Take-Home Examination

Due: Thursday, Feb. 27, 2004 at 930 a.m., the beginning of class. Late submissions will not be graded.

Instructions: This exam contains 5 questions, worth totally 100 points. Since this is an exam, *you should not discuss these problems with anyone else.* You can use your textbook and your class notes and handouts, but *you are not to consult any other material when working on this exam.*

Unless otherwise specified in the question, you can refer to algorithms or results from your class notes or the text-book without reproducing the full algorithm or proof, but please give explicit pointers to such results, such as the page number in the textbook or handout, or the date of the lecture which covered the material.

Good presentation counts! For full credit your solutions should be clear, concise, and legible in addition to being correct.

Declaration: I declare that I have not taken or given any help during this exam, and I have followed all instructions given above.

Name:__________________________ **Signature (required):**__________________________

Please staple this signed page with your solutions.

1. (20 points) A ski rental agency has m pairs of skis, where the height of the ith pair of skis is s_i. There are n skiers who wish to rent skis, where the height of the ith skier is h_i. Ideally, each skier should obtain a pair of skis whose height matches her/his own height as closely as possible. We would like to assign skis to skiers so that the sum of the absolute differences of the heights of each skier and her/his skis is minimized.

   (a) Assuming that $m = n$, design a greedy algorithm for the problem. Prove the correctness of your algorithm.

   (b) Assuming that $m > n$, design a dynamic programming algorithm for the problem.

2. (20 points) A statistician is given the annual salaries of the n residents of a city and asked to return a list that gives for each resident R the largest number i such that R's salary is in the top $1/2^i$ fraction of all of the salaries in the city. The initial list of salaries is given in an arbitrary (unsorted) order.

   Design and analyze an efficient algorithm for the statistician. Make your algorithm as efficient as you can, in terms of its asymptotic worst-case running time.

3. (20 points) Given a set of points $p[0..n]$ on the plane, design an efficient algorithm to find the pair $p[i], p[j]$ that are farthest from each other, that is, the Euclidean distance between them is the largest among all pairs. Argue that your algorithm achieves the best performance.

4. (20 points) Suppose you are given a graph $G = (V, E)$, with a cost $c[e]$ on each edge e. We view the costs as quantities that have been measured experimentally, subject to possible errors in measurement. Thus, the minimum spanning tree one computes for G may not in fact be the "real" minimum spanning tree. Given error parameters $\epsilon > 0$ and $k > 0$, and a specific edge e that is not in the minimum spanning tree, you would like to be able to make a claim of the following form:

   > (P) Even if the cost of each edge were to be changed by at most ϵ (either increased or decreased), and the costs of k of the edges other than e were further changed to arbitrarily different values, the edge e would still not belong to any minimum spanning tree of G.

   Such a property provides a type of guarantee that e is not likely to be belong to the minimum spanning tree, even assuming significant measurement error (unless the cost $c[e]$ of the edge e contains significant error).

   (a) Give a polynomial time algorithm that takes G, e, and ϵ, and decides whether or not property P holds for e with $k = 0$.

   (b) Give a polynomial time algorithm that takes G, e, ϵ, and k, and decides whether or not property P holds for e.

5. (20 points) Suppose after solving a shortest path problem, you realize that you underestimated each edge length by k units. Suggest an $O(E)$ algorithm for solving the original problem with the correct edge lengths. The running time of your algorithm should be independent of the value of k.