  $1 - P_{suc}(n)$ . Thus the average number of departures per slot (i.e. the departure rate) is equal to  $P_{suc}(n)$ .

5

In the figure below, the average arrival rate and the average departure rate as a function of  $n$  is plotted. (Regardless of the backlog, we are assuming that the average arrival rate of new packets is  $S$ .)



When the departure rate is less than the arrival rate, the number of backlogged packets will tend to increase. When the departure rate is more than the arrival rate, the number of backlogged packets will tend to decrease.

As shown in the figure, for a given arrival rate, two equilibrium points can be identified. When the backlog increases beyond unstable equilibrium point, then it tends to increase without limit and the departure rate drops to 0.

6

While the above analysis assumed an infinite number of nodes, a similar type of analysis can be done for a finite number of nodes. In this case the same type of behavior can be shown.

Choosing  $q$  small increases the backlog at which instability occurs (since  $G(n) = \lambda + nq$ ), but also increases delay (since the mean time between transmission attempts is  $1/q$ ).

To improve the performance and balance between these considerations, one can attempt to estimate the backlog from the feedback and use this to adjust the retransmission probability. (Decreasing  $q$  as  $n$  is estimated to increase)

One way of doing this is called the *(binary) exponential back-off* technique.

With this technique, each time a node attempts a transmission and a collision occurs, the node divides the transmission probability,  $q$ , by 2.

(This technique is still unstable for the infinite node model, but works reasonably well for a finite number of nodes. There are elaborate techniques that produce stable algorithms even in the infinite node case.)

The figure below compares the average queueing delay for a stabilized Aloha system and the average delay for TDM systems with  $m=8$  and  $m=16$  nodes. In all cases the total arrival rate is the same and arrivals are assumed to be Poisson. At low arrival rates Aloha has much lower average delay than TDMA, but as the arrival rate approaches  $1/e$ , the delay blows up. TDMA can achieve throughputs up to 1 packet/slot, but the delay increases linearly with number of slots. The delay for stabilized Aloha depends on the overall arrival rate and is essentially independent of the number of nodes.

