Utility-Based Power Control for a Two-Cell CDMA Data Network

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***Abstract*—Power allocation across users in two adjacent cells is studied for a code-division multiple access (CDMA) data service. The forward link is considered and cells are modeled as one-dimensional with uniformly distributed users and orthogonal signatures within each cell. Each user is assumed to have a utility function that describes the user's received utility, or willingness to pay, for a received signal-to-interference-plus-noise ratio (SINR). The objective is to allocate the transmitted power to maximize the total utility summed over all users subject to power constraints in each cell. It is first shown that this optimization can be achieved by a pricing scheme in which each base station announces a price per unit transmitted power to the users, and each user requests power to maximize individual surplus (utility minus cost). Setting prices to maximize total revenue over both cells is also considered, and it is shown that, in general, the solution is different from the one obtained by maximizing total utility. Conditions are given for which independent optimization in each cell, which leads to a Nash equilibrium (NE), is globally optimal. It is shown that, in general, coordination between the two cells is needed to achieve the maximum utility or revenue.**

***Index Terms*—CDMA, data service, forward link, power control, pricing, resource allocation, revenue, surplus, utility.**

I. INTRODUCTION

IN A WIRELESS data network, limited resources, such as power and bandwidth, must be assigned to a set of user requests. In a cellular system, this allocation can take place among multiple cells and must account for channel variations across users. Here, we consider forward-link power allocation across a static set of users in two adjacent cells in a wireless code-division multiple access (CDMA) data network.

Our approach to resource allocation is based on maximizing the overall utility of the network. That is, we assume that each user has a utility function that describes received utility, or willingness to pay, as a function of quality of service (QoS), defined as received signal-to-interference-plus-noise ratio (SINR). The total utility for a given allocation of resources is then the sum of the received utilities over the users. The primary advantage of this approach is that it can directly account for differences in perceived utility across the users. In general, by maximizing total utility, more resources are generally allocated to those users with higher utilities.

This work is a continuation of our prior work in [1] and [2]. In [1], utility-based forward-link power allocation across voice users in a single-cell is considered, and in [2], this was extended to two adjacent interfering cells. In that work, the utility function is assumed to be a step function, i.e., the user receives a fixed utility provided that the rate, or SINR, exceeds a threshold. For the data service considered here, the utility function is assumed to be an increasing and concave function of the data rate. This leads to qualitatively different results from the voice service considered earlier. For example, in contrast to the two-cell voice model, the optimal power distribution for a data service is no longer characterized by a single threshold, or radius of active users, and maximizing revenue can lead to substantially different power allocations than maximizing utility. Other related work on utility-based power allocation for wireless networks has been presented in [3]–[10]. The emphasis in that work is on conserving power at the handset, which leads to a different model formulation than the one considered here. Power allocation based on revenue maximization for a single cell has been analyzed in [10]–[12].

We consider two scenarios. 1) Users announce their utility functions to the base station, which then performs the power allocation. (For example, there may be a small set of utility functions associated with particular classes of service.) In this scenario, the total utility is maximized by a pricing scheme. Namely, each base station announces a price per unit power, and each user requests an amount of power to maximize individual surplus (received utility minus cost). 2) Users need not provide the utility functions to the base station, and the base station sets the price per unit power to maximize total revenue.

A symmetric one-dimensional (1-D) two-cell model is considered in which the users are uniformly spread over the two cells. All users have the same utility function. Because signatures within each cell are orthogonal, interference is received only from the base station in the adjacent cell. We formulate optimal power allocation problems across users in the two cells in the large system limit as the number of users and processing gain approaches infinity with fixed ratio. This model is simple enough to be tractable, yet, it captures the effect of intercell interference on intracell power allocations. In this way, we are able to characterize the optimal power distribution across users and infer certain properties.

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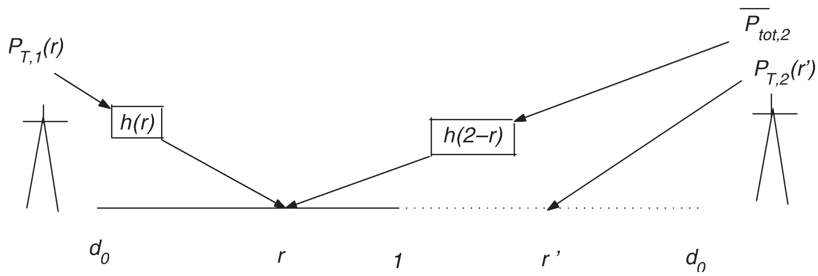


Fig. 1. 1-D two-cell model.

We also specify the Nash equilibrium (NE) of the power control game, in which the base stations (the players in the game) each maximize their own utility or revenue independently of the other base stations. We show that when the power budget in each cell is sufficiently small, the NE corresponds to the maximum achievable utility or revenue. In that case, the incremental increase in utility in one cell, due to an associated increase in power, is larger than the incremental decrease in utility in the other cell due to the increase in adjacent-cell interference. In contrast, if the power budget in each cell is sufficiently large, then the maximum utility is achieved when one cell uses all available power and the other cell uses only part of the available power. In that case, increasing the power in the other cell decreases the overall utility. That is, the increase in utility in that cell is less than the utility decrease in the other cell caused by the increase in interference. We therefore conclude that an optimal allocation of power across the two cells in general requires coordination between the two cells.

Similar types of results are shown when the base stations allocate power to maximize revenue. A key difference from utility maximization, however, is that with sufficiently large power budgets, both cells may not use all of the available power (i.e., the network may withhold resources).

Although our analysis pertains to a simple 1-D two-cell model, we expect that our main conclusion regarding the value of coordination applies to more realistic models. Namely, the tradeoff between utility and other-cell interference in a two-dimensional (2-D) multicell model will be qualitatively similar to the 1-D tradeoff examined here.

In the next section, we present the two-cell CDMA system model, and in Section III, we formulate the large system power allocation problem. In Sections IV and V, we state properties of the power allocations, which maximize total utility and revenue, respectively. Conclusions are stated in Section VI. The Appendix contains the proofs of theorems and lemmas.

II. System Model

The 1-D model of two adjacent cells is illustrated in Fig. 1. The cell radius is normalized to 1, and d_0 is a small reference distance from the base station. As in [2], we assume a static set of users in each cell, which are uniformly distributed along the line from d_0 to 1, and consider a large system limit as the number of users per cell K, and the number of available codes per cell M, each to infinity with fixed ratio, or load K/M. This model captures the effect of other-cell interference and is sufficiently tractable to allow a characterization of optimal power allocations across the two cells.[1]

We assume that the received power is determined solely by the distance from the transmitter. All codes within a cell are assumed to be orthogonal, so that interference is generated only from the adjacent cell. Treating the signatures from the adjacent cell as random independent identically distributed (i.i.d.), and assuming matched filter receivers, we can write the SINR for a user at distance r from base station i as

$$\xi_i(r) = \frac{P_{\mathrm{T},i}(r)}{A_i(r)}$$
$$\text{where} \quad A_i(r) = \frac{\sigma^2 + \overline{P}_{\mathrm{tot},j} h(2-r)}{h(r)}, \qquad i \neq j \tag{1}$$

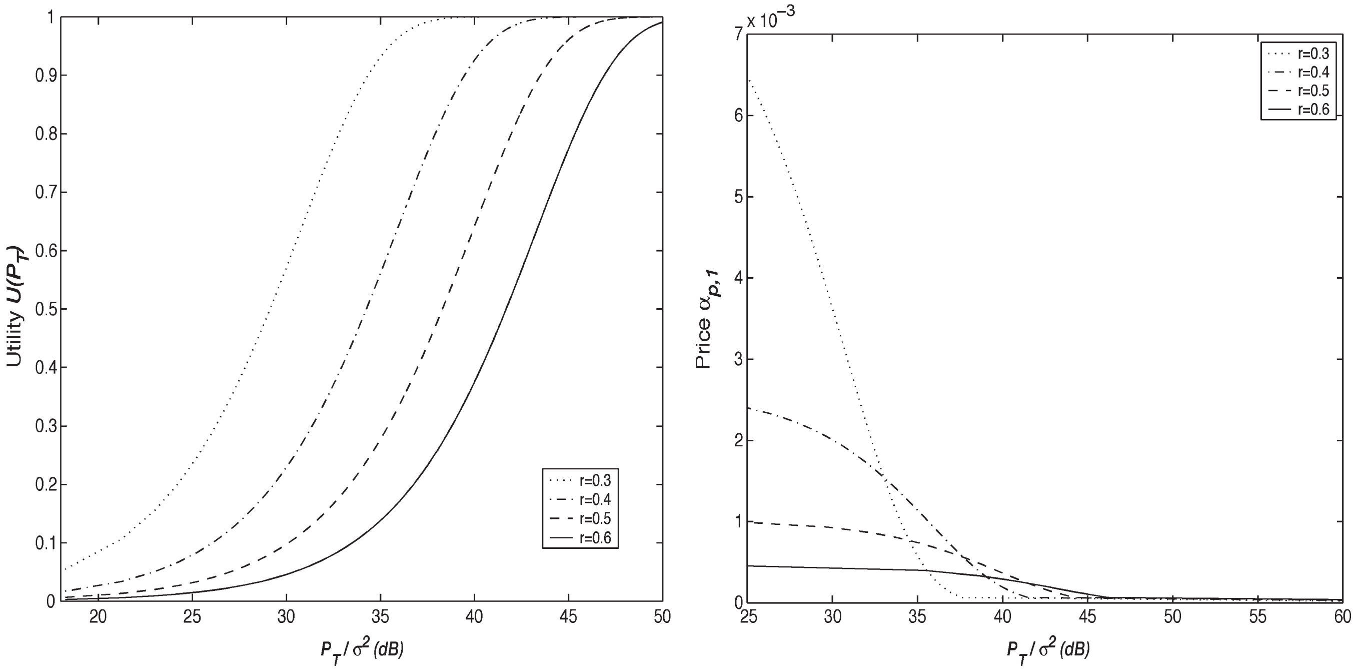
where $P_{\mathrm{T},i}(r)$ is the transmitted power from base station i designated for the user at distance r, $h(r)$ is the attenuation, or path loss function, σ^2 is the noise level, $\overline{P}_{\mathrm{tot},j}$ is the total power per code transmitted from cell $j \neq i$, defined as

$$\overline{P}_{\mathrm{tot},j} = \frac{K}{M(1-d_0)} \int_{d_0}^{1} P_{\mathrm{T},j}(r')dr' \tag{2}$$

and $\overline{P}_{\mathrm{tot},j} h(2-r)$ is the interference power. Here, we assume that the processing gain is equal to the number of codes M. As r increases, both the attenuation of the desired signal and the interference increase, so that more transmitted power is required to achieve a target SINR.

We assume that each user is assigned a utility function $U(\cdot)$, which is an increasing concave function of the received SINR [13], i.e., $U'(\xi) > 0$ and $U''(\xi) < 0$ for all $\xi \geq 0$. This corresponds to the situation in which QoS is measured by data rate, and the data rate is proportional to SINR. We consider two scenarios, which lead to different interpretations for the assigned utility functions. In the first scenario, the utility functions are assigned by the service provider in order to capture a desired tradeoff between efficiency and fairness. Since the service provider knows the utility function for each active user, it can allocate resources to maximize total (sum) utility. Different utility functions might correspond to different classes of service. In the second scenario, the utility function for each user is private information and reflects the amount the user is

[1]A direct application of this 1-D model is specific to base stations along a highway, where interference from base stations farther than the nearest interfering base station is ignored.



(a) (b)

Fig. 2. (a)Utility versus transmitted power for users at different distances. (b) Demand function for users at different distances.

willing to pay for a given data rate. In that case, the service provider can allocate resources to maximize its revenue.

In what follows, we make the simplifying assumption that all users are assigned the same utility function $U(\cdot)$. (In the first scenario previously mentioned, this corresponds to all users being in the same priority class.) This is for analytical convenience. Many of the following results can be easily extended to the situation in which users are assigned different utility functions.

Because of channel variations, which appear in the SINR expression (1), different users derive different utilities for the same transmitted power. This is illustrated in Fig. 2(a), which shows utility as a function of transmitted power for users at different locations in cell 1. For this example, the utility function is

$$U_{\exp}(\xi) = u_0\left[1 - \exp\left(-\frac{\xi}{\lambda}\right)\right] \quad (3)$$

where $u_0 = 1$, $\lambda = 13$, the attenuation $h(r) = (0.1/r)^4$, and $\overline{P}_{\text{tot},2}/\sigma^2 = 40$ dB. For the same transmitted power, users closer to the base station receive a higher SINR, and hence, more utility. The total utility per code for cell i is

$$U_{\text{tot},i} = \frac{K}{M(1-d_0)} \int_{d_0}^{1} U\left[\xi_i(r)\right] dr \quad (4)$$

where $\xi_i(r)$ is given by (1).

III. Utility Maximization

A power allocation across users is defined by the transmitted power functions $P_{\text{T},i}(r)$, $d_0 \le r \le 1$, $i = 1, 2$. Our objective is to find a power allocation that maximizes the total utility per code over the two cells, subject to the power constraints in each cell. That is, we wish to

$$\text{Problem U1}: \max_{P_{\text{T},1}(r),\, P_{\text{T},2}(r)} U_{\text{tot}} = U_{\text{tot},1} + U_{\text{tot},2} \quad (5)$$

$$\text{subject to}: \ \overline{P}_{\text{tot},i} \le \mathcal{P}, \quad i = 1, 2 \quad (6)$$

where $\mathcal{P}$ is the total available power. When the power constraint in cell i is satisfied with equality, we call cell i power limited. Here, we implicitly assume that M is large enough so that the load $K/M \le 1$, which guarantees that the K users can be assigned orthogonal codes. A code constraint could also be imposed, but is omitted to simplify the following discussion.

The maximum utility can be achieved via the following pricing scheme:

1) Base station i announces a price per unit transmitted power, $\alpha_{p,i}$, $i = 1, 2$.
2) Each user in cell i requests the transmitted power, which maximizes individual surplus (utility minus cost), $U(\xi) - \alpha_{p,i} P_{\text{T},i}(r)$.

Given the prices $\alpha_{p,i}$, $i = 1, 2$, the SINR allocation, which determines the power allocation, is given by

$$U'\left[(\xi_i^*(r)\right] = \alpha_{p,i} A_i(r) \quad \text{or} \quad \xi_i^*(r) = V^{-1}[\alpha_{p,i} A_i(r)] \quad (7)$$

where $A_i(r)$ is defined in (1) and $V(\xi) = U'(\xi)$.

Theorem 1: There exist prices $\alpha_{p,i}$, $i = 1, 2$, for which the power allocation produced by the preceding pricing scheme achieves the maximum two-cell utility.

Proof: The Lagrangian for the utility maximization problem is

$$L\left[P_{\text{T},1}(r), P_{\text{T},2}(r); \mu_{p,1}, \mu_{p,2}\right] = U_{\text{tot},1} + U_{\text{tot},2} + \mu_{p,1}(\mathcal{P} - \overline{P}_{\text{tot},1}) + \mu_{p,2}(\mathcal{P} - \overline{P}_{\text{tot},2}) \quad (8)$$

where $\mu_{p,1}$ and $\mu_{p,2}$ are the Lagrange multipliers. Setting the variation of L with respect to $P_{\mathrm{T},i}(r)$ equal to zero gives the optimality condition

$$\begin{aligned}\mu_{p,i} &= \frac{U'(\xi)}{A_i(r)} + \frac{\partial U_{\mathrm{tot},j}}{\partial \overline{P}_{\mathrm{tot},i}}, \quad \text{if } P_{\mathrm{T},i}(r) > 0 \\ &> \frac{U'(\xi)}{A_i(r)} + \frac{\partial U_{\mathrm{tot},j}}{\partial \overline{P}_{\mathrm{tot},i}}, \quad \text{if } P_{\mathrm{T},i}(r) = 0 \end{aligned} \tag{9}$$

where $A_i(r)$ is given by (1). Furthermore, $\mu_{p,i} > 0$ if $\mathcal{P} = \overline{P}_{\mathrm{tot},i}$, and $\mu_{p,i} = 0$ if $\overline{P}_{\mathrm{tot},i} < \mathcal{P}$. Comparing with the preceding pricing scheme and (7), the optimal price is therefore $\alpha_{p,i} = \mu_{p,i} - \partial U_{\mathrm{tot},j}/\partial \overline{P}_{\mathrm{tot},i}$, where $\partial U_{\mathrm{tot},j}/\partial \overline{P}_{\mathrm{tot},i}$ can be interpreted as an externality price, which cell i pays to account for the interference caused to cell j. ■

The utility maximization problem is therefore equivalent to finding the prices $\alpha_{p,i}$, $i = 1, 2$, which maximize total utility subject to the power constraints

$$\text{Problem U2}: \max_{\alpha_{p,1},\alpha_{p,2}} U_{\mathrm{tot}} = U_{\mathrm{tot},1} + U_{\mathrm{tot},2} \tag{10}$$

$$\text{subject to}: \ \overline{P}_{\mathrm{tot},i} \le \mathcal{P}, \quad i = 1, 2. \tag{11}$$

We will see that the optimal prices may not be the same, even for identical cells. We remark that for a single cell with fixed interference, Theorem 1 holds where the price α_p, which maximizes the utility in that cell subject to the power constraint is unique.

From (7), the demand for transmitted power at distance r as a function of price is

$$D_P(r; \alpha_{p,i}) = \begin{cases} A_i(r)V^{-1}\left[A_i(r)\alpha_{p,i}\right], & \text{if surplus} \ge 0 \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

and

$$\begin{aligned}\overline{P}_{\mathrm{tot},i} &= \frac{K}{M(1-d_0)} \int_{d_0}^{1} D_P(r; \alpha_{p,i}) dr \\ &= \frac{K}{M(1-d_0)} \int_{d_0}^{1} A_i(r)\xi_i^*(r) dr \end{aligned} \tag{13}$$

$$U_{\mathrm{tot}} = \frac{K}{M(1-d_0)} \int_{d_0}^{1} \left[U\left(\xi_1^*(r)\right) + U\left(\xi_2^*(r)\right)\right] dr \quad i = 1, 2. \tag{14}$$

For a user at fixed distance r, the demand for transmitted power decreases with price $\alpha_{p,i}$. For any given price, as r increases (i.e., a user moves away from the base station), $A_i(r)$, the received interference, increases, and the SINR $V^{-1}[A_i(r)\alpha_{p,i}]$ decreases. Hence, from (12), the transmitted power may decrease as r increases.

Fig. 2(b) shows the demand function corresponding to the exponential utility function (3) for users at different locations with the same parameters as in Fig. 2(a). The demand depends on the location r and increases as the price $\alpha_{p,1}$ decreases.

We emphasize that when each user in cell i requests an amount of power according to (7), the power distribution $P_{\mathrm{T},i}(r)$ depends only on $\overline{P}_{\mathrm{tot},i}$ and $\overline{P}_{\mathrm{tot},j}$, $j \ne i$. That is, it does not depend on how the power is distributed over users in cell j. Hence, $(\overline{P}_{\mathrm{tot},1}, \overline{P}_{\mathrm{tot},1})$ also specifies the total utility $(U_{\mathrm{tot},1}, U_{\mathrm{tot},2})$. Furthermore, for fixed $\overline{P}_{\mathrm{tot},j}$ both $\overline{P}_{\mathrm{tot},i}$ and $U_{\mathrm{tot},i}$ are determined by the price $\alpha_{p,i}$.

In the Appendix, we compute the derivatives $\partial U_{\mathrm{tot},i}/\partial \overline{P}_{\mathrm{tot},j}$ and $\partial \alpha_{p,i}/\partial \overline{P}_{\mathrm{tot},j}$ (for $j = i$ and $j \ne i$), which are needed to prove many of the following theorems. This computation shows that $\partial \alpha_{p,i}/\partial \overline{P}_{\mathrm{tot},i} < 0$ and $\partial \xi_i^*(r)/\partial \overline{P}_{\mathrm{tot},i} > 0$. That is, as the total transmitted power in cell i increases, the price in cell i decreases, and each user requests more power, which increases the SINR. Furthermore, $\partial U_{\mathrm{tot},i}/\partial \overline{P}_{\mathrm{tot},i} > 0$ and $\partial U_{\mathrm{tot},i}/\partial \overline{P}_{\mathrm{tot},j} < 0$, $j \ne i$, i.e., as $\overline{P}_{\mathrm{tot},i}$ increases, the total utility in cell i increases, and the total utility in cell j decreases due to the increase in interference.

IV. Properties of the Optimal Allocation

In this section, we state some properties of the optimal two-cell power allocation. We first consider the case where the base stations do not coordinate their power allocations. That is, each base station attempts to maximize its own utility in the presence of fixed interference from the other cell. This can be viewed as a power control game with noncooperative players (the base stations) [13]. To specify the corresponding NE, we start by setting the price $\alpha_{p,i}$ in cell i to maximize the utility $U_{\mathrm{tot},i}$ subject to the power constraint $\overline{P}_{\mathrm{tot},i} \le \mathcal{P}$.

With fixed interference, the received utility at any r increases with the transmitted power. Hence, the total utility in the cell is maximized by using all available power. That is

$$\frac{\partial U_{\mathrm{tot},1}}{\partial \overline{P}_{\mathrm{tot},1}} = \frac{K}{M(1-d_0)} \int_{d_0}^{1} U'\left[\xi_1^*(r)\right] \frac{\partial \xi_1^*(r)}{\partial \overline{P}_{\mathrm{tot},1}} dr > 0 \tag{15}$$

since $U'[\xi_1^*(r)] > 0$ and $\partial \xi_1^*(r)/\partial \overline{P}_{\mathrm{tot},1} > 0$. Therefore, $U_{\mathrm{tot},1}$ is maximized when cell 1 is power limited, i.e., $\overline{P}_{\mathrm{tot},1} = \mathcal{P}$.

Theorem 2: Any solution to the two-cell utility maximization problem U1 has the property that at least one cell is power limited.

If the two cells are constrained to be symmetric, namely, $\overline{P}_{\mathrm{tot},1} = \overline{P}_{\mathrm{tot},2}$ and $\alpha_{p,1} = \alpha_{p,2}$, then (34) reduces to $\partial U_{\mathrm{tot}}/\partial \overline{P}_{\mathrm{tot},i} > 0$. In that case, to maximize total utility, both cells must be power limited.

We are interested in determining when the NE is globally optimal, in the sense of maximizing $U_{\mathrm{tot}} = U_{\mathrm{tot},1} + U_{\mathrm{tot},2}$.

Theorem 3: There exists a $P_{L,U}$ such that for any power constraint $\mathcal{P} \le P_{L,U}$, the NE achieves the maximum total utility.

Since from Theorem 2, at least one cell is power limited at the global optimum, finding the two-cell optimal power allocation can be achieved by fixing $\overline{P}_{\mathrm{tot},2} = \mathcal{P}$ and varying the total transmitted power in cell 1. Fig. 3(a) shows the total utility U_{tot} versus $\overline{P}_{\mathrm{tot},1}/\sigma^2$ for small $\mathcal{P}$ with $\overline{P}_{\mathrm{tot},2} = \mathcal{P}$ and the

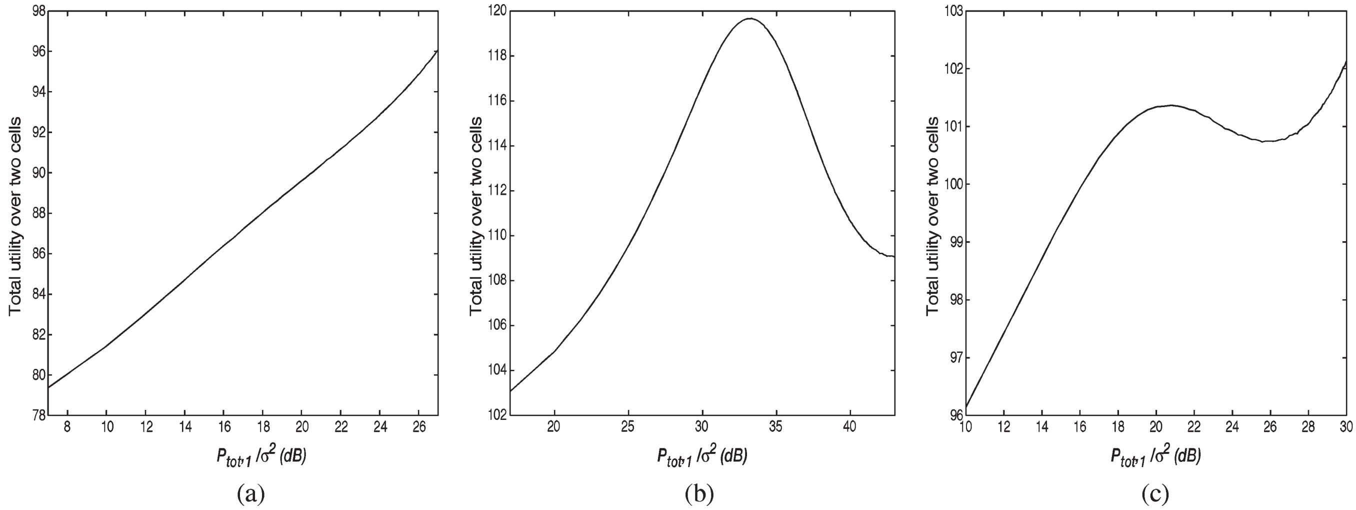


Fig. 3. Total utility versus $\overline{P}_{\mathrm{tot},1}/\sigma^2$. (a) Small $\mathcal{P}$ ($\mathcal{P}/\sigma^2 = 500$ or 27 dB). (b) Large $\mathcal{P}$ ($\mathcal{P}/\sigma^2 = 43$ dB). (c) $P_{L,U} < \mathcal{P} < P_{H,U}$ ($\mathcal{P}/\sigma^2 = 30$ dB). Parameters are $u_0 = 100$, $\lambda = 13$, $K/M = 1$, and the path loss function $h(r) = (0.1/r)^2$.

exponential utility function (3). For the parameters shown, the total utility monotonically increases with $\overline{P}_{\mathrm{tot},1}$, so that the optimal allocation occurs at $\overline{P}_{\mathrm{tot},1} = \overline{P}_{\mathrm{tot},2} = \mathcal{P}$, which corresponds to the NE.

A complete characterization of conditions for which the NE is optimal or suboptimal appears to be difficult. However, we can gain insight by comparing the NE with the case $\overline{P}_{\mathrm{tot},1} = \mathcal{P}$ and $\overline{P}_{\mathrm{tot},2} = 0$. We will refer to the corresponding optimal power allocation as the one-cell solution, and the corresponding total utility as U_{OC}. The total utility corresponding to the NE is U_{NE}. In what follows, we assume that the utility function $U(\cdot)$ has the additional property that $\lim_{\xi\to\infty} U'(\xi) = 0$.

Theorem 4: There exists a $P_{H,U}$ such that if $\mathcal{P} > P_{H,U}$ and $U(\infty) > 2U[\xi_\infty(d_0)]$, then $U_{\mathrm{OC}} > U_{\mathrm{NE}}$.

Theorem 4 states that when the total power constraint in each cell becomes large, the NE does not achieve the maximum utility. Hence, achieving the maximum utility requires the two cells to coordinate, i.e., one cell must reduce its power. Note that the same result applies when the maximum power is fixed and we vary the cell radius. That is, as the cells move closer together, the total utility associated with the NE eventually decreases due to interference, and the total utility can be increased by reducing the power in one cell.

The condition in Theorem 4 is true for any unbounded utility function, that is, when $U(\xi) \to \infty$ as $\xi \to \infty$. Examples of unbounded increasing concave utility functions are

$$U_{\mathrm{pl}}(\xi) = u_0\xi^\beta \text{ where } \beta < 1 \quad \text{and} \quad U_{\log}(\xi) = u_0 \log(1+\xi) \tag{16}$$

where u_0 is a constant. For the exponential utility function defined in (3), the condition in Theorem 4 gives the following corollary.

Corollary 1: For the exponential utility function, there exist $P_{H,U}$ and $\lambda_{H,U}$ such that if $\mathcal{P} > P_{H,U}$ and $\lambda > \lambda_{H,U}$, then $U_{\mathrm{OC}} > U_{\mathrm{NE}}$.

The thresholds $P_{H,U}$ and $\lambda_{H,U}$ depend in a complicated way on the path loss function $h(r)$, the noise variance σ^2, and the load. Large λ in (3) implies that a relatively large increase in transmitted power is needed to increase the utility significantly. Furthermore, large $\mathcal{P}$ implies high interference. As $\mathcal{P}$ increases, the marginal increase in utility in cell i due to increasing $\overline{P}_{\mathrm{tot},i}$ is therefore outweighed by the marginal decrease in utility in the other cell due to the increase in interference.

Fig. 3(b) shows the total utility U_{tot} versus $\overline{P}_{\mathrm{tot},1}/\sigma^2$ for large $\mathcal{P}$ when cell 2 is power limited ($\overline{P}_{\mathrm{tot},2} = \mathcal{P}$). For the parameters shown, the total utility has a unique maximum at $\overline{P}_{\mathrm{tot},1}/\sigma^2 < \mathcal{P}/\sigma^2$. For this example, the total utility at the NE ($\overline{P}_{\mathrm{tot},1} = \overline{P}_{\mathrm{tot},2} = \mathcal{P}$) is about 8% less than the global maximum.

If $P_{L,U} < \mathcal{P} < P_{H,U}$, then it is not easy to determine if the NE is globally optimal. An example is shown in Fig. 3(c). For the parameters shown, there are two stationary points in the U_{tot} versus $\overline{P}_{\mathrm{tot},1}/\sigma^2$ curve, one corresponding to a local maximum, and the other corresponding to a local minimum. The global maximum occurs at the NE.

A. Unequal Loads

When the two cells have different loads, distributed optimization again leads to an NE with power-limited cells. In contrast to the equal-load case, the two cells are not identical at the NE. Let K_i be the number of users in cell i, and $L_i = K_i/M$ be the corresponding load.

Theorem 5: If $L_1 > L_2$, then $U_{\mathrm{tot},1} > U_{\mathrm{tot},2}$ at the NE.

When $\mathcal{P}$ is small, whether or not the NE is optimal depends on the difference between L_1 and L_2. Namely, the NE is optimal when $|L_1 - L_2|$ is sufficiently small. As $|L_i - L_j|$ increases, the total utility over the two cells becomes dominated by $U_{\mathrm{tot},i}$, and the power pair $(\overline{P}_{\mathrm{tot},1}, \overline{P}_{\mathrm{tot},2})$, which maximizes the total utility moves away from the NE ($\overline{P}_{\mathrm{tot},1} = \overline{P}_{\mathrm{tot},2} = \mathcal{P}$). Namely, $\overline{P}_{\mathrm{tot},i} = \mathcal{P}$ and $\overline{P}_{\mathrm{tot},j} < \mathcal{P}$.

For large $\mathcal{P}$, there exists a unique optimum, instead of the two optima for the equal-load system. At the optimum, the cell with the larger load is power limited, and the cell with the smaller load is not power limited. The NE is again suboptimal.

V. REVENUE MAXIMIZATION

We now consider the power allocation across the two cells that results from maximizing revenue. That is, the objective is to set the prices $\alpha_{p,1}$ and $\alpha_{p,2}$ to maximize the total revenue per code, $R_{\rm tot}$, subject to the power constraints in each cell, i.e.,

$$\text{Problem R}: \max_{\alpha_{p,1},\alpha_{p,2}} R_{\rm tot} = R_{\rm tot,1} + R_{\rm tot,2} \tag{17}$$

$$\text{subject to}: \overline{P}_{{\rm tot},i} \le \mathcal{P}, \quad i = 1,2 \tag{18}$$

where $R_{{\rm tot},i} = \alpha_{p,i}\overline{P}_{{\rm tot},i}$ is the revenue in cell i.

A. Nash Equilibrium

In analogy with the NE associated with utility maximization, we can again define an NE for the power control game in which each base station allocates power to maximize revenue. We start by maximizing revenue in cell i in the presence of fixed interference from cell j, specified by $\overline{P}_{{\rm tot},j}$. We have

$$\frac{\partial R_{{\rm tot},i}}{\partial \overline{P}_{{\rm tot},i}} = \overline{P}_{{\rm tot},i}\frac{\partial \alpha_{p,i}}{\partial \overline{P}_{{\rm tot},i}} + \alpha_{p,i} = \alpha_{p,i}\left(\frac{1}{e(\overline{P}_{{\rm tot},i},\alpha_{p,i})} + 1\right) \tag{19}$$

where

$$e(\overline{P}_{{\rm tot},i},\alpha_{p,i}) = \frac{\text{Fractional change in } \overline{P}_{{\rm tot},i}}{\text{Fractional change in } \alpha_{p,i}} = \frac{\alpha_{p,i}}{\overline{P}_{{\rm tot},i}}\frac{\partial \overline{P}_{{\rm tot},i}}{\partial \alpha_{p,i}} \tag{20}$$

is the price elasticity of demand for power [13]. Note that $\partial\overline{P}_{{\rm tot},i}/\partial\alpha_{p,i} < 0$, hence, $e(\overline{P}_{{\rm tot},i},\alpha_{p,i}) < 0$. The demand is elastic (inelastic) when $e(\overline{P}_{{\rm tot},i},\alpha_{p,i}) < -1 (e(\overline{P}_{{\rm tot},i}, \alpha_{p,i}) > -1)$. Elastic demand indicates that the demand is price sensitive, and $\partial R_{{\rm tot},i}/\partial\overline{P}_{{\rm tot},i} > 0$. The reverse is true for inelastic demand, and the revenue is maximized when $e(\overline{P}_{{\rm tot},i},\alpha_{p,i}) = -1$.

In contrast with utility maximization, maximizing revenue in the presence of fixed interference does not always lead to a power-limited cell. Characterizing the optimal power allocation for general utility functions appears to be difficult, so that in what follows, we restrict our attention to the exponential, power-law, and log utility functions in (3) and (16).

Lemma 1: For the exponential utility function, the one-cell revenue is maximized by choosing $\overline{P}_{\rm tot,1} = \min(\overline{P}^*, \mathcal{P})$, where $\overline{P}^* = (K/M(1-d_0))\int_r \lambda A_1(r)dr$. For the power-law and log utility functions, revenue is maximized when $\overline{P}_{\rm tot,1} = \mathcal{P}$.

For the power-law utility function, the maximum revenue is

$$R_{{\rm tot},i} = \frac{Ku_0\beta}{M(1-d_0)}\int_{d_0}^{1}\left(\frac{\alpha_{p,i}A_i(r)}{u_0\beta}\right)^{\frac{\beta}{\beta-1}} dr. \tag{21}$$

As the price $\alpha_{p,i} \to 0$, the demand for transmitted power goes to infinity, and so does the revenue in cell i. Therefore,

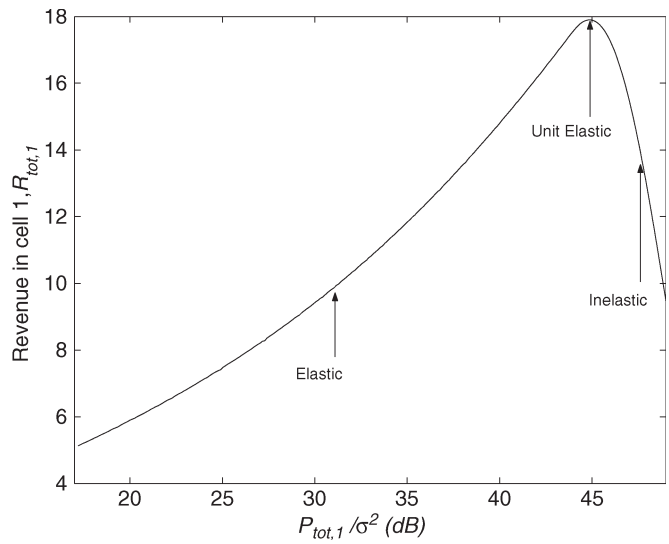


Fig. 4. Single-cell revenue $R_{\rm tot,1}$ versus $\overline{P}_{\rm tot,1}/\sigma^2$. The path loss is $h(r) = (d_0/r)^4$, and the parameters for the exponential utility function are $u_0 = 100$ and $\lambda = 13$.

$R_{{\rm tot},i} \to \infty$ as the available power $\mathcal{P} \to \infty$. In contrast, for the log utility function, the maximum revenue is

$$R_{{\rm tot},i} = \frac{Ku_0}{M} - \frac{K}{M(1-d_0)}\int_{d_0}^{1}\alpha_{p,i}A_i(r)dr \le \frac{Ku_0}{M}. \tag{22}$$

As $\alpha_{p,i} \to 0$, or equivalently, $\mathcal{P} \to \infty$, $R_{{\rm tot},i} \to Ku_0/M$ (a constant).

Fig. 4 shows $R_{\rm tot,1}$ versus $\overline{P}_{\rm tot,1}/\sigma^2$ for the exponential utility function with fixed interference from cell 2. There is a unique maximum at $\overline{P}_{\rm tot,1}/\sigma^2 = 44.8$ dB, which corresponds to the optimal power allocation, unless $\mathcal{P}/\sigma^2 < 44.8$ dB, in which case the cell is power limited.

Fig. 5(a) and (b) shows received SINR (or equivalently, data rate) and transmitted power $P_{\rm T,1}(r)$ versus distance for a single cell with fixed interference. Curves are shown for the path loss function $h(r) = (d_0/r)^\alpha$ with different values of α. In each case, the price is set to maximize the revenue. Fig. 5(a) shows that the data rate decreases monotonically as the users move away from the base station. Fig. 5(b) shows that the maximum transmitted power is not requested by users at the cell boundary. Namely, as users move away from the base station, the data rate decreases moderately, and the transmitted power increases with r. As the users approach the cell boundary, the data rate decreases sharply, and the transmitted power decreases with r.

Lemma 1 allows us to characterize the NE for the different utility functions considered. Specifically, we define a reaction curve for cell i as the optimal (revenue maximizing) $\overline{P}_{{\rm tot},i}$ as a function of $\overline{P}_{{\rm tot},j}$, $j \ne i$. This is illustrated in Fig. 6 for the exponential utility function. According to (51), the optimal power in cell i, $\overline{P}^*$, is a linear function of $\overline{P}_{{\rm tot},j}$, $j \ne i$, for $\overline{P}^* < \mathcal{P}$, and is constant for $\overline{P}^* > \mathcal{P}$. The NE is the intersection of the two reaction curves for cells 1 and 2. Fig. 6(a) and (b) shows reaction curves for two different sets of parameters. In Fig. 6(a), the NE is at the corner point B, i.e., $\overline{P}_{\rm tot,1} = \overline{P}_{\rm tot,2} = \mathcal{P}$. In Fig. 6(b), the NE is at the interior point A.

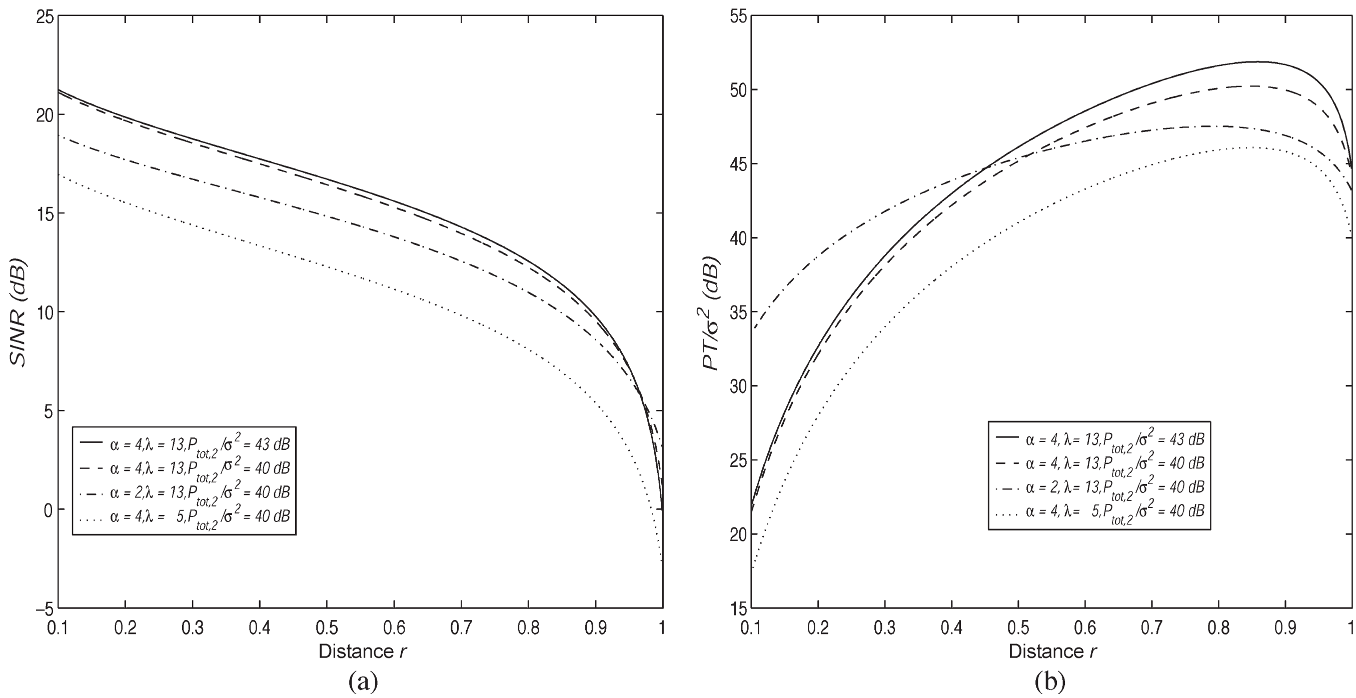


Fig. 5. (a) SINR versus distance from the base station. (b) Transmitted power versus distance.

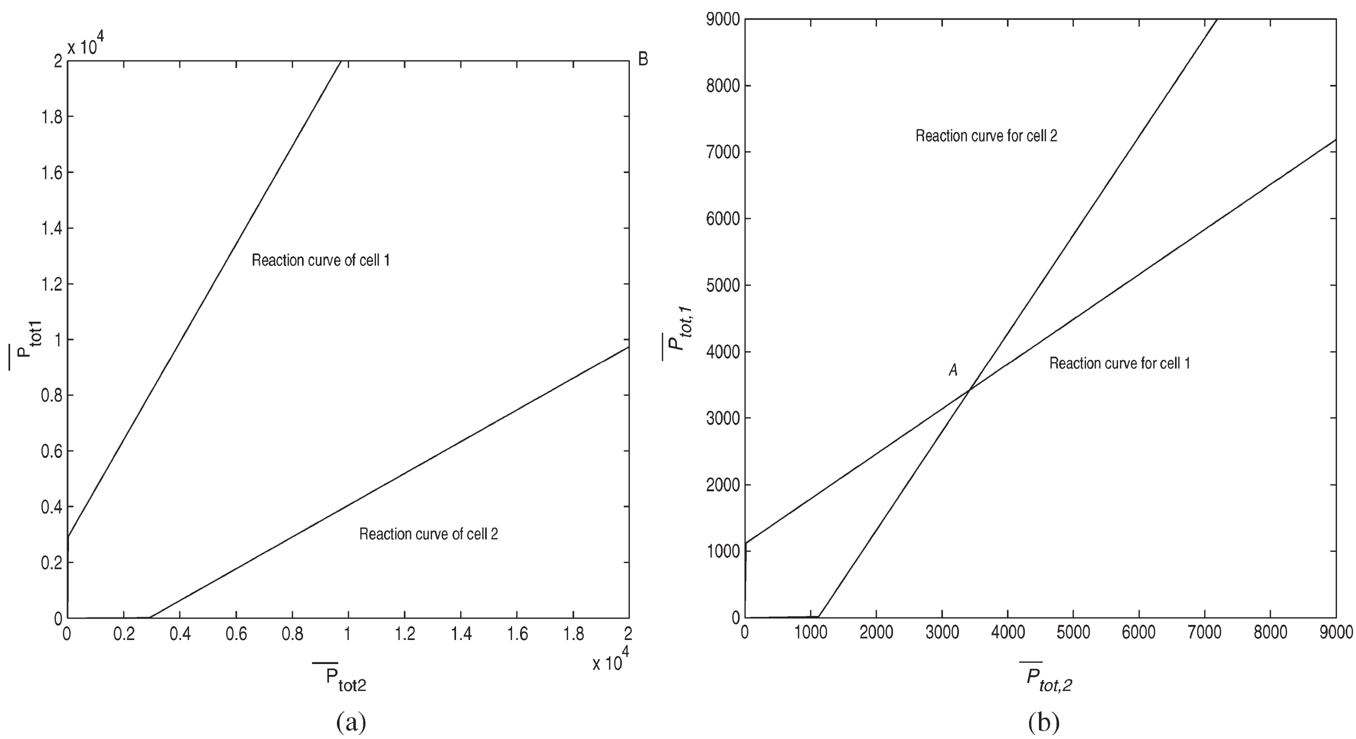


Fig. 6. Reaction curves for single-cell revenue maximization. Parameters used to generate (a) are $K/M = 1$, $h(r) = (0.1/r)^4$, $\mathcal{P}/\sigma^2 = 53$ dB, and $\lambda = 13$. For (b) $\mathcal{P}/\sigma^2 = 49.5$ dB and $\lambda = 5$.

The power allocations at the NE are summarized by the following theorem.

Theorem 6: For the power-law and log utility functions, the NE is at the corner point. For the exponential utility function, the NE is at the corner point if $(K/M(1-d_0)) \int_{d_0}^{r^*} (\lambda h(2-r)/h(r)) dr \geq 1$. Otherwise, there exists a P_0 such that for $\mathcal{P} < P_0$, the NE is at the corner point, and for $\mathcal{P} > P_0$, the NE is at an interior point $\overline{P}_{\text{tot},2} = \overline{P}_{\text{tot},2} < \mathcal{P}$.

Proof: This follows directly from Lemma 1. For the exponential utility function, if $K \int_{d_0}^{1} \lambda h(2-r)/h(r) dr / M(1-d_0) \geq 1$, then in the absence of power constraints, the reaction curves do not intersect, as shown in Fig. 6(a). If $K \int_{d_0}^{1} \lambda h$

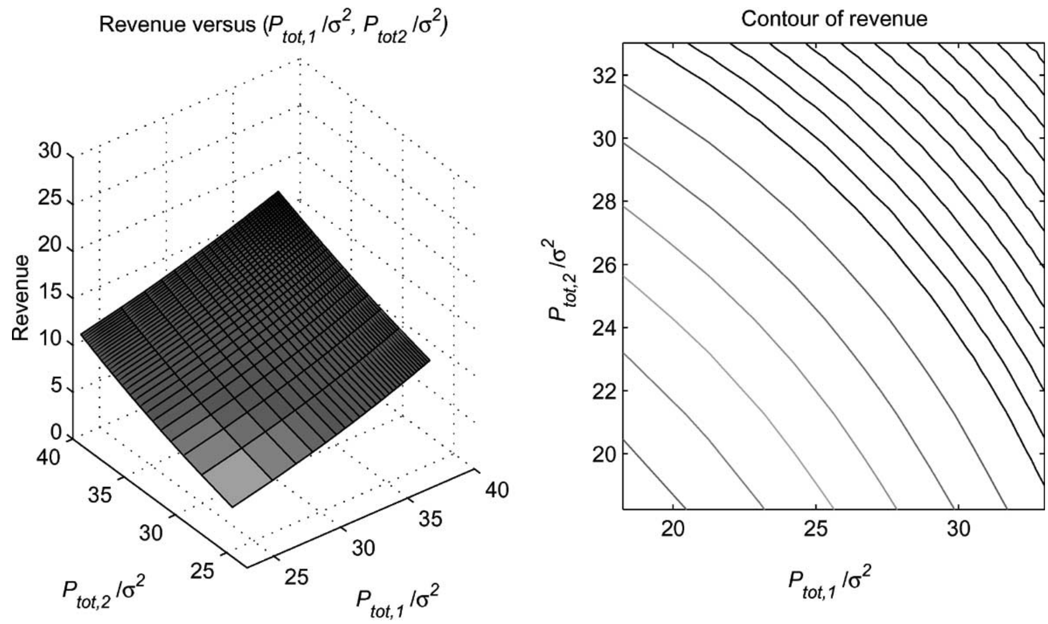


Fig. 7. Total revenue versus ($\overline{P}_{\text{tot},1}/\sigma^2$, $\overline{P}_{\text{tot},2}/\sigma^2$), and revenue contours for small $\mathcal{P}$. Parameters are $u_0 = 100$, $\lambda = 5$, $K/M = 1$, $\mathcal{P}/\sigma^2 = 33$ dB, and $h(r) = (0.1/r)^4$.

$(2-r)/h(r)dr/M(1-d_0) < 1$, then the reactions curves can intersect, as shown in Fig. 6(b), where the value of $\overline{P}_{\text{tot},i}$ at the intersection point corresponds to P_0. ■

B. Two-Cell Power Allocation

We now consider revenue maximization over the two cells. The following theorem is analogous to Theorem 3.

Theorem 7: There exists a $P_{L,R}$ such that for any $\mathcal{P} \leq P_{L,R}$ the total revenue over the two cells is maximized when $\overline{P}_{\text{tot},1} = \overline{P}_{\text{tot},2} = \mathcal{P}$.

Fig. 7 shows the total revenue over two cells versus ($\overline{P}_{\text{tot},1}/\sigma^2$, $\overline{P}_{\text{tot},2}/\sigma^2$), and the corresponding revenue contours when the total available power $\mathcal{P}$ is small. The revenue contours show that the total revenue increases with $\overline{P}_{\text{tot},i}/\sigma^2$ for any fixed $\overline{P}_{\text{tot},j}/\sigma^2$. The maximum revenue occurs at the NE, $\overline{P}_{\text{tot},1} = \overline{P}_{\text{tot},2} = \mathcal{P}$.

When the total available power $\mathcal{P}$ is large, the corner point $\overline{P}_{\text{tot},1} = \overline{P}_{\text{tot},2} = \mathcal{P}$ is often not optimal, and the exact solution depends on the specific utility function. Our results are summarized by the following three theorems.

Theorem 8: For the power-law utility function, the power allocation that maximizes utility also maximizes revenue.

Proof: For any $\alpha_{p,i}$, the received SINR for users at distance r is given in (7), and the utility per code in cell i is

$$\begin{aligned} U_{\text{tot},i} &= \frac{K}{M(1-d_0)} \int_{d_0}^{1} U\left[\xi_i^*(r)\right] dr \\ &= \frac{K}{M(1-d_0)} \int_{d_0}^{1} u_0 \left(\frac{\alpha_{p,i} A_i(r)}{u_0 \beta}\right)^{\frac{\beta}{\beta-1}} dr \\ &= \frac{R_{\text{tot},i}}{\beta} \end{aligned} \tag{23}$$

where $R_{\text{tot},i}$ is given by (21). ■

Hence, Theorem 4 applies in this case, and for large $\mathcal{P}$, the NE is not optimal. Theorem 8 presents a special case in which utility maximization is equivalent to revenue maximization. In general, these two optimization problems give different power allocations.

Theorem 9: For the log utility function, the power allocation that maximizes revenue has at least one power-limited cell. If $(K/M(1-d_0)) \int_{d_0}^{1} (h(2-r)/h(r))dr > 1$, then there exists a $P_{H,R}$ such that if $\mathcal{P} > P_{H,R}$, then the optimal power allocation has only one power-limited cell.

Although maximizing either utility or revenue with the log utility function gives one power-limited cell (according to Theorems 4 and 9), the power allocation that maximizes total revenue is generally different from the allocation that maximizes total utility.

Theorem 10: For the exponential utility function, there exist $P_{H,R}$ and $\lambda_{L,R}$ such that if $\mathcal{P} > P_{H,R}$ and $\lambda < \lambda_{L,R}$, then the maximum revenue does not occur at the NE.

Comparing with Corollary 1, which states the analogous result for utility maximization, here, the condition is that λ must be sufficiently small, whereas in Corollary 1, λ must be sufficiently large. That is, small λ implies that the requests for power, which maximize surplus are relatively small. Furthermore, for large $\mathcal{P}$, it can be shown that the price decreases nearly exponentially as $\mathcal{P}$ increases. Consequently, the marginal increase in revenue (i.e., price times total power) due to increasing $\mathcal{P}$ is outweighed by the marginal decrease in revenue due to the decrease in the price. As in Corollary 1, the thresholds $P_{H,R}$ and $\lambda_{L,R}$ depend on the path loss function $h(r)$, noise variance σ^2, and load.

Fig. 8 shows an example of total revenue versus ($\overline{P}_{\text{tot},1}/\sigma^2$, $\overline{P}_{\text{tot},2}/\sigma^2$) when $\mathcal{P}$ is large. In contrast to the solutions for the power-law and log utility functions, the maximum revenue occurs at an interior point of the constraint region, i.e., $\overline{P}_{\text{tot},1} < \mathcal{P}$ and $\overline{P}_{\text{tot},2} < \mathcal{P}$. In this example, the global

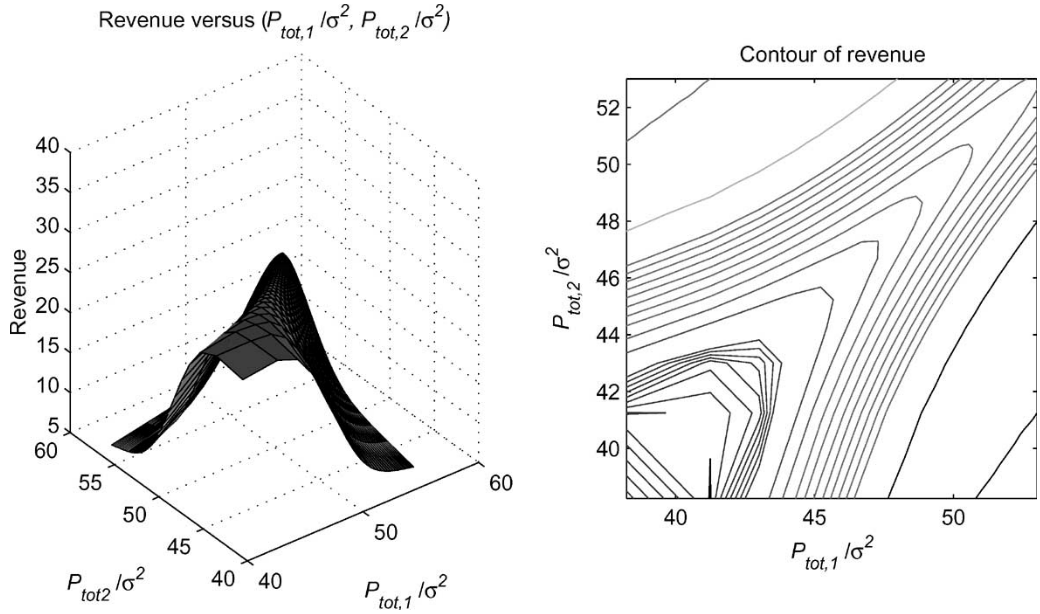


Fig. 8. Revenue versus ($\overline{P}_{\rm tot,1}/\sigma^2$, $\overline{P}_{\rm tot,2}/\sigma^2$) and revenue contours for large $\mathcal{P}$ ($\mathcal{P}/\sigma^2 = 53$ dB). The remaining parameters are the same as in Fig. 7.

optimum does not coincide with either the corner point or the NE shown in Fig. 6(b), which is at $\overline{P}_{\rm tot,1}/\sigma^2 = \overline{P}_{\rm tot,2}/\sigma^2 = 45.26$ dB.

VI. Conclusion

We have studied forward-link power allocation across users in two 1-D, adjacent CDMA cells. Utility maximization can be achieved by a pricing scheme in which the base station announces a price per unit transmitted power, and each user requests the transmitted power, which maximizes surplus. For data traffic, which is represented by an increasing concave utility function, the prices that maximize utility are generally different from the prices that maximize revenue. In either case, the nature of the optimal power allocation depends on the total power constraint. When the available transmitted power falls below a threshold, the two cells use up all of the power, which corresponds to uncoordinated, or distributed, optimization NE. When the available power becomes large, in general, the two cells must coordinate to maximize total utility. Specifically, one cell must reduce the total transmitted power below the power constraint. This can be accomplished by raising the price per unit power in one cell relative to the other cell. When maximizing revenue, both the NE and the optimal power allocation can be in the interior of the constraint region. An example shows that these two points do not generally coincide.

Extending the analysis presented here for the 1-D model to a 2-D cellular model is generally difficult. Still, we expect that many of the results presented here apply to such models. For example, the necessity of intercell coordination to maximize total utility in a particular 2-D model is observed numerically in [16].

Extensions of the results presented here to system models with mixed voice and data traffic and stochastic arrivals and departures are presented in [14] and [15]. Other extensions and generalizations of the model presented here include nonuniform user distributions within the cells, different utility functions across users, and different types of path loss functions (e.g., random). Although an analysis of those more general models may be difficult, performance measures such as utility, revenue, and total rate can be evaluated numerically. Such studies may provide additional insight into optimal resource allocation in more practical situations.

Appendix

The proofs that follow make use of the following derivatives with respect to $\overline{P}_{\rm tot,i}$ and $\overline{P}_{\rm tot,j}$, $i \neq j$. From (7), we have

$$U''\left[\xi_i^*(r)\right]\frac{\partial \xi_i^*(r)}{\partial \overline{P}_{\rm tot,i}} = A_i(r)\frac{\partial \alpha_{p,i}}{\partial \overline{P}_{\rm tot,i}} \tag{24}$$

$$U''\left[\xi_i^*(r)\right]\frac{\partial \xi_i^*(r)}{\partial \overline{P}_{\rm tot,j}} = A_i(r)\frac{\partial \alpha_{p,i}}{\partial \overline{P}_{\rm tot,j}} + \frac{\alpha_{p,i}h(2-r)}{h(r)} \tag{25}$$

and, from (13), we have

$$1 = \frac{K}{M(1-d_0)}\int_{d_0}^{1} A_i(r)\frac{\partial \xi_i^*(r)}{\partial \overline{P}_{\rm tot,i}}dr \tag{26}$$

$$0 = \frac{K}{M(1-d_0)}\int_{d_0}^{1}\left[A_i(r)\frac{\partial \xi_i^*(r)}{\partial \overline{P}_{\rm tot,j}} + \frac{\xi_i^*(r)h(2-r)}{h(r)}\right]dr. \tag{27}$$

Since $\partial\alpha_{p,i}/\partial\overline{P}_{\text{tot},i}$ and $\partial\alpha_{p,i}/\partial\overline{P}_{\text{tot},j}$ are independent of r, from (24)–(27), we have

$$\frac{\partial\alpha_{p,i}}{\partial\overline{P}_{\text{tot},i}} = \frac{1}{\frac{K}{M(1-d_0)}\int_{d_0}^{1}\frac{[A_i(r)]^2}{U''\left[\xi_i^*(r)\right]}dr}$$

$$\frac{\partial\alpha_{p,i}}{\partial\overline{P}_{\text{tot},j}} = -\frac{\int_{d_0}^{1}\left[\xi_i^*(r)\frac{h(2-r)}{h(r)}+\frac{\alpha_{p,i}h(2-r)A_i(r)}{h(r)U''\left[\xi_i^*(r)\right]}\right]dr}{\int_{d_0}^{1}\frac{[A_i(r)]^2}{U''\left[\xi_i^*(r)\right]}dr}. \quad (28)$$

Note that (28) implies that $\partial\alpha_{p,i}/\partial\overline{P}_{\text{tot},i} < 0$, as expected. Since $U''[\xi_i^*(r)] < 0$, (24) implies $\partial\xi_i^*(r)/\partial\overline{P}_{\text{tot},i} > 0$.

From (4) and (7), we have

$$\frac{\partial U_{\text{tot},i}}{\partial\overline{P}_{\text{tot},i}} = \frac{K}{M(1-d_0)}\int_{d_0}^{1} U'\left[\xi_i^*(r)\right]\frac{\partial\xi_i^*(r)}{\partial\overline{P}_{\text{tot},i}}dr$$

$$= \frac{K}{M(1-d_0)}\int_{d_0}^{1}\alpha_{p,i}A_i(r)\frac{\partial\xi_i^*(r)}{\partial\overline{P}_{\text{tot},i}}dr \quad (29)$$

$$\frac{\partial U_{\text{tot},j}}{\partial\overline{P}_{\text{tot},i}} = \frac{K}{M(1-d_0)}\int_{d_0}^{1} U'\left[\xi_j^*(r)\right]\frac{\partial\xi_j^*(r)}{\partial\overline{P}_{\text{tot},i}}dr$$

$$= \frac{K}{M(1-d_0)}\int_{d_0}^{1}\alpha_{p,j}A_j\frac{\partial\xi_j^*(r)}{\partial\overline{P}_{\text{tot},i}}dr \quad (30)$$

and combining these with (26) and (27) gives

$$\frac{\partial U_{\text{tot}}}{\partial\overline{P}_{\text{tot},i}} = \frac{\partial U_{\text{tot},i}}{\partial\overline{P}_{\text{tot},i}} + \frac{\partial U_{\text{tot},j}}{\partial\overline{P}_{\text{tot},i}}$$

$$= \alpha_{p,i} - \alpha_{p,j}\frac{K}{M(1-d_0)}\int_{d_0}^{1}\frac{\xi_j^*(r)h(2-r)}{h(r)}dr. \quad (31)$$

A. Proof of Theorem 2

Suppose that the two-cell maximum utility is achieved at $\overline{P}_{\text{tot},1} = P_1^*$, $\overline{P}_{\text{tot},2} = P_2^*$, and neither of the cells is power limited at the maximum, that is, $P_1^* < \mathcal{P}$ and $P_2^* < \mathcal{P}$. Since the total utility U_{tot} is differentiable, we have $\partial U_{\text{tot}}/\partial\overline{P}_{\text{tot},i} = 0$ and $\partial U_{\text{tot}}/\partial\overline{P}_{\text{tot},j} = 0$ at $(\overline{P}_{\text{tot},1} = P_1^*, \overline{P}_{\text{tot},2} = P_2^*)$. From (13)

$$\overline{P}_{\text{tot},i} = \frac{K}{M(1-d_0)}\int_{d_0}^{1}\frac{\sigma^2 + \overline{P}_{\text{tot},j}h(2-r)}{h(r)}\xi_i^*(r)dr$$

$$> \frac{K}{M(1-d_0)}\int_{d_0}^{1}\frac{\overline{P}_{\text{tot},j}h(2-r)}{h(r)}\xi_i^*(r)dr \quad (32)$$

or

$$\frac{\overline{P}_{\text{tot},i}}{\overline{P}_{\text{tot},j}} > \frac{K}{M(1-d_0)}\int_{d_0}^{1}\frac{\xi_i^*(r)h(2-r)}{h(r)}dr. \quad (33)$$

From (31) and (33)

$$\frac{\partial U_{\text{tot}}}{\partial\overline{P}_{\text{tot},i}} = \alpha_{p,i} - \alpha_{p,j}\frac{K}{M(1-d_0)}\int_{d_0}^{1}\frac{\xi_j^*(r)h(2-r)}{h(r)}dr$$

$$> \alpha_{p,i} - \alpha_{p,j}\frac{\overline{P}_{\text{tot},j}}{\overline{P}_{\text{tot},i}}. \quad (34)$$

Since $\partial U_{\text{tot}}/\partial\overline{P}_{\text{tot},1} = 0$ when U_{tot} is maximized, from (34), we have $\alpha_{p,1} - \alpha_{p,2}P_2^*/P_1^* < 0$. Furthermore, $\partial U_{\text{tot}}/\partial\overline{P}_{\text{tot},2} = 0$ gives $\alpha_{p,2} - \alpha_{p,1}(P_1^*/P_2^*) < 0$, which is a contradiction, hence, at least one cell must be power limited.

B. Proofs of Theorems 3 and 7

To prove Theorem 3, it suffices to show that $\partial U_{\text{tot}}/\partial\overline{P}_{\text{tot},i} > 0$ at $(\overline{P}_{\text{tot},1} = 0, \overline{P}_{\text{tot},2} = 0)$, i.e., the marginal utility is positive at this point. Since $\partial U_{\text{tot}}/\partial\overline{P}_{\text{tot},i}$ is a continuous function of $\overline{P}_{\text{tot},i}$, $i = 1, 2$, it follows that there exists a $P_{L,U}$ such that $\partial U_{\text{tot}}/\partial\overline{P}_{\text{tot},i} > 0$ for $\overline{P}_{\text{tot},i} < P_{L,U}$, $i = 1, 2$. Hence, to maximize utility in this region, we must choose $\overline{P}_{\text{tot},1} = \overline{P}_{\text{tot},2} = \mathcal{P}$ for any $\mathcal{P} \leq P_{L,U}$.

At $(\overline{P}_{\text{tot},1} = 0, \overline{P}_{\text{tot},2} = 0)$, we have $\xi_i^*(r) = \xi_j^*(r) = 0$ for any distance r. Therefore, from (31), we have $\partial U_{\text{tot}}/\partial\overline{P}_{\text{tot},i} = \alpha_{p,i}$. For small enough $\mathcal{P}$, we must have $\alpha_{p,i} > 0$, so that $\partial U_{\text{tot}}/\partial\overline{P}_{\text{tot},i} > 0$.

To prove Theorem 7, it suffices to show that $\partial R_{\text{tot}}/\partial\overline{P}_{\text{tot},i} > 0$ at $(\overline{P}_{\text{tot},1} = 0, \overline{P}_{\text{tot},2} = 0)$. Taking the derivative of R_{tot} with respect to $\overline{P}_{\text{tot},i}$ gives

$$\frac{\partial R_{\text{tot}}}{\partial\overline{P}_{\text{tot},i}} = \overline{P}_{\text{tot},i}\frac{\partial\alpha_{p,i}}{\partial\overline{P}_{\text{tot},i}} + \alpha_{p,i} + \overline{P}_{\text{tot},j}\frac{\partial\alpha_{p,j}}{\partial\overline{P}_{\text{tot},i}}. \quad (35)$$

At $(\overline{P}_{\text{tot},1} = 0, \overline{P}_{\text{tot},2} = 0)$, this reduces to $\partial R_{\text{tot}}/\partial\overline{P}_{\text{tot},i} = \alpha_{p,i} > 0$.

C. Proof of Theorem 4

To compare U_{OC} with U_{NE} we need the following Lemma.

Lemma 2: At the NE, $\lim_{\mathcal{P}\to\infty}\alpha_{p,i}\mathcal{P} = C$ where $0 \leq C < \infty$.

Proof: For large $\mathcal{P}$, σ^2 becomes negligible in the definition of $A_i(r)$ in (1). Combining with the expression for $\overline{P}_{\text{tot},i}$ in (13), we can write

$$\left|1 - \frac{K}{M(1-d_0)}\int_{d_0}^{1}\frac{h(2-r)}{h(r)}V^{-1}\left[\frac{\alpha_{p,i}\mathcal{P}h(2-r)}{h(r)}\right]dr\right| \to 0$$

$$i = 1, 2 \quad (36)$$

as $\mathcal{P} \to \infty$. By assumption, $\lim_{x\to\infty}V^{-1}(x) = 0$, hence, $\alpha_{p,i}\mathcal{P}$ must converge to a constant as $\mathcal{P} \to \infty$. ■

As $\mathcal{P} \to \infty$, Lemma 2 implies that the SINR for a user at distance r converges to

$$\xi_\infty(r) = V^{-1}\left(\frac{Ch(2-r)}{h(r)}\right). \quad (37)$$

At the NE, $U_{\text{tot},1} = U_{\text{tot},2}$, and from (7) and (14), the total utility satisfies

$$\left| U_{\text{NE}} - \frac{2K}{M(1-d_0)} \int_{d_0}^{1} U\left[V^{-1}\left(\frac{Ch(2-r)}{h(r)}\right)\right] dr \right| \to 0 \quad (38)$$

as $\mathcal{P} \to \infty$.

For the one-cell solution, $\overline{P}_{\text{tot},1} = \mathcal{P}$, $\overline{P}_{\text{tot},2} = 0$, and from (1) and (13), we have

$$\overline{P}_{\text{tot},1} = \mathcal{P} = \frac{K}{M(1-d_0)} \int_{d_0}^{1} \frac{\sigma^2}{h(r)} V^{-1}\left(\frac{\alpha_{p,1}\sigma^2}{h(r)}\right) dr. \quad (39)$$

Since $U_{\text{tot},2} = 0$, the total utility is

$$U_{\text{OC}} = \frac{K}{M(1-d_0)} \int_{d_0}^{1} U\left[V^{-1}\left(\frac{\alpha_{p,1}\sigma^2}{h(r)}\right)\right] dr. \quad (40)$$

As $\mathcal{P} \to \infty$ (39), the price $\alpha_{p,1} \to 0$, and by assumption, the received SINR $V^{-1}(\alpha_{p,1}\sigma^2/h(r)) \to \infty$ for $d_0 < r < 1$, so that

$$\lim_{\mathcal{P}\to\infty} U_{\text{OC}} = \frac{K}{M} U(\infty). \quad (41)$$

To find a condition for which $U_{\text{NE}} < U_{\text{OC}}$, we can write

$$\frac{2K}{M(1-d_0)} \int_{d_0}^{1} U\left[\xi_\infty(r)\right] dr < \frac{2K}{M(1-d_0)} \int_{d_0}^{1} U\left[\xi_\infty(d_0)\right] dr = \frac{2K}{M} U\left[\xi_\infty(d_0)\right]. \quad (42)$$

Since $U(\cdot)$ is monotonically increasing, (38) and (42) imply that $U_{\text{NE}} < U_{\text{OC}}$ when $U(\infty) > 2U[\xi_\infty(d_0)]$.

D. Proof of Theorem 5

Considering cell 1, we have

$$\mathcal{P} = \frac{L_1}{1-d_0} \int_{d_0}^{1} A_1(r)\xi_1^*(r) dr$$

$$U_{\text{tot},1} = \frac{L_1}{1-d_0} \int_{d_0}^{1} U\left[\xi_1^*(r)\right] dr. \quad (43)$$

Taking derivatives with respect to L_1 gives

$$0 = \frac{1}{1-d_0} \int_{d_0}^{1} A_1(r)\xi_1^*(r) dr + \frac{L_1}{1-d_0} \int_{d_0}^{1} A_1(r) \frac{\partial \xi_1^*(r)}{\partial L_1} dr \quad (44)$$

$$\frac{\partial U_{\text{tot},1}}{\partial L_1} = \frac{1}{1-d_0} \int_{d_0}^{1} U\left[\xi_1^*(r)\right] dr + \frac{L_1}{1-d_0} \int_{d_0}^{1} U'\left[\xi_1^*(r)\right] \frac{\partial \xi_1^*(r)}{\partial L_1} dr \quad (45)$$

and combining these with (7) gives

$$\frac{\partial U_{\text{tot},1}}{\partial L_1} = \frac{1}{1-d_0} \int_{d_0}^{1} \left(U\left[\xi_1^*(r)\right] - U'\left[\xi_1^*(r)\right] \xi_1^*(r)\right) dr. \quad (46)$$

Since $U(\cdot)$ is increasing concave, we have $U[\xi_1^*(r)]/\xi_1^*(r) > U'[\xi_1^*(r)]$, so that $\partial U_{\text{tot},1}/\partial L_1 > 0$. That is, the utility in each cell increases with load.

E. Proof of Lemma 1

For the power-law and log utility functions, from (20), (13), and (28), the elasticities are, respectively

$$e_{\text{pl}}(\overline{P}_{\text{tot},i}, \alpha_{p,i}) = \frac{1}{\beta - 1} \quad (47)$$

$$e_{\log}(\overline{P}_{\text{tot},i}, \alpha_{p,i}) = -\frac{1}{1 - \frac{\alpha_{p,i}}{u_0(1-d_0)} \int_{d_0}^{1} A_i(r) dr} < -1. \quad (48)$$

By assumption, $\beta < 1$, hence, $e_{\text{pl}}(\overline{P}_{\text{tot},i}, \alpha_{p,i}) < -1$, or equivalently, $\partial R_{\text{tot},1}/\partial \overline{P}_{\text{tot},1} > 0$ so that the revenue is maximized by setting $P_{\text{tot},1} = \mathcal{P}$. For the log-utility function, from (13)

$$\overline{P}_{\text{tot},i} = \frac{K}{M(1-d_0)} \int_{d_0}^{1} A_i(r) \left(\frac{\alpha_{p,i} A_i(r)}{u_0 \beta}\right)^{\frac{1}{\beta-1}} dr > 0 \quad (49)$$

hence, $e_{\log}(\overline{P}_{\text{tot},i}, \alpha_{p,i}) < -1$.

For the exponential utility function, we can compute

$$\frac{\partial R_{\text{tot},1}}{\partial \overline{P}_{\text{tot},1}} = \alpha_{p,1}\left(1 - \frac{M(1-d_0)\overline{P}_{\text{tot},1}}{K \int_{d_0}^{r_i^*} \lambda A_1 dr}\right) \quad (50)$$

where $r_i^* = \min(1, r_i')$ and $A_i(r_i')\alpha_{p,i} = 1$. If the power constraint is not binding, then we must have $\partial R_{\text{tot},1}/\partial \overline{P}_{\text{tot},1} = 0$, which gives

$$\overline{P}^* = \frac{K}{M(1-d_0)} \int_{d_0}^{r_1^*} \lambda A_1(r) dr. \quad (51)$$

If $\overline{P}^* > \mathcal{P}$, then $\partial R_{\text{tot},1}/\partial \overline{P}_{\text{tot},1} > 0$ for $0 < \overline{P}_{\text{tot},1} < \mathcal{P}$, so that to maximize revenue, $\overline{P}_{\text{tot},1} = \mathcal{P}$.

F. Proof of Theorem 9

Suppose the two-cell maximum revenue is achieved at the interior point $\overline{P}_{\text{tot},1} = P_1^* < \mathcal{P}$ and $\overline{P}_{\text{tot},2} = P_2^* < \mathcal{P}$. Since the total revenue R_{tot} is differentiable, we must have $\partial R_{\text{tot}}/\partial \overline{P}_{\text{tot},i} = 0$ at $(\overline{P}_{\text{tot},1} = P_1^*, \overline{P}_{\text{tot},2} = P_2^*)$, where $\partial R_{\text{tot}}/\partial \overline{P}_{\text{tot},i}$ is given by (35). Combining (7), (13), and (28) with (35) gives

$$\begin{aligned} \frac{\partial R_{\text{tot}}}{\partial \overline{P}_{\text{tot},i}} &= \frac{1}{u_0(1-d_0)} \\ &\quad \times \left[\alpha_{p,i}^2 \int_{d_0}^{1} A_i(r) dr - \alpha_{p,j}^2 \overline{P}_{\text{tot},j} \int_{d_0}^{1} \frac{h(2-r)}{h(r)} dr \right] \\ &> \frac{\alpha_{p,i}^2 - \alpha_{p,j}^2}{u_0(1-d_0)} \overline{P}_{\text{tot},j} \int_{d_0}^{1} \frac{h(2-r)}{h(r)} dr. \end{aligned} \quad (52)$$

Setting $\partial R_{\text{tot}}/\partial \overline{P}_{\text{tot},i} = 0$ implies $\alpha_{p,i} < \alpha_{p,j}$, which cannot be true for each $i = 1, 2$, hence, at least one cell must be power limited.

To prove the second statement in the theorem, in analogy with the proof of Theorem 4, we again compare the revenue at the NE with the one-cell solution, corresponding to $\overline{P}_{\text{tot},1} = \mathcal{P}$ and $\overline{P}_{\text{tot},2} = 0$. According to Lemma 2, as $\mathcal{P} \to \infty$, $R_{\text{tot},i} = \alpha_{p,i}\mathcal{P}$ converges to a constant, which satisfies (36). At the NE, $R_{\text{tot},1} = R_{\text{tot},2}$, and the total revenue is $R_{\text{NE}} = ((2Ku_0/M)/(1 + (K/M(1-d_0)) \int_{d_0}^{1} (h(2-r)/h(r)) dr))$. For the one-cell solution, $R_{\text{tot},2} = 0$, and $R_{\text{tot},1}$ is given by (22). As $\mathcal{P} \to \infty$, $\alpha_{p,1} \to 0$, and the total revenue is $R_{\text{OC}} = Ku_0/M$. If $(K/(M(1-d_0))) \int_{d_0}^{1} (h(2-r)/h(r)) dr > 1$, then $R_{\text{NE}} < R_{\text{OC}}$.

G. Proof of Theorem 10

The derivatives $\partial \alpha_{p,i}/\partial \overline{P}_{\text{tot},i}$ and $\partial \alpha_{p,j}/\partial \overline{P}_{\text{tot},i}$ can be computed from (28), and combining it with (35) gives

$$\begin{aligned} \frac{\partial R_{\text{tot}}}{\partial \overline{P}_{\text{tot},i}} &= -\frac{M(1-d_0)\alpha_{p,i}\overline{P}_{\text{tot},i}}{K \int_{d_0}^{r_i^*} \lambda A_i(r) dr} + \alpha_{p,i} \\ &\quad - \frac{\alpha_{p,j}\overline{P}_{\text{tot},j} \int_{d_0}^{r_i^*} \frac{\lambda h(2-r)}{h(r)} \left[\ln\left(\frac{\lambda A_j(r) \alpha_{p,j}}{u_0} \right) + 1 \right] dr}{\int_{d_0}^{r_j^*} \lambda A_j(r) dr}. \end{aligned}$$

At the NE, we have $\alpha_{p,i} = \alpha_{p,j}$ and $\overline{P}_{\text{tot},i} = \overline{P}_{\text{tot},j} = \mathcal{P}$. Letting $\mathcal{P} \to \infty$, $A_i(r) \to \mathcal{P}h(2-r)/h(r)$ and

$$\begin{aligned} \frac{\partial R_{\text{tot}}}{\partial \overline{P}_{\text{tot},i}} &\to -\frac{M(1-d_0)\alpha_{p,i}}{K \int_{d_0}^{r_i^*} \lambda \frac{h(2-r)}{h(r)} dr} + \alpha_{p,i} \\ &\quad - \frac{\alpha_{p,i} \int_{d_0}^{r_i^*} \frac{h(2-r)}{h(r)} \left[\ln\left(\frac{\lambda A_i(r) \alpha_{p,i}}{u_0} \right) + 1 \right] dr}{\int_{d_0}^{r_i^*} \frac{h(2-r)}{h(r)} dr}. \end{aligned} \quad (53)$$

When λ is small enough, the first term on the right dominates, and $\partial R_{\text{tot}}/\partial \overline{P}_{\text{tot},i} < 0$. We therefore conclude that for large enough $\mathcal{P}$, the revenue decreases with $\mathcal{P}$, and the maximum revenue does not occur when the two cells are power limited.

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