ECE 333: Introduction to Communication Networks
Fall 2001

Lecture 14: Medium Access Control II

- Aloha throughput analysis
- Slotted Aloha
- Stability

Last lecture we began discussing multiple access protocols for allocating a shared broadcast channel. We classified these protocols as either static or dynamic, and further classified dynamic protocols as contention-based or perfectly scheduled. We then began discussing a basic contention-based approach, the pure Aloha protocol. Recall, under the Aloha protocol user transmit, when a frame arrives. If a collision occurs they wait a random amount of time and then retransmit. Because of this such protocols are also called ***random access protocols***.

In this lecture, we will look at a simplified analysis that predicts the maximum throughput of the Aloha protocol. We then look at an improvement to the basic Aloha protocol and discuss the issue of ***stability*** for this type of protocol.

Aloha Throughput

To analyze the throughput of Aloha, we consider a model with **an infinite number of users** that generate **fixed length frames.** We assume that each new frame belongs to a new user, and that the overall arrival of frames is a Poisson process with rate λ frames/sec. Let $\boldsymbol{T}$ indicate the time to transmit one frame. It will be convenient to normalize time by $\boldsymbol{T}$. Let $\boldsymbol{S}$ be arrival rate of frames in "frame times" of $\boldsymbol{T}$ seconds, i.e.

$$S = \lambda T$$

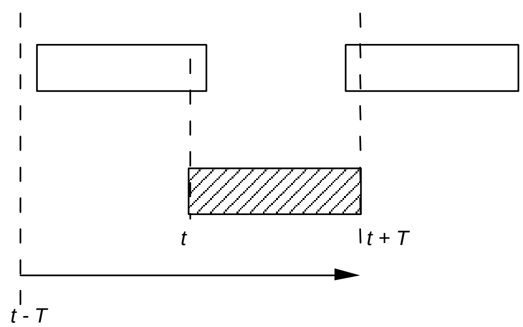
A transmission will be attempted at any time if either a new frame arrives or a frame is retransmitted. Assume that the start of transmissions at any time is also modeled as a Poisson process with rate $\boldsymbol{G}$ attempts per frame time. (This assumption can be shown to be a reasonable approximation for some common retransmission strategies.)

Thus the number of transmission attempts in a frame time will be a Poisson random variable, with mean $\boldsymbol{G}$ (attempts/frame time). Note that $S \leq G$, with equality only if there are no collisions. Assuming the system is stable then the average rate of successful transmission must equal the average arrival rate. Thus, denoting the probability of a successful transmission by P_0 , we have

$$S = GP_0.$$

Probability of Success

If a transmission starts at some time *t,* there is an interval of time around *t* during which if another transmission starts, it will result in a collision. The length of this contention interval is twice the frame time, as shown below.



Thus the probability a frame is successfully transmitted, P_0, is the probability no other frames begin transmission during the contention window. This is the probability of 0 arrivals from a Poisson process with rate $\boldsymbol{S}$, during 2 frame times. From the properties of the Poisson process (cf. Lec. 12), the number of arrivals in 2 frame times is a Poisson random variable with mean 2$\boldsymbol{S}$. Thus

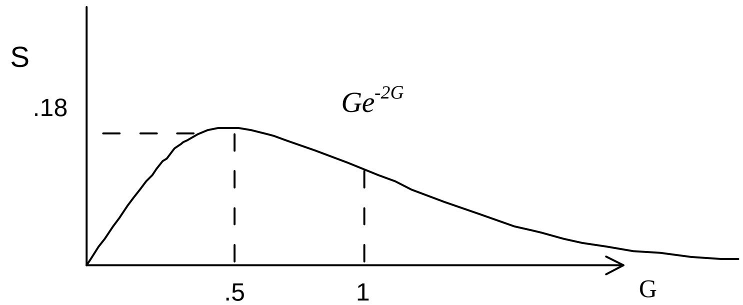
$$P_0 = \frac{(2G)^0 e^{-2G}}{0!} = e^{-2G}.$$

Aloha Throughput

From the above we have

$$S = GP_0 = Ge^{-2G}$$

This is plotted as a function of G below.



The maximum of this curve represents the maximum throughput for a stable system. To find the maximum we set the derivative of S(G) equal to zero as follows:

$$\frac{dS}{dG} = \frac{d}{dG} Ge^{-2G} = e^{-2G} - 2Ge^{-2G} = 0$$

Solving we find that $G^* = 0.5$, and thus the maximum throughput is $S_{\max} = 1/(2e) \approx 0.18$.

To summarize, we have considered a simplified analysis of a pure Aloha, and found that the maximum throughput is limited to be at most 1/(2e).

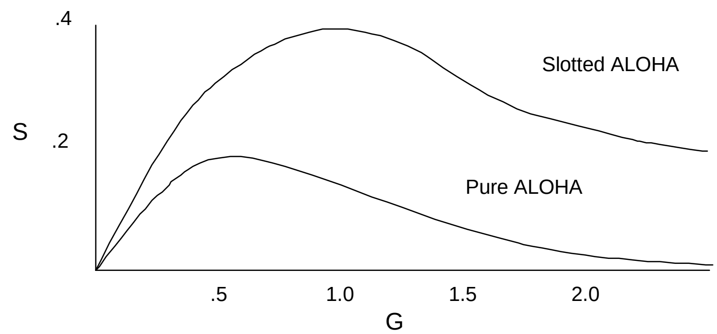
Note have not taken into account how the offered load changes with time, or specified the details of how the retransmission time is adjusted.

Despite these shortcomings, this analysis does correctly predict the maximum throughput of a pure Aloha system.

We will next look at an improvement on this system, and then return to address some of the above issues.

Slotted Aloha (Roberts '72)

Slotted Aloha, as the name implies, changes the protocol from continuous time to slotted time. Specifically, we view the time axis a sequence of slots of length ***T***, where one frame can be sent in each slot. The transmitters are assumed to all be synchronized so that all transmissions start at the beginning of a slot. When a frame arrives be transmitted during a slot, it is queued until the beginning of the next slot. Thus a frame only contends with frames generated during the same slot; this reduces the contention period from 2 frame times to 1 frame time. Repeating the above argument we now find $P_0 = e^{-G}$, and $S = Ge^{-G}$. In this case, the maximum throughput is $1/e$ (0.36) and occurs at $G = 1$. This is twice the maximum throughput of pure Aloha.



Stability of Slotted Aloha

As with the pure Aloha system, we have still not taken into account the dynamics of the system. That is how G changes with time. Notice as the number of backlogged packets increases, G will increase, which will generate further changes in the number of backlogged packets.

Now we look closer at this for a slotted Aloha system. In the following we still assume a model with an infinite number of nodes as above and that the average arrival rate per slot is denoted by ***S*.** We also make the following additional assumptions:

- Assume that when a packet arrives, it is transmitted in the next slot.
- If a transmission has a collision, the node becomes **backlogged**.
- Backlogged nodes transmit in each slot with probability ***q*** until successful.

Let $\boldsymbol{n}$ denote the number of backlogged packets.

Let $G(n)$ = the average transmission attempt rate given that there are n backlogged packets. This is the average number of new arrivals plus the average number of retransmission attempts, i.e.,

$$G(n) = S + nq$$

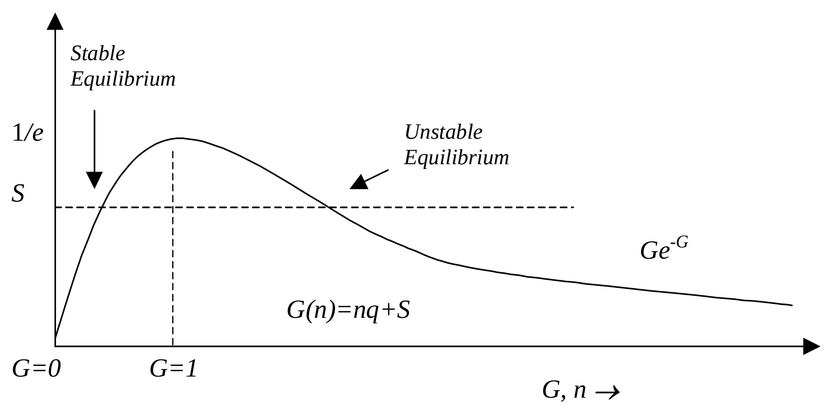
As we assumed above, the number of attempted transmissions per slot when n packets are backlogged can be shown to be approximately a Poisson random variable of mean $G(n)$.

The probability of a successful transmission in a slot with n packets backlogged the probability one transmission occurs. Using the Poisson assumption, this is given by:

$$P_{suc}(n) \approx G(n)e^{-G(n)}.$$

In a slot, either 1 packets departs the system with probability $P_{suc}(n)$, or 0 packets depart with probability 1- $P_{suc}(n)$. Thus the average number of departures per slot (i.e. the departure rate) is equal to $P_{suc}(n)$.

In the figure below, the average arrival rate and the average departure rate as a function of n is plotted. (Regardless of the backlog, we are assuming that the average arrival rate of new packets is S.)



When the departure rate is less than the arrival rate, the number of backlogged packets will tend to increase. When the departure rate is more than the arrival rate, the number of backlogged packets will tend to decrease.

As shown in the figure, for a given arrival rate, two equilibrium points can be identified. When the backlog increases beyond unstable equilibrium point, then it tends to increase without limit and the departure rate drops to 0.

While the above analysis assumed an infinite number of nodes, a similar type of analysis can be done for a finite number of nodes. In this case the same type of behavior can be shown.

Choosing q small increases the backlog at which instability occurs (since $G(n) = \lambda + nq$), but also increases delay (since the mean time between transmission attempts is $1/q$).

To improve the performance and balance between these considerations, one can attempt to estimate the backlog from the feedback and use this to adjust the retransmission probability. (Decreasing q as n is estimated to increase)

One way of doing this is called the ***(binary) exponential back-off*** technique.

With this technique, each time a node attempts a transmission and a collision occurs, the node divides the transmission probability, q, by 2.

(This technique is still unstable for the infinite node model, but works reasonably well for a finite number of nodes. There are elaborate techniques that produce stable algorithms even in the infinite node case.)

The figure below compares the average queueing delay for a stabilized Aloha system and the average delay for TDM systems with m=8 and m=16 nodes. In all cases the total arrival rate is the same and arrivals are assumed to be Poisson. At low arrival rates Aloha has much lower average delay than TDMA, but as the arrival rate approaches $1/e$, the delay blows up. TDMA can achieve throughputs up to 1 packet/slot, but the delay increases linearly with number of slots. The delay for stabilized Aloha depends on the overall arrival rate and is essentially independent of the number of nodes.